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Optimal Control of Power Kites for Wind Power Production

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Abstract

Ground based wind energy systems have reached the peak of their capacity. Wind instability, high cost of installations and small power output of a single unit are some of the the limitations of the current design. In order to become competitive the wind energy industry needs new methods to extract energy from the wind.

The Earth's surface creates a boundary layer effect on the wind that increases its speed with altitude. In fact, with altitude the wind is not only stronger, but steadier. In order to capitalize these strong streams new extraction methods were proposed. One of these solutions is to drive a generator using a tethered kite. This concept allows very large power outputs per unit.

The major goal of this work is to study a possible trajectory of the kite in order to maximize the power output using an optimal control software - Imperial College London Optimal Control Software (ICLOCS), model and optimize it.

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Acronyms

HWE	High Wind Energy
WT	Wind Turbine
WTG	Wind Turbine Generator
OC	Optimal Control
OCP	Optimal Control Problem
ICLOCS	Imperial College London Optimal Control Software
MATLAB	Software for numerical computation, visualization and programming
CVODES	Ordinary Differential Equations Solver with Sensitivity Analysis Capabilities
IPOPT	Library for large scale nonlinear optimization of continuous systems (Interior Point Optimizer)
ODE	Ordinary Differential Equations

Chapter 1

Introduction

1.1 Context

In wind resides a potential solution to the current power demand. Despite having been used for centuries to grind grain, pump water, propel sailing ships across oceans, among others, wind has not yet reached its full potential [9]. The current technology based on wind mills is far from being a competitive alternative against the fossil sources. Indeed, the current renewable sources will not contribute with more than 20% within the next 15 to 20 years. Even though wind mills are the largest supplier of renewable energy (excluding hydro power plants), when compared, the generated power density per km^2 of a wind mill, it is 200 to 300 times lower than that of a thermal plant [10]. This is due to a number of contributing factors. For instance, a wind turbine generator (WTG) requires heavy foundations and huge blades which represents a massive investment and leads to a significant ground occupation. Another factor is the variability of wind speed at low altitudes.

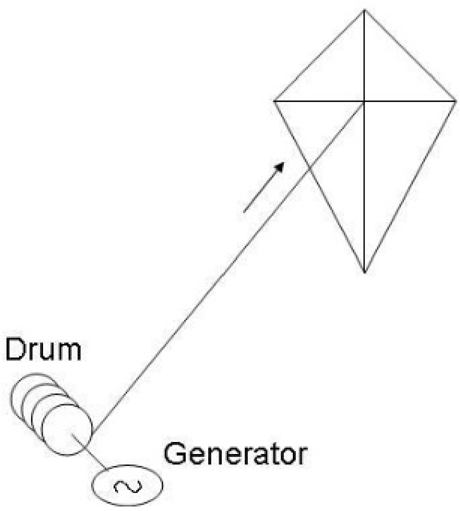


Figure 1.1: Simple illustration of a power kite [1].

In order to overcome such limitations the development of technology that could take advantage of the fact that wind speed generally increases with altitude, where stronger and steadier streams are available, began [11, 1]. The basic idea is to collect the potential energy of these streams with a tethered airfoil, like a power kite (Figure 1.1). The energy transferred by the kite lines would be transformed at ground level by converting the mechanical power into electric power.

1.2 Motivation

In the current scenario, kites have become one of the most promising applications to generate electric power. The kite is connected to the ground to a drum capable of paying the cable in and out. Power is generated during the reel-out phase, which is when the line is at higher tension, and spent during the reel-in when the line is at lower tension. The tension can be controlled by pulling the kite lines. By doing this, it will be possible to control the forces on the kite and subsequently, the kite angle of attack. As the kite angle is increased, the tension also increases and the kite is lifted, generating power. The opposite happens when the angle is decreased. The biggest the gap in the tension during both phases, the biggest the power output of the system [6]. It is this ability to be manoeuvred that is most appealing [12].

The aim of this MSc is to be able to control one of this kites in order to enhance its power production.

It is expected that a wind power generator of this type will have a power production capacity per unit 5000 to 10000 times larger than a conventional WTG. The installation and rent costs per unit will be lower and no propeller noise will be emitted [9].

1.3 Objectives

This master thesis was proposed to study the optimal means of flying a kite to maximize its power output.

To reach this goal the following objectives are considered:

- **R**eview technologies and principles of high wind power extraction.
- **S**tudy of the kite cinematic and dynamic model.
- **S**imulation of trajectories.
- **R**esolution of the OCP in order to maximize the power output.
- **S**ensitivity analysis of the proposed solution.

