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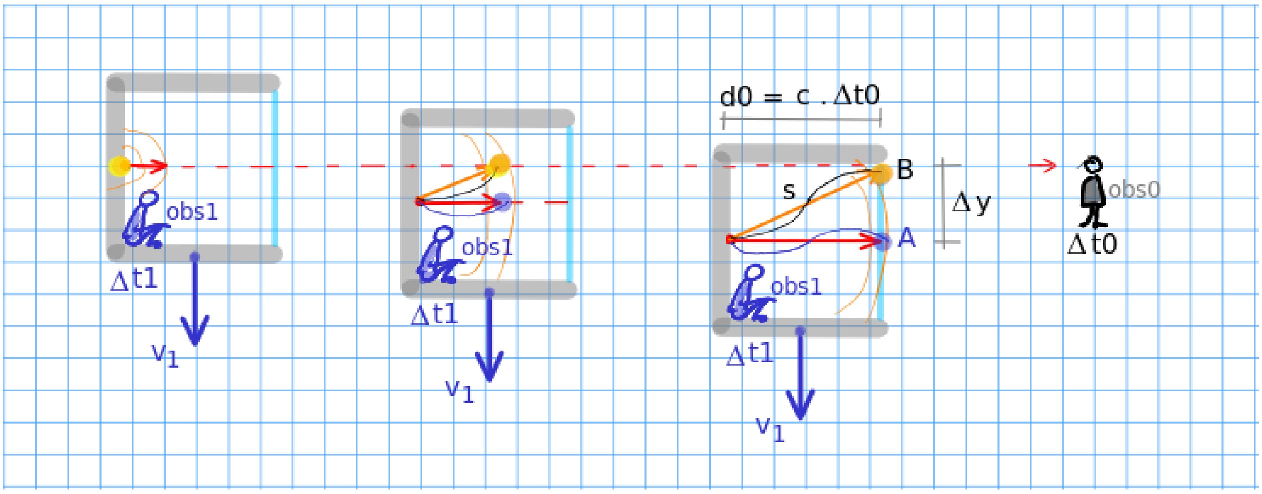
Einstein's Train With Blind Observers

Space-Time contraction/dilatation in a train with blind observers



Physics, Engineering, Modelling and Simulation

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ABSTRACT

Special Relativity, fist propose by Einstein in the beginning of the twentieth century, is one of the pearls of modern Physics, as we know, since for the first time the concept of a universal *Time* has been challenged. Time-dilatation and time-compression could at those times only be regarded as non-

senses, or as unsupportable provocations, if not jokes. The explanation of the concept to the academic community by a consistent mathematical deduction had, however, to be generally accepted, since in effect no mistake has been detected in the formulation proposed by Einstein. It deals, however, with the *perception* of events by different observers, which, in the simplest case, are considered located at different inertial reference frames. That is, all them have a constant velocity in respect to one another.

Here we will start by addressing two cases like that in order to explain Einstein's reasoning, but later we will also consider a case where blind observers are added to the experiment. The results will probably be surprising for some, and obvious for others.

1. **Einstein's Train**

Einstein's Train is an exercise of imagination, usually called a *thought-experiment*, which is frequently used by those who dedicate to theoretical science in order to help test and validate their physical models and/or mathematical proposals. Einstein, in particular, in my opinion because he was an exceptional dreamer driven by unusual intuition, seemed to have a predilection for this method, and naturally used it in all its power. In effect, that was the only way he could in his times “experiment” with velocities of the order of the speed of light (*c*). Although several different imaginary experiments can be used to explain the dependency of the perception of *Time* and *simultaneity* on the relative velocity of the inertial observers, we will start by the original scheme presented by Einstein himself (fig.1).