1.4 Document Structure

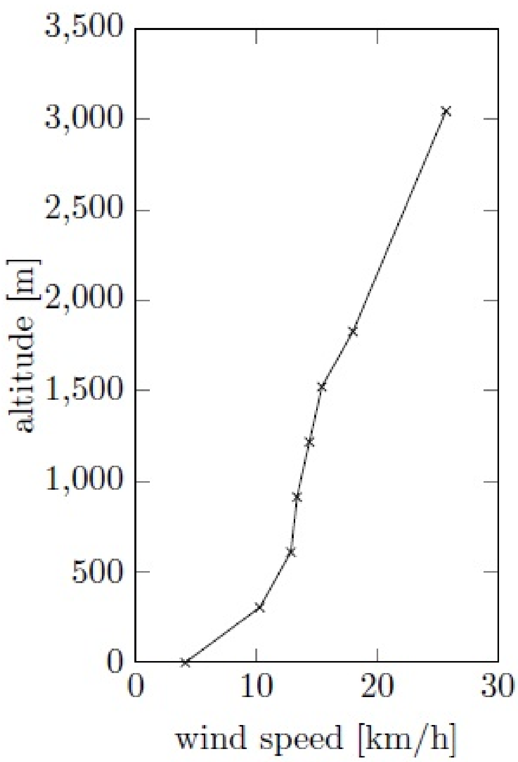
This document is organized as follows. In Chapter 2 the state of art and all the related work in this field is presented, as well as the kite model. Chapter 3 introduces the methodology and the steps that will be followed to accomplish all goals. The work plan for this dissertation is delineated in Chapter 4. In Chapter 5 the conclusions of this work are drawn.

Chapter 2

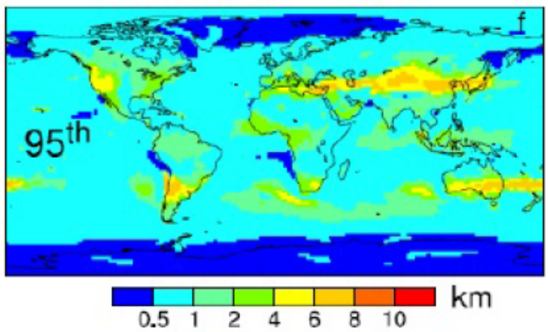
State of the Art

In this Chapter, the wind conditions at high altitude are studied. Afterwards, some solutions and methods to extract power from the high wind are discussed. In the following section a simplified system model of the kite is presented and finally the currently used power kite actuators are introduced.

2.1 High Wind Energy



(a) Average wind velocity profile. Brussels 30-05-2012.



(b) Altitude of wind power density(95th percentile of annual average).

Figure 2.1: Airborne wind energy systems can profit from higher wind speeds at altitude [2].

The power extracted from a WTG strongly depends on the wind speed. Unfortunately, the wind is relatively slow close to the ground and, sometimes, is non-existent what makes the power production less efficient.

The Earth's surface creates a boundary layer effect so that wind speed increases generally with altitude. Although the true wind patterns depend on a variety of factors like the Earth's rotation, the solar flux, among others, the wind in the troposphere and stratosphere is mostly present even when there is no wind at ground level. With steadier and more predictable winds, high-altitude wind has an advantage over wind near the ground.

As it can be seen in Figure 2.1 there are two major jet streams: the Polar Front Jet and the Sub-Tropical Jet, both located between 30 and 40 degrees in each hemisphere. The wind speed in the jet streams is permanent and can reach 40 m/s and in some parts even more. This is a very significant wind source, since the near the surface wind is on the order of the 5 m/s. Moreover, the power generated by the WTG's not only increase with the wind speed, but rather by the cube of it [6, 9].

2.2 High Wind Energy Systems

The idea of using high wind energy (HWE) goes as back as the 1930s. In the late 1960s, a company named Sheldahl, Inc. placed a generator on a tethered balloon and generated about 350 Watts of power [13]. In the mid 70s several proposals placed to the National Science Foundation and the Energy Resources Development Administration in the United States, were denied because of the possible hazards to aircraft and were classified as non profitable. This has not stopped further research, and advances in technology caused this idea to persist in a way that proposals for such systems are still being made [1]. In this section, such proposals will be presented.

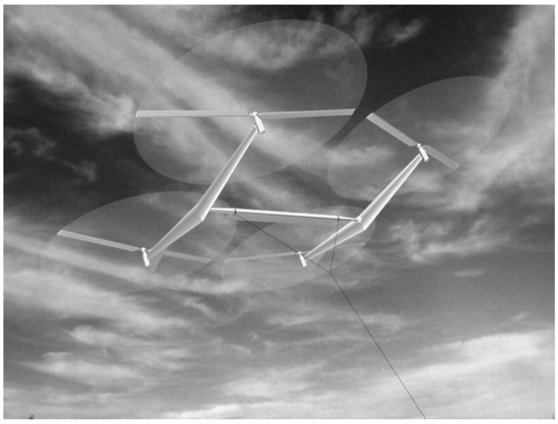
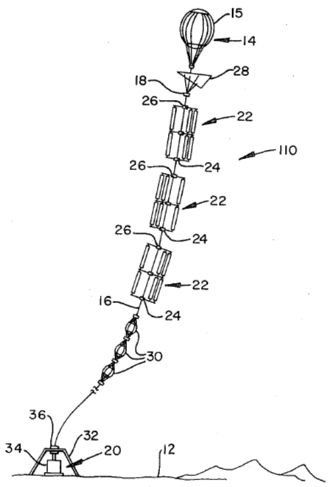
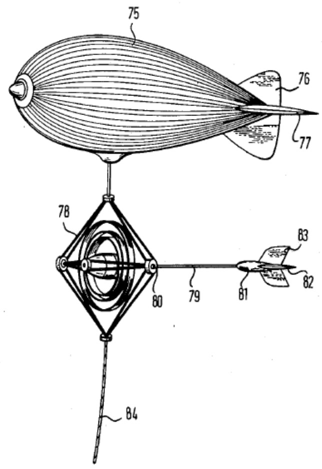