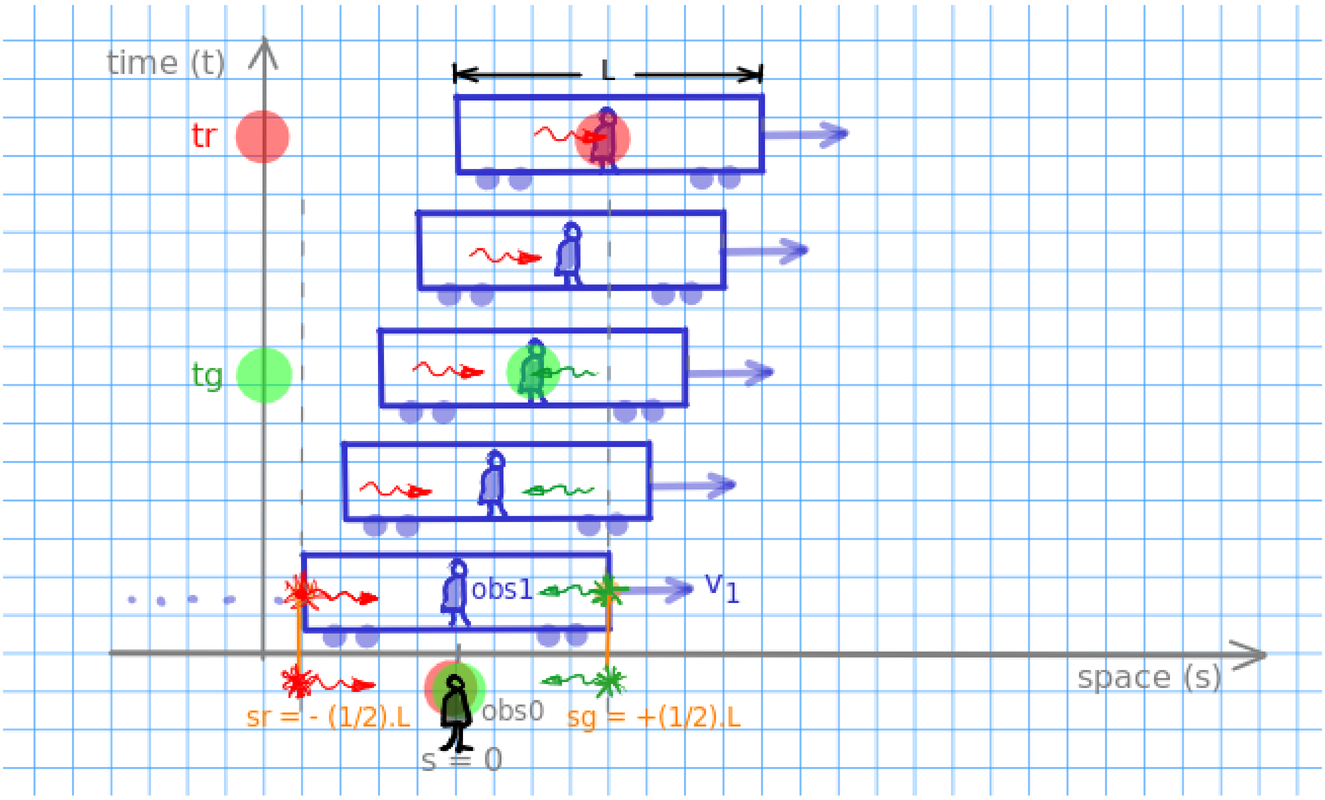

Fig. 1 Einstein's Train imaginary experiment (adapted to use colour information). In the precise moment that a train with speed (*v1*) passes in front of an observer at rest (*obs0*) two pulses of light are emitted at the same time at both extremes of the train. Since the light propagates with a certain finite speed, the observer at rest (*obs0*) will perceive both signals reaching him at the same time (simultaneous events), while the observer inside the train (*obs1*) will first receive the signal of the front of the train (at a time *tg*) and only then the one emitted from the rear (at a time *tr*). Thus, for the running observer those events are perceived as not-simultaneous. Einstein defended that this was due to the velocity of the train. Because of this phenomenon, perception of *Time* can also be different for the two observers, he suggests.

Let us use the scheme of figure 1 to compute the relation between the perception of those events by each of the observers. Noticing that there are four elements in this system to which we can associate individual equations of motion: the two observers, and the two pulses. Writing these equations using Newton's physics, we can say that:

$s0(t) = 0 + 0.t =$ equation of motion of (*obs0*) at rest
$s1(t) = 0 + v1.t =$ equation of motion of (*obs1*) moving
$sr(t) = sr + c.t =$ equation of motion of *red pulse*
$sg(t) = sg - c.t =$ equation of motion of *green pulse*
(1)

Notice that *(L)* is the length of the train, therefore we will have, as represented in figure 1, $sr = -(1/2).L$ and $sg = +(1/2).L$. And these are the locations where the two light pulses occur at time ($t=0$). In figure 1 we represent both the *red* and the *green* pulses, both in the *resting-world* and the *moving-world*[1], but in effect they are only two pulses, not four. Notice, however, it is not clear in which *world* the two pulses are emitted. Is it important to know it? Well, if the source of the pulses are in the *resting-world* their initial speed is null, while if

[1] We like to use the term "world" instead of "frame", since it gives a better impression that the observer may experience different perspectives in different frames.

they are fixed to the train they will have the same initial speed as the speed of the train. But let us assume such an issue is not relevant for the case... As time passes, the system is moving to the right along the space axis (*s*) and up along the time axis (*t*). We can see by the figure that the *red-pulse* and the *green-pulse* reach at the same time (not specified in the figure) the observer at rest (*obs0*), while the *green-pulse* reaches the moving observer (*obs1*) at time (*tg*) and only later this observer will receive the *red-pulse* at time (*tr*).

The important now is to compute all these times. Let us use the equations of motion previously written:

$$
\begin{aligned}
&s0(t) = 0 + 0.t \\
&s1(t) = 0 + v1.t \\
&sr(t) = sr + c.t \\
&sg(t) = sg - c.t
\end{aligned}
\tag{2}
$$

And first focus on what happens with the observer at rest. We know that in the instant that the *green-pule* reaches (*obs0*) the two will have to be in the same position. That is, $sg(t) = s0(t)$. And this is equivalent to write:

$$sg - c.t = 0 + 0.t = 0$$
$$t = sg/c = t0g \qquad (3)$$

Where we defined (*t0g*) as the instant of time the *green-pulse* reaches (*obs0*)... Now, for the *red-pulse*, we will have *sr(t) = s0(t) = 0*:

$$sr + c.t = 0 + 0.t = 0$$
$$t = - sr/c = t0r \qquad (4)$$

With (*t0r*) being the instant the *red-pulse* reaches (*obs0*)... from this we can very simple compute the difference of these two instants and call it ($\Delta t0$):

$$\Delta t0 = t0r - t0g = (1/c).(-sr + sg) \qquad (5)$$

Since in the present case we have (*sr* = -*sg*), this difference will always vanish:

$$\Delta t0 = 0 \Rightarrow \textit{simultaneous events for (obs0)} \qquad (6)$$

Let us now make the same computations for the moving observer (*obs1*). When the *green-pulse* reaches her, *sg(t) = s1(t)*, or:

$sg - c.t = 0 + v1.t$

$t = sg/(c+v1) = t1g$ (7)

And when the *red-pulse* reaches here, $sr(t) = s1(t)$:

$sr + c.t = 0 + v1.t$

$t = - sr/(c - v1) = t1r$ (8)

Thus the difference between the two will be:

$\Delta t1 = t1r - t1g = -sr/(c - v1) - sg/(c + v1)$ (9)

As again ($sr = -sg$), we will have:

$\Delta t1 = sg/(c - v1) - sg/(c + v1)$ (10)

$\Delta t1 = sg.\{1/(c - v1) - 1/(c + v1)\}$

$\Delta t1 = sg.\{(c + v1)/(c^2 - v1^2) - (c - v1)/(c^2 + v1^2)\}$

$\Delta t1 = [sg/(c^2 - v1^2)].\{c + v1 - c + v1\}$

$\Delta t1 = [sg/(c^2 - v1^2)].2.v1$

$\Delta t1 = sg.2.v1/(c^2 - v1^2)$

$\Delta t1 = (sg.2.v1/c^2).\{1/(1 - v1^2/c^2)\}$ (11)

Remembering that ($sg = L/2$), we get:

$$\Delta t1 = (L\,.v1/c^2).\{1/(1 - v1^2/c^2)\}$$

$$\Delta t1 = (L\,/c).(v1/c).\{1/(1 - v1^2/c^2)\} \quad (12)$$

And this obviously shows that the moving observer will not understand the two events as simultaneous. If the velocity of the *moving-world*, however, will be null ($v1=0$), that is, if the *moving-world* will also be at rest, then $\Delta t1 = 0$ and the two events will also be perceived as simultaneous. We suppose that Einstein concluded that relative velocities between observers may change their perception on certain events in time, and that such a phenomenon may lead to the possibility of *Time* contraction/dilatation.