Figure 2.2: Quadruple rotor arrangement [3].

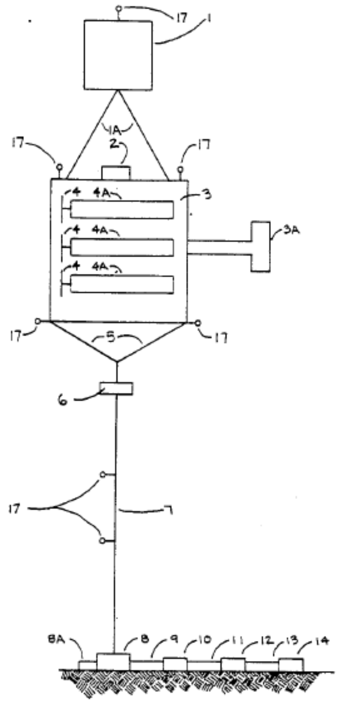
In 1983, Riedler and Riegler proposed using a WTG mounted on a tethered balloon. Six symmetrically arranged turbines were attached to the balloon just behind its center of gravity [14]. A system consisting of a platform with rotary wings to provide power from the jet stream was



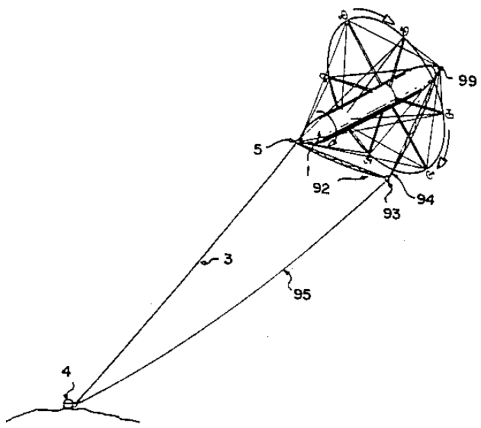
(a) Wind driven high altitude power apparatus [16].



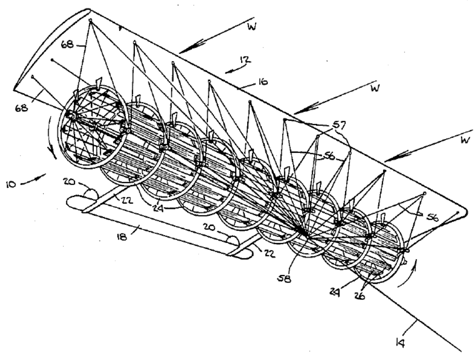
(b) Wind driven power plant [17].



(c) Wind generator kite system [18].



(d) Airship-floated wind turbine [19].



(e) Drum kitecraft [20].

Figure 2.3: Examples of high wind systems with lifted generators.

also studied. The power would be transmitted to the ground by means of a tethered cable with aluminium conductors [15]. Recently, this concept was studied and improved. The main design uses an airframe with two or more rotors inclined at a controlled angle to the wind stream (Figure 2.2). The rotors not only are used to produce electricity but also to keep the frame up in the air. The power is transmitted to the ground via an aluminium-Spectra composite (Spectra is a high strength fibre). The frame would have to land from time to time depending on the wind speed [3].

In 1978, Fri and Hise suggested a tethered balloon with wind driven rotors fixed along the length of a flexible power shaft. The lower end of the shaft is connected to a ground supported energy conversion device (Figure 2.3a) [16].

Another system consisted in a tethered aerostat carrying at least one rotor and a current generator coupled to it (Figure 2.3b) [17].

In 1984, Pugh proposed a wind power generator suspended in the air like a kite and connected to the ground by high strength cables. The small wind generators would be mounted in the kite and raised to the desired altitude by a gas balloon (Figure 2.3c) [18].

One year later, a system using an airship floated wind turbine was proposed. It was connected to the ground by two tethers, through which the power was conducted (Figure 2.3d) [19].

Later, 1987, an invention named kitecraft was secured to a ground tether supporting a cylindrical drum. The drum consists in many wheels interconnected by airfoils. Wind action on the airfoils rotates the drum turning the generator to provide electric power (Figure 2.3e) [20].

All these systems had the drawback of having the power generator up in the air. This increased significantly the weight of the system and consequently the difficulty of launching, lifting and keep it flying. An alternative was first introduced in 2001, where generator was placed on the ground and the cable was moved through the generator by a high lifting body, like a kite or a balloon(Figure 2.4). The wings on the upside operate at high angle of attack in order to generate high lift to pull the cable upwards. On the downside, the wings have an angle of attack as small as possible to reduce the drag. The difference in tension causes the cable to be pulled through the generator, generating power. The wings would be made of a light material so in case of a fall, no harm will come to the people or buildings around [4].

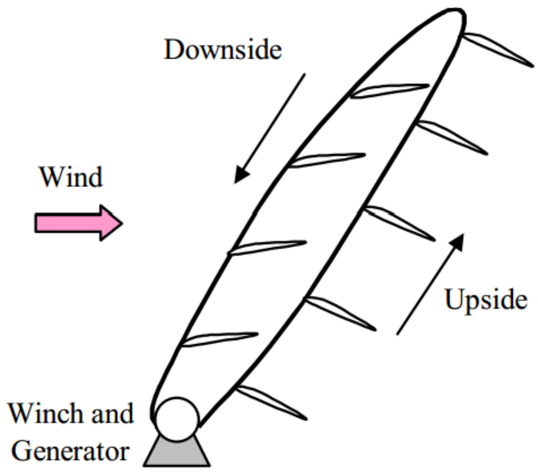


Figure 2.4: The original laddermill concept [4].

An alternative concept of the laddermill consists in a single cable being pulled through a generator. Instead of being pulled continuously around the generator, it would be pulled up and down it. This is, indeed, a much lighter-weight option and a lot simpler [21].

It is clear that the systems differ in one main aspect: the location of the generator. Newer proposals place it on the ground instead of up in the atmosphere. In the next section the laddermill model will be studied in more detail.

2.3 The Laddermill

The laddermill concept uses lifting bodies, like kites, flying into the higher regions of the atmosphere. In order to have a better perspective of the problem a simpler model of the laddermill will be considered. This model consists of only one lifting body as in Figure 1.1. The actual laddermill will have several kites connected to the cable. The lower part of the cable, about 10 %, is wound around a drum. The tension created by the kite pulls the generator up to produce power. To maximize the output, the kite trajectory must be controlled. This can be achieved by lowering the attack angle during the descent and the opposite during the ascending. One possible ideal trajectory resembles a the figure of number eight as can be seen in Figure 2.5.

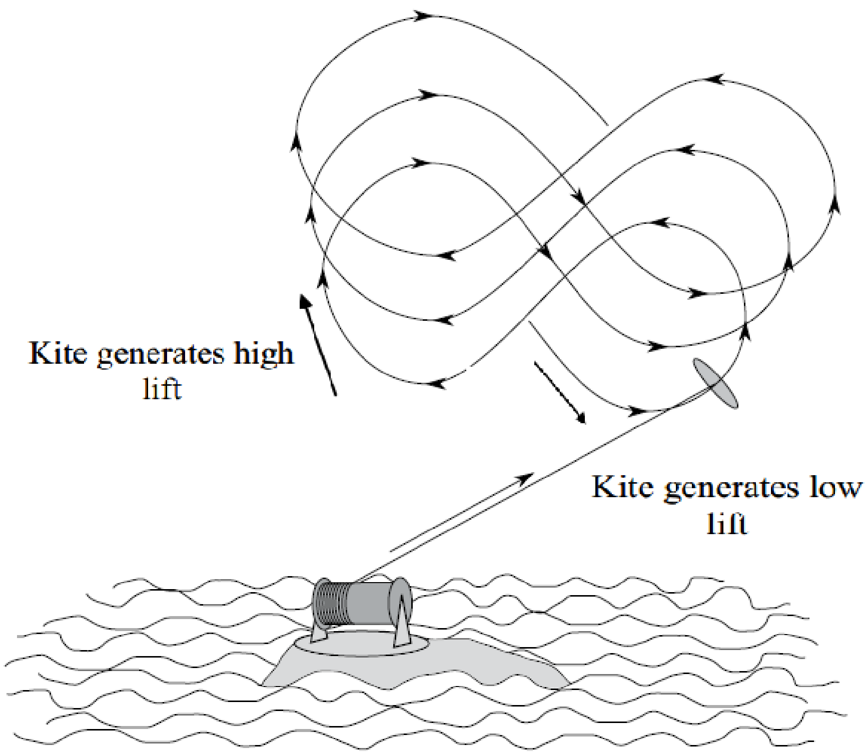


Figure 2.5: A kite generator and a possible ideal trajectory [5].

The higher the difference between the tension during the descend and the ascend, the higher the power output and consequently, the efficiency. In order to study a possible optimal trajectory, the kite model must be studied. A possible model is presented in the following section.