First comment, before making a real calculation: all this have been deduced supposing one observer in a *resting-world* and the other observer in a *moving-world*. For the first observer time between events was null, for the second observer such a time was not null. Thus, we may expect relative motion to originate certain time disturbances, which may probably interfere with the measures made in each world. Velocity is what is responsible for such disturbances, we could say… but… let us therefore consider again the same scheme, but time with the train at rest. At rest, but not precisely centred relatively to the points where the light pulses happen. That is, the center of the train will not be at (*s=0*) but at some distance to the right of it (*s01*) in between (*s=0*) and (*sg*). The new equations of motion will be:

$$s0(t) = 0 + 0.t$$
$$s1(t) = s01 + 0.t$$
$$sr(t) = sr + c.t$$
$$sg(t) = sg - c.t \tag{13}$$

Obviously, (*obs0*) will still understand the two signals as being simultaneous. But the observer (*obs1*)... well... the *green-pulse* reaches (*obs1*) when:

$$sg - c.t = s01$$
$$t = (sg - s01)/c = t1g \tag{14}$$

And the *red-pulse* when:

$$sr + c.t = s01$$
$$t = (-sr + s01)/c = t1r$$
$$(-sr = sg) \;\; => \;\; t1r = (sg + s01)/c \tag{15}$$

Therefore:

$$\Delta t1 = t1r - t1g = (sg + s01)/c - (sg - s01)/c \qquad (16)$$

$$\Delta t1 = (1/c).(sg + s01 - sg + s01)$$

$$\Delta t1 = 2 \,.\, s01/c \qquad (17)$$

The second observer, even now that she at rest, still does not perceive the two events as simultaneous!

This let us infer that in effect it is not the velocity that transforms *space-time* properties and make those events appear "strange"; the source of this phenomenon is the fact that *information* is transferred from events to observers by means of a finite, and supposed constant, speed. It is coming more from an artefact than from a property of *space-time* itself. In effect, there is no deformation of the Newtonian space, what happens is a sort of illusion provoked by the process of transfer of *information*. In reality, we can say for both cases that events are always separated, but the observer in the center of coordinates is in a very special situation, therefore he perceives them as simultaneous. If he would position himself slightly to the right or to the left, he would automatically loose such an illusion. We may so deduce that it is only necessary the two observers are in two different positions of the space for that they will perceive differently events happening simultaneously. Of course a relative velocity between the two observer will ensure this, and even make such an illusion be dependent on the speed.

2. Time contraction/dilatation

Let us now suppose that each observer has a perfect watch. The resting observer (*obs0*) measures his time (*t0*) and the moving observer (*obs1*) measures her time (*t1*). When the middle point of he train moving with velocity (*v1*) passes in front of the observer at rest both watches are reset to (*t=t0=t1=0*) and both light pulses are emitted. But none of the observers knows anything about those pulses. If they could communicate with each other by means of instantaneous messages their conversation could be:

(obs0): my clock is zero.
(obs1): mine too. What are we doing here?
(obs0): I don't know... they offered me the watch.
(obs1): lol, to me too. We must register all we detect.
(obs0): yep. Did you already detect anything?
(obs1): nop. You?
(obs0): nop.

(obs1): ah! Green light flash in the front of the train.
(obs0): are you sure? There is nothing happening yet!
(obs1): absolutely sure! But now is dark again...
(obs0): maybe it is psychological, you know...
(obs1): no no, I am sure!

(obs0): waw, maybe you are right!
(obs1): why?
(obs0): a green-pulse from front, and a red-pulse from rear!
(obs1): two pulses?! Maybe your psycho... lol.
(obs0): I swear!

(obs1): wait, I also received now a red-pulse from rear!
(obs0): And I start no to trust in you...
(obs1): at least we have new watches...