2.3.1 Dynamic Model the Tethered Kite System

In order to have a better knowledge of the behaviour of the tethered kite system, it is necessary to study a simplified model. In this model the tether will be considered inelastic and straight. For trajectory purposes the kite dynamics are not considered. The forces on the kite are only considered as drag and lift, whose magnitudes are functions of the kite angle of attack. The coordinates of the tether are described by spherical coordinates (r, θ, ϕ), where r represents the tether length, θ describes the angle from the winch to the kite in the XZ plane (Figure 2.6) and ϕ represents the angle from the winch to the kite in the crosswind direction (XY plane).

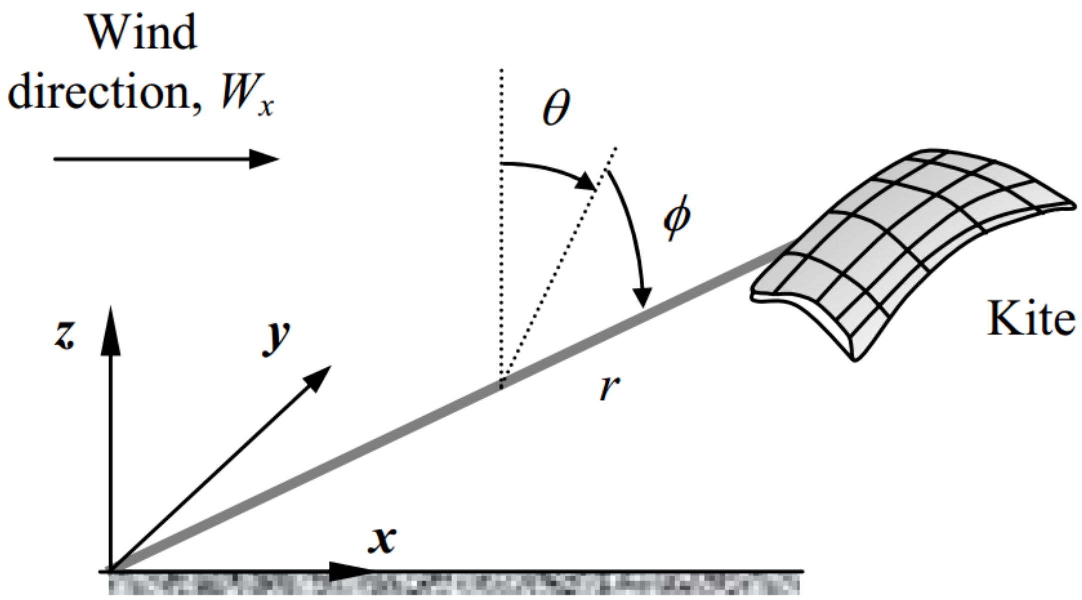


Figure 2.6: Simplified tethered kite model [1].

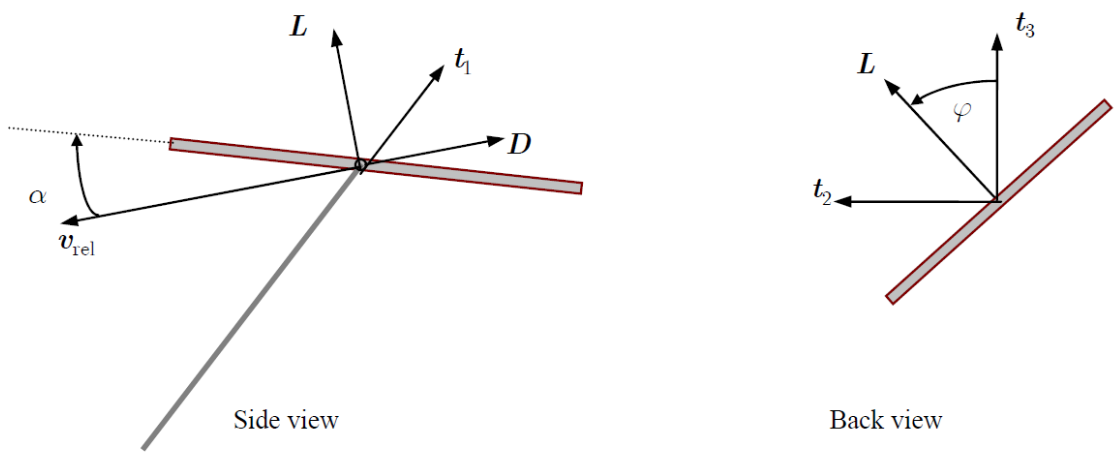


Figure 2.7: Assumed lift and drag forces on the kite [6].

The equations of motion for the simplified system are the following:

$$Q_\theta = \left(m_{kite} + \frac{\rho_c r}{3}\right) r^2[\ddot{\theta}\cos^2\phi - 2\dot{\theta}\dot{\phi}\sin\phi\cos\phi] + 2\left(m_{kite} + \frac{\rho_c r}{2}\right) r\dot{r}\dot{\theta}\cos^2\phi - \left(m_{kite} + \frac{\rho_c r}{2}\right) rg\cos\phi\sin\theta \tag{2.1}$$

$$Q_\phi = \left(m_{kite} + \frac{\rho_c r}{3}\right) r^2\ddot{\phi} + 2\left(m_{kite} + \frac{\rho_c r}{2}\right) r\dot{r}\dot{\phi} + \left(m_{kite} + \frac{\rho_c r}{3}\right) r^2\dot{\theta}^2\sin\phi\cos\phi - \left(m_{kite} + \frac{\rho_c r}{2}\right) rg\sin\phi\cos\theta \tag{2.2}$$

$$Q_r = (m_{kite} + \rho_c r)\ddot{r} + \frac{1}{2}\rho_c\dot{r}^2 - \left(m_{kite} + \frac{\rho_c r}{2}\right)(r\dot{\phi}^2 + r\dot{\theta}\cos^2\phi) + (m_{kite} + \rho_c r)g\cos\phi\cos\theta \tag{2.3}$$

m_{kite} is the mass of the kite, ρ_c is the cable density, g is the gravitational acceleration, and Q represents the generalised forces acting on the system (distributed tether drag, kite lift and drag). The kite forces are assumed to be dependent on the kite angle of attack, a, and the velocity roll angle μ (Figure 2.7) [1].

2.4 Kite Control Actuators

Remote control of power kites is achieved through the use of some mechanisms. Despite several kites optimized for laddermill use, the kites currently used in research are mostly derive from commercial available surfing kites. A surfing kite is usually controlled using a handle bar. Steering lines are attached to both sides of the bar, leading up to the tips of the kite. This enables the surf to control both the angle of attack and to steer the kite in a certain direction. The same principles are applied to the remote controlled kite.

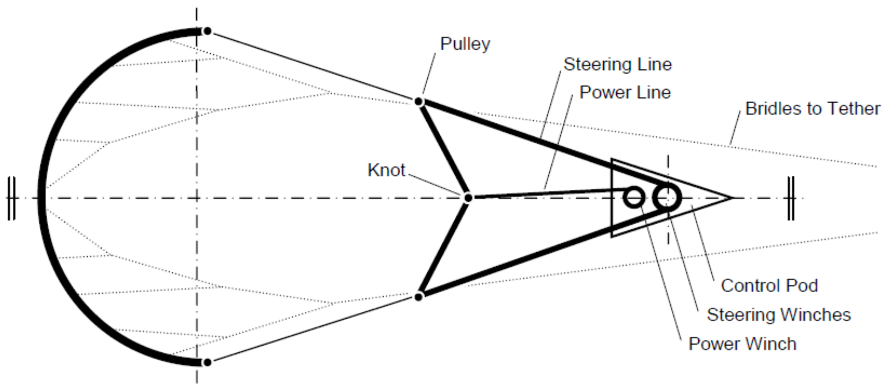


Figure 2.8: Kite with airborne steering mechanism [7].

The control pod of a typical system in represented in Figure 2.8. It contains two motors, one for steering the kite and one for controlling the power setting. In this system, there are two winches on the steering motor, each wind up one end of the steering line, so that when one end is released, the other is pulled. As an asymmetric wingtip load is applied, the kite begins to turn. Only one winch is connected to the power motor. Winding up the power line applies a symmetrical load to the wingtips, which leads to an increase in the angle of attack [7].

Figure 2.9: Wingtip AoA control [1].

Another possible way of controlling the angle of attack is by installing a winch servo (Figure 2.9). This cause additional drag on that side together with a side force, causing the kite to yaw.

Figure 2.10: Drag flap control mechanism [8].

The control system in Figure 2.9 uses a small servo to activate drag flaps on the side of the kite. This control mechanism only works on a stable kite because the force generated is quite small. The drag force creates a deviation from the equilibrium, causing the kite to yaw.

The latest control mechanism steers the kite by changing the attachment point of the steering line on the kite. The slide mechanism has been demonstrated to work very well on many different types of surf kites (Figure 2.11) [1].

Figure 2.11: Slide mechanism on the kite [1].

An important question is the location of the control pod. Implementing the actuators as part of the ground station eliminates additional drag and weight, which have positive effects on the aerodynamic properties of the kite (Figure 2.12a).

On the other hand, an airborne control pod (Figure 2.12b) has minimal mechanical delay between activation of the winches and the dynamic response of the wing. Another advantage is the ability to be controlled if the main tether ruptures. It is also easier to reel in and out a single main tether than both steering lines, while still enabling full control over the kite [7, 22].

(a) Ground control pod [23].

(b) Airborne control pod [22].

Figure 2.12: Different types of control pods.

2.5 Discussion

In this chapter, the research work that so far has been done in the field of power extraction from high wind was presented . Many systems were presented that could be used to make this power source profitable. As it could be seen, the main difference between different systems resides in the place where is the generator placed, on the ground or up in the atmosphere. Placing the generator in the ground reduces the weight of the system making it easier to lift and safer is case of a crash.

Even though the laddermill is still an emerging topic, already some research has been done in this field. This system has the advantage of having a ground generator and the materials used to build the kites are light, which makes it safer without compromising its efficiency.