We see from this little conversation that both observers start to not trust in their measures, unless they previous have been informed that *space-time* may be deformed in different worlds, and probably such a deformation could even influence the *ticks* of their watches... but... let us use another well known thought-experiment in order to try better address such a problem:

Consider now that the watches of the two observers (and we may now forget the train), are absolutely equal and they work based on the emission of laser pulses that are reflected by two parallel mirrors (fig.2) distant of (*d0*). In the literature, usually the watch is consider to *tick* each time de pulse returns to the first mirror, where it was emitted from, after travelling the distance (*2.d0*). But we think it is much simpler to understand this system if we consider that the watch *ticks* each time the pulse reaches one of the mirrors, that is, after travelling the distance (*d0*). Notice that both watches are considered aligned by the vertical, and the velocity of the observer (*obs1*) has only horizontal component.

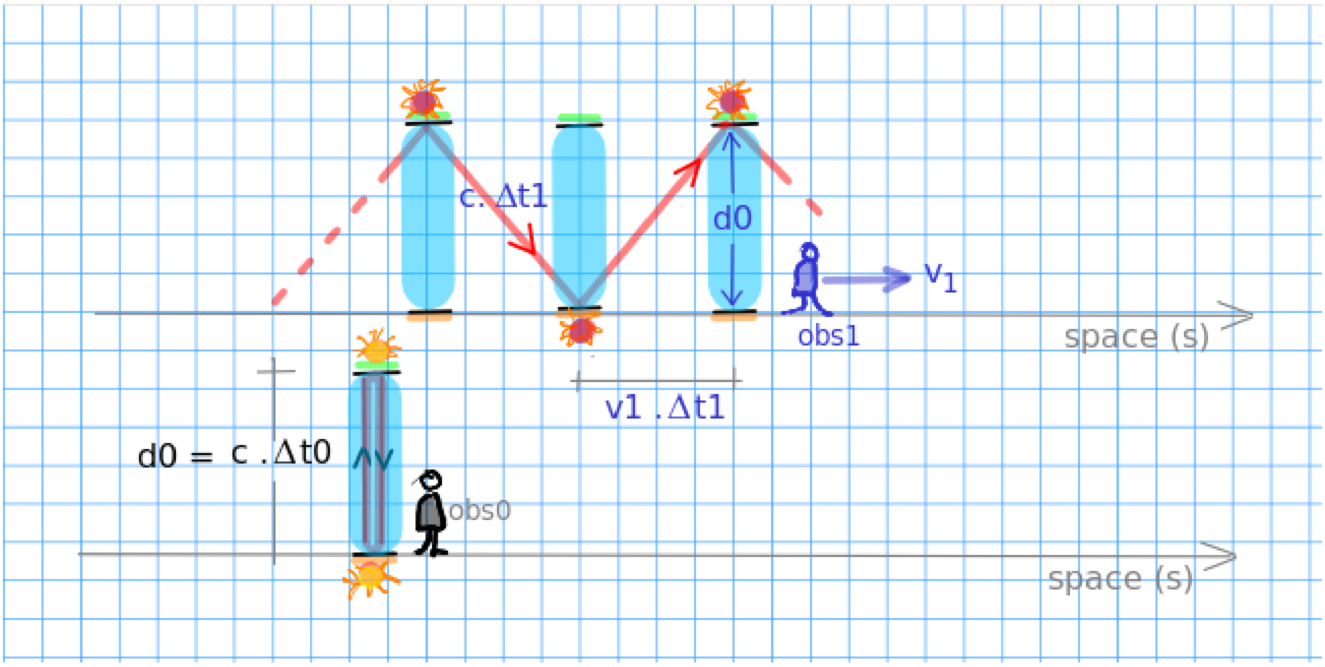

Fig. 2 Einstein's imaginary experiment (simplified) for deducing time compression and time dilatation. An observer at rest (*obs0*) measures his time (*t0*) based on a vertical laser watch that *ticks* each time the laser pulse reaches one of its mirrors. The vertical length of the watch is (*d0*). The time between ticks (*Δt0*) represents how this observer measures what happens in his *resting-world*. A second observer (*obs1*) travelling along the horizontal with velocity (*v1*) uses a perfect copy of the same watch. But the observer at rest deduces something odd: the watch of the moving observer *ticks* slower...