The simplified kite model presented will be used to study an optimal trajectory of the kite in order to maximize the power output.

Chapter 3

Methodology

This chapter presents the proposed methodology and a description of the steps to be followed during the development of this research.

After reviewing the state of the art and studying the kite model for high wind power extraction, simulations will be carried out to determine an optimal trajectory to optimize the power output of the kite system. At last, a sensibility analysis will be done to check the system performance and efficiency.

3.1 Optimal Control Problem

Optimal control (OC) is one of several applications and extensions of the calculus of variations. It deals with finding control in functions or control feedback gains that minimize a performance index with different equations constraints [24].

In this case, there is a need to deal with nonlinear equations. OC of nonlinear systems is one of the most active subjects in control theory. One of the main difficulties with classic OC theory is that to determine optimal control for a nonlinear system. There is rarely an analytical solution although several numerical computation approaches have been proposed [25].

In this MSc, OC will be used to calculate optimal trajectories for nonlinear dynamic systems, particularly, for an aircraft.

An algorithm will be used to maximize the average power gained by the system described in 2.1, 2.2, 2.3, minimizing a cost function by subjecting the system to some constraints. The cost function and the constraints are yet to be determined.

3.2 Solving the Problem using ICLOCS

The OCP will be solved using the Imperial College London Optimal Control Software (ICLOCS). It was developed by Paola Falugi, Eric Kerrigan and Eugene van Wyk and is implemented in MATLAB. This software allows users to define and solve optimal control problems with general path and boundary constraints and free or fixed final time. It is also possible to include constant design parameters as unknowns.The following optimal control problems fall within the scope of the code:

$$
\begin{aligned}
& min_{u(t),t_f,p,x_0} J(x(\cdot),u(\cdot),p,t_F) \\
& \text{subject to} \\
& \dot{x} = f(x(t),u(t),p,t) && \forall t \in [t_0,t_F] \\
& x(t_0) = x_0 && \forall t \in [t_0,t_F] \\
& g_L \leq g(x(t),u(t),p,t) \leq g_U && \forall t \in [t_0,t_F] \\
& \phi_L \leq \phi(x_0,x_f,u_0,u_f,p,t_f) \leq \phi_U && \forall t \in [t_0,t_F] \\
& x_L \leq x(t) \leq x_U && \forall t \in [t_0,t_F] \\
& u_L \leq u(t) \leq u_U && \forall t \in [t_0,t_F] \\
& p_L \leq p \leq p_U
\end{aligned}
\tag{3.1}
$$

where $u_0 \triangleq u(t_0), x_f \triangleq x(t_f)$ and $u_f \triangleq u(t_f)$. Here the cost function is defined as

$$
J(x(\cdot),u(\cdot),p,t_f) \triangleq \int_{t_0}^{t_f} L(x(t),u(t),p,t)dt + E(x_0,x_f,u_0,u_f,p,t_f) \tag{3.2}
$$

where $E(\cdot)$ is the cost associated with the boundary state and $L(\cdot)$ the stage cost function. The arguments over which the cost function can be minimised are the time-varying control input signals $u(\cdot)$, the initial state x_0, the final time t_f and a set of parameters p that are constant during the phase. The function $g(\cdot)$ describes general path constraints and $\phi(\cdot)$ imposes boundary conditions at the beginning and end of the phase.

As a first step, the user defined OCP is transcribed to a static optimisation problem by either direct multiple shooting or direct collocation methods. The direct multiple shooting formulation requires the solution of initial value problems that can be determined using the open source sensitivity solver package CVODES. The direct collocation formulations discretize the system dynamics using implicit Runge-Kutta formulae and can also be used to incorporate discrete time problems. Once the OCP has been transcribed it can be solved with a selection of nonlinear constrained optimisation algorithms given by the open source code IPOPT or MATLAB's own solver. The derivatives of the ODE right-hand side, cost and constraint functions are also required for the optimisation and are either estimated numerically or supplied analytically [27, 26].

3.3 Analysis of results

Finally, results will be processed and analysed. A sensibility analysis will me made in order to test the robustness of the solution proposed. The results will also be compared with other wind power extraction systems to compare the performance of the different systems.

Chapter 4

Workplan

In this chapter a plan of the tasks to be developed in the next semester is presented in order to achieve all the objectives previously introduced in Chapter 1. The following work plan and a estimation of the number of weeks needed to complete each task is proposed:

- **17/02 - 10/03** — Becoming familiar with the kite cinematic and dynamic model. Getting acquainted with the optimal control software that is going to be used (ICLOCS and IPOPT).
- **10/03 - 10/04** — Trajectory simulations. Detailed characterisation of the optimal control problem.
- **10/04 - 02/06** — Resolution of optimal control problem in order to maximize the power output. Sensitivity analysis of the proposed solution. If time permits, study of the control system dynamics (control signals).
- **02/06 - 23/06** — Writing of the dissertation.