If we position ourselves as the (*obs0*) looking at the moving observer (*obs1*), the fist thing we may notice is that the path of the laser pulse in the *moving-world* is not restricted to vertical motion. Instead, it progresses in the space as a triangular path with characteristics depending on the velocity (*v1*). We know, however, that the vertical component of this path must precisely reflect the laser pulse path of our resting watch, since the two watches do not move in that direction (vertical axis). We may also understand that the laser pulse of (*obs1*) will have to travel a diagonal distance between mirrors, and such a distance must be travelled at the speed of light (*c*) during a time interval (*Δt1*) which we don't know yet. But since the watch is moving in the horizontal with a velocity (*v1*), by figure 2 we also understand that the horizontal distance between consecutive *ticks* is simply (*v1.Δt1*). Knowing this, and using the Pythagoras theorem, we can finally write a relation between (*d0*), (*v1.Δt1*), and (c .*Δt1*). That is:

$$(c\,.\Delta t1)^2 = (v1.\Delta t1)^2 + (d0)^2 \qquad (18)$$

$$(c\,.\Delta t1)^2 - (v1.\Delta t1)^2 = (d0)^2$$

$$(\Delta t1)^2\,.\,(c^2 - v1^2) = (d0)^2$$

$$(\Delta t1)^2 = (d0)^2 / (c^2 - v1^2)$$

But we also know, from our watch, that *(c .Δt0 = d0)*, therefore we may write:

$$\Delta t1^2 = c^2 . \Delta t0^2 / (c^2 - v1^2)$$
$$\Delta t1^2 = \Delta t0^2 / (1 - v1^2/c^2)$$
$$\Delta t1 = \Delta t0 / \sqrt{(1 - v1^2/c^2)} \qquad (19)$$

This is the well known expression found by Einstein. It tells us that the observer at rest (*obs0*) has the impression the watch of the moving observer (*obs1*) is running slower, since the time interval between ticks (*Δt1*) is longer than that of his watch (*Δt0*)... and such is an effect due to velocity.

3. **Two new observers: blind**

Let us now make some calculations by using real numbers. Suppose the moving observer has a speed of around *144 km/s = 40 m/s = (v1)*, and both watches generate a *tick* each *(Δt0 = 10 s)*, when at rest. Since light propagates with the same speed in the two worlds, being $(c = 3 . 10^8 \ m/s)$, we can deduce that:

$$\Delta t1 = 10 / \sqrt{(1 - 40^2/(9.10^{16}))}$$

$\Delta t1 = 10 / \sqrt{(1 - 177{,}8 \cdot 10^{-16})}$

$\Delta t1 = 10$ *if we don't have a calculator with at least 16 digits*

That is, the two observers will feel the time flowing practically the same way, since the difference is so small!

But, let us now add two new observers to the system, both blind, and adapt slightly Einstein's experiment to their condition. Each watch is now prepared to emit an *ultra-sound-pulse* perceived only by the blind observers, each time it ticks. This way, the blind observers can perfectly be synchronized with their mates in any measure they decide to do. We can say, even, that their perception of the flow of time should be precisely the same, as they cohabit the same space and the *light-pulses* are made to drive the *ultra-sound-pulses*. One of the blinds will stay next to the observer at rest, while the second blind joins the moving observer, and sits next to her. Since the speed of propagation of ultrasounds in the air (*330 m/s*) is the same in the two worlds, we may use the same expression (19) to compute the difference on the perception of time for the blind observers:

$\Delta t1 = 10 / \sqrt{(1 - 40^2/330^2)}$

$$\Delta t1 = 10 / \sqrt{(1 - 0{,}0178)}$$
$$\Delta t1 = 10 / \sqrt{0{,}982}$$
$$\Delta t1 = 10{,}181 \ s \qquad (20)$$