Chapter 5

Conclusion

This master's dissertation arises in the context of power extraction from high altitude wind. The goal is to study a possible optimal trajectory to improve the efficiency of a laddermill. This report includes the preliminary work of preparation for the dissertation.

As it could be seen in Figure 2.1 Portugal is in the latitude zone where the strongest jet streams are present. This means that it could be a good investment for the energy sector to invest in this field of research, which looks very promising.

After reviewing the state of the art of high wind energy systems we concluded that this is actually an emerging research topic with plenty of open problems.

With the work developed during this course of Preparation for the MSc Dissertation it was possible to get in touch with the topic and the related research problems and challenges. It was also possible to design the best approach for the work that will be performed during the next semester.

References

[1] Bas Lansdorp and Paul Williams. The laddermill-innovative wind energy from high altitudes in holland and australia. In *Wind Power*, 2006.

[2] Mathias Clinckemaillie. An experimental set-up for energy generation using balanced kites. Master's thesis, KU Leuven, 2012.

[3] BW Roberts and DH Shepard. Unmanned rotorcraft to generate electricity using upper atmospheric winds. In *Australian International Aerospace Congress*, 2003.

[4] Wubbo J Ockels. Laddermill, a novel concept to exploit the energy in the airspace. *Aircraft design*, 4(2):81–97, 2001.

[5] I Argatov, P Rautakorpi, and R Silvennoinen. Estimation of the mechanical energy output of the kite wind generator. *Renewable Energy*, 34(6):1525–1532, 2009.

[6] Paul Williams, Bas Lansdorp, and Wubbo Ockesl. Optimal crosswind towing and power generation with tethered kites. *Journal of guidance, control, and dynamics*, 31(1):81–93, 2008.

[7] Cross Product. C. 4 miscellaneous. *Automatic Flight Control of Tethered Kites for Power Generation*, 2012.

[8] Bas Lansdorp, B Remes, and WJ Ockels. Design and testing of a remotely controlled surfkite for the laddermill. In *World Wind Energy Conference, Melbourne, Australia*, 2005.

[9] Alexander Bolonkin. Using of high altitude wind energy. *Smart Grid and Renewable Energy*, 2(2):75–85, 2011.

[10] Massimo Canale, Lorenzo Fagiano, and Mario Milanese. Kitegen: A revolution in wind energy generation. *Energy*, 34(3):355–361, 2009.

[11] Massimo Canale, Lorenzo Fagiano, and Mario Milanese. High altitude wind energy generation using controlled power kites. *Control Systems Technology, IEEE Transactions on*, 18(2):279–293, 2010.

[12] Boris Houska and Moritz Diehl. Optimal control of towing kites. In *Decision and Control, 2006 45th IEEE Conference on*, pages 2693–2697. IEEE, 2006.

[13] MS Manalis. Airborne windmills and communication aerostats. *Journal of Aircraft*, 13(7):543–544, 1976.

[14] G Riegler, W Riedler, and E Horvath. Transformation of wind energy by a high-altitude power plant. *Journal of Energy*, 7(1):92–94, 1983.

[15] CAJ Fletcher. On the rotary wing concept for jet stream electricity generation. *Journal of Energy*, 7(1):90–92, 1983.

[16] Charles Max Fry and Henry W Hise. Wind driven, high altitude power apparatus, April 11 1978. US Patent 4,084,102.

[17] Alberto Kling. Wind driven power plant, February 14 1978. US Patent 4,073,516.

[18] Paul F Pugh. Wind generator kite system, December 4 1984. US Patent 4,486,669.

[19] William K Watson. Airship-floated wind turbine, January 1 1985. US Patent 4,491,739.

[20] David H Shepard. Power generation from high altitude winds, April 21 1987. US Patent 4,659,940.

[21] Bas Lansdorp and WJ Ockels. Comparison of concepts for high-altitude wind energy generation with ground based generator. In *Proceedings of the NRE 2005 Conference, Beijing, China*, pages 409–417, 2005.

[22] Uwe Fechner and Roland Schmehl. Design of a distributed kite power control system. In *Control Applications (CCA), 2012 IEEE International Conference on*, pages 800–805. IEEE, 2012.

[23] L Fagiano, AU Zgraggen, M Khammash, and M Morari. On control of tethered wings for airborne wind energy. In *American Control Conference (ACC), 2013*, pages 1430–1435. IEEE, 2013.

[24] Wen-Hua Chen, Donald J Ballance, and Peter J Gawthrop. Optimal control of nonlinear systems: a predictive control approach. *Automatica*, 39(4):633–641, 2003.

[25] Arthur E Bryson Jr. Optimal control-1950 to 1985. *Control Systems, IEEE*, 16(3):26–33, 1996.

[26] Andreas Waechter, Carl Laird, F Margot, and Y Kawajir. Introduction to ipopt: A tutorial for downloading, installing, and using ipopt, 2009.

[27] Paola Falugi, Eric Kerrigan, and Eugene van Wyk. Imperial college london optimal control software user guide (iclocs), 2010.