And so we discover that there is much more dilatation of time for blind observers that for normal observers, even if they occupy or travel in the same conditions of their fellows... and naturally the questions arise: why a blind person in a train moving at velocity (*v1*) will have to see her laser-sonic-watch running much slower that the same laser-sonic-watch of her fellow sited next to her? Are we still in the domain of Physics, or we are already entering Psycho-physics? Since the two moving observers are next to each other, do they notice any strangeness in their persons? Why the normal observer at rest sees almost nothing changed on what happens in the train, while his blind friend obviously sees what happens in the train as if happening in slow motion? Would it be a judicious decision to consider *velocity* deforms *space* and *time*, and use such an idea to develop more complex physical theories like, for example, gravitation? What to expect of the simulation of reality based on those models if they would suddenly present us with strange things as *dark-matter* or *dark-energy*, or even *black-holes*... could all that be simply artefacts produced by those models and not the true reality?

4. **Another approach**

One of the issues we consider strange in Einstein's Special Relativity is the fact light is simply seen as a corpuscular element moving in straight-line, as in Geometrical Optics, when it was already known at that time light is an electromagnetic wave, therefore it could be treated as a wave. Our approach here will precisely go in this direction. We will imagine an experiment similar to that proposed by Einstein, but also analyse it by means of wave mechanics.

This time let us imagine an observer (*obs1*) inside a cabin falling at a constant velocity (*v1*) in respect to a resting observer (*obs0*), as depicted in the next figure (fig.3). As in the previous examples, both observers have the same type of watches, which at rest *tick* each (*Δt0*). While the cabin falls, however, the resting observer suspects the watch of the moving observer will *tick* in a different way, and calls it (*Δt1*). Knowing the horizontal length of the cabin to be (*d0*) and that a pulse of a *wave* as been emitted in its direction at time (*t=0*) in a position of the space exactly in front of his eyes, he tries to understand how the watch of is fellow is ticking...

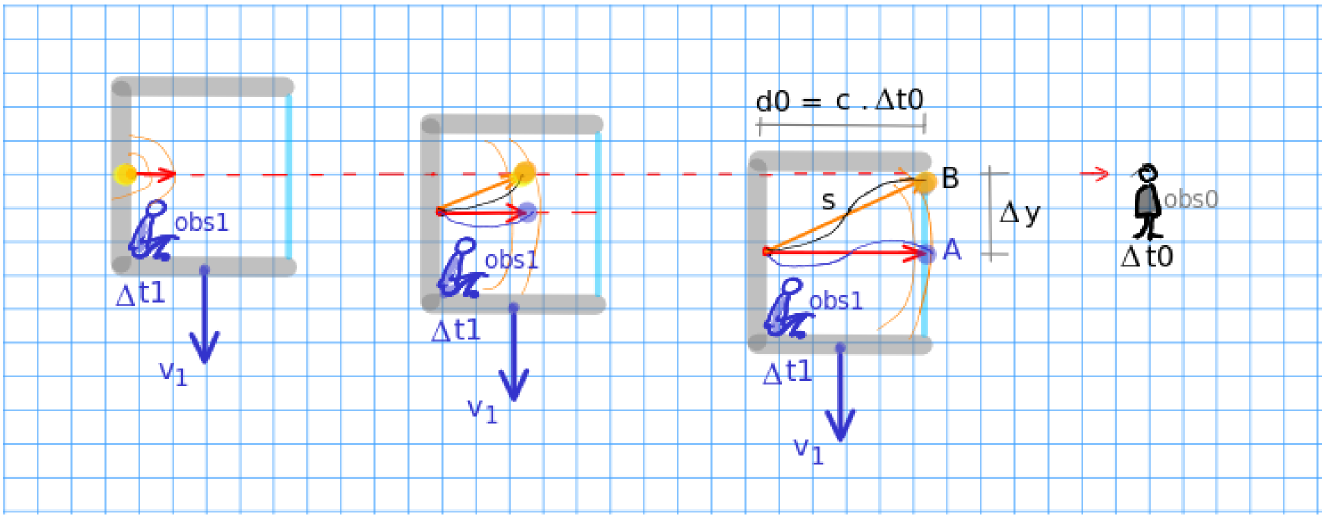

Fig. 3 Our imaginary experiment for studying time compression and time dilatation. An observer at rest (*obs0*) measures his time based on a watch that *ticks* each (*Δt0*) seconds. A second observer (*obs1*) falling along the vertical with velocity (*v1*) uses a perfect copy of the same watch, and emits a wave pulse that propagates with speed (c) in the direction of the observer at rest. Time is considered to ev olve from left to right. The figure represents three instants of time.

It is obvious that if the two observers would be at rest in respect to each other they would both see the pulse emitted on the left of the cabin reaching the window of the cabin at the position (*A*), therefore (*d0* = *c.Δt0)*. When the cabin is in motion, however, the observer at rest will detect the pulse at the position (*B*), separated (*Δy*) from (*A*) after travelling an apparent distance (*s*) along the diagonal. He knows[2] that such a distance has been also travelled with the speed of the wave (c), but

[2] Notice that we are considering the speed of the wave solely dependent on the medium of propagation, and in both cases the medium is the same.

not during the same time interval ($\Delta t0$), since ($s>d0$). He therefore assumes such a time to be ($\Delta t1$), which represents two consecutive *ticks* of the watch of the moving observer. Thus, he will deduce that ($s = c.\Delta t1$). By using Pythagoras theorem, he then writes:

$$s^2 = d0^2 + \Delta y^2 \qquad (21)$$

$$(c.\Delta t1)^2 = (c.\Delta t0)^2 + (v1.\Delta t1)^2$$

$$c^2.\Delta t1^2 - v1^2.\Delta t1^2 = c^2.\Delta t0^2$$

$$\Delta t1^2 (1 - v1^2/c^2) = \Delta t0^2$$

$$\Delta t1 = \Delta t0 / \sqrt{(1 - v1^2/c^2)} \qquad (22)$$

Leading to the same result as before (equation (19)). But if we suppose those pulses coming from a *wave*, we can also understand that the frequency of the original wave is ($f0 = 1/\Delta t0$) and the frequency perceived by the observer at rest is ($f1 = 1/\Delta t1$). For simplicity, let us suppose that we would choose a source of waves with an original (at rest) wavelength ($\lambda 0 = d0$), as shown in figure 3. Since the process of transference of *information* between the two observers must ensure no new information is added to the original signal, it is obvious that we must have the resting observer perceiving (s) as a single wavelength too. That is, ($\lambda 1 = s$). Of course, both waves have to be related by the common speed of propagation:

$$c = \lambda 1 \,.\, f1 = \lambda 0 \,.\, f0 \qquad (23)$$

$$\lambda 1 \,.\, f1 = \lambda 1 / \Delta t1 = (\lambda 1 / \Delta t0).\sqrt{(1 - v1^2/c^2)} = \lambda 0 / \Delta t0$$
$$(\lambda 1 / \Delta t0).\sqrt{(1 - v1^2/c^2)} = \lambda 0 / \Delta t0$$
$$\lambda 1 = \lambda 0 / \sqrt{(1 - v1^2/c^2)} \qquad (24)$$

As expected, the resting observer is also receiving a signal from a longer wavelength than that emitted by the source, precisely as when in presence of a *Doppler* effect... Could it be that Einstein was simply describing a *Doppler* effect while considering light as a corpuscle? Will in reality a blind person travelling in a high speed train become younger than her blind fellows at rest? We bet not!... Maybe she would seem to get younger while she was moving away, but then she would seem getting older when returning back to their fellows again. And the sum of these two contributions will be null.

5. **Conclusions**

The physico-mathematical deductions presented in this article have made us questioning the validity of the assumption that *velocity* may be responsible for the deformation of *space-time*, as proposed by Einstein's Special Relativity, and for the change in the perception of simultaneity of events by different observers in reference frames of motion. Our impression is these are illusions created by the finite speed of propagation of *information*. Analysing Einstein's train with blind observers made also suspect that such effects are similar to the usual *Doppler* effects resulting from the propagation of waves.

Author's Biography:

J. Manuel Feliz-Teixeira, graduated in Physics, Masters in Mechanical Engineering and Doctorate in Sciences of Engineering from the University of Porto. He has been dedicated to various fields of knowledge and several industrial projects. More recently, he became mainly interested in lecturing Physics and studying mainly *electromagnetism* and *gravitation* by means of rethinking classical principles.