Incoherently coupled soliton pairs in biased photorefractive crystals

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We show that incoherently coupled soliton pairs are possible in biased photorefractive crystals, under steady-state conditions. These solitons can propagate in bright-bright, dark-dark, as well as in bright-dark configurations. Such soliton pairs can be established provided that the carrier beams share the same polarization, wavelength, and are mutually incoherent. Relevant examples are provided where the photorefractive crystal is of the strontium barium niobate type. The characteristics and stability properties of these soliton states are also discussed in detail. © *1996 American Institute of Physics.* [S0003-6951(96)04213-3]

Recently, optical spatial solitons at μW power levels have been successfully observed in photorefractive (PR) materials.[1–4] In these experiments self-trapping was also found to take place in both transverse dimensions.[1,3] Thus, such PR solitons show considerable promise towards implementing all-optical beam switching and/or processing devices. Of particular interest are the so-called screening solitons which occur in steady state when an external bias voltage is appropriately applied to a PR crystal.[5,6] Such soliton states are possible provided that the optical beam is linearly polarized. Very recently, vector solitons involving the two polarization components of an optical beam have also been predicted in biased PR materials.[7,8] Depending on the symmetry class of the appropriate crystal and its orientation, these solitary beams were found to obey a self-coupled or a cross-coupled system of nonlinear evolution equations.[7] Self-coupled vector solitons can be realized only in certain noncentrosymmetric crystals, such as that of $4mm$ and $3m$ class, where the perturbed diagonal permittivity terms must be equal. This can be achieved either by off-axis propagation (at a specific propagation angle) or by temperature tuning.[7] On the other hand, cross-coupled solitons require phase matching between the two polarizations, which can only be obtained in specific phase matched geometries.[7]

In this letter, we show that a new type of incoherently coupled soliton pairs is possible in biased photorefractive crystals under steady-state conditions. These soliton states can be established provided that their carrier beams share the same polarization, wavelength, and are mutually incoherent. Such solitons can propagate in bright-bright, dark-dark, as well as in bright-dark configurations. Moreover, they can be readily realized in a simple experimental setup where the two mutually incoherent optical beams (of the same wavelength and polarization) propagate collinearly, in which case they experience equal effective electrooptic coefficients. Note, that since the two sources are mutually incoherent, no phase matching arrangement is then required. The functional form and characteristics of these self-trapped pairs are discussed and their stability properties are also considered. Relevant examples are provided where the photorefractive crystal is assumed to be of the strontium barium niobate ($Sr_xBa_{1-x}Nb_2O_6$,SBN) type.

To start, let us consider a SBN crystal with its optical c-axis oriented in the x direction. Two laser beams $\mathscr{E}_1$ and $\mathscr{E}_2$ of the same frequency[9] but mutually incoherent are used to illuminate the crystal. The two wave fronts may actually originate from the same laser as long as their differential path is kept beyond the source coherence length. The polarization of both optical beams is assumed to be parallel to the x axis. The two optical beams can also be slightly misaligned at the input so as to differentiate them in the output plane. Note, that a similar setup has been previously employed to study soliton waveguiding in a $Bi_{12}TiO_{20}$ crystal.[10,11] The two wave fronts propagate collinearly along the z axis and are allowed to diffract only along the x direction. In essence, our diffraction model is one dimensional and any y-dependent perturbations have been implicitly omitted in our analysis.[12] Furthermore, the external bias electric field is applied in the x direction (i.e., the c axis). In this case, the perturbed refractive index along the x axis[13] (for both beams) is given by $n_e'^2 = n_e^2 - n_e^4 r_{33} E_s$ where n_e is the unperturbed extraordinary index of refraction and $\mathbf{E}_s = E_s \hat{x}$ is the static space charge field in this PR crystal. For purposes of simplicity, the PR material is also assumed to be lossless. The optical fields are then expressed as usual[5,6] in terms of slowly varying envelopes (ϕ,ψ), that is $\mathscr{E}_1 = \hat{x}\phi(x,z)\exp(ikz)$ and $\mathscr{E}_2 = \hat{x}\psi(x,z)\exp(ikz)$. k is the propagation constant given by $k = k_0 n_e = (2\pi/\lambda_0) n_e$ and λ_0 is the common free-space wavelength. In that case, it can be readily shown[6] that the (ϕ,ψ) envelopes obey the following evolution equations:

$$i\phi_z + \frac{1}{2k}\phi_{xx} - \frac{k_0(n_e^3 r_{33} E_s)}{2}\phi = 0, \tag{1}$$

$$i\psi_z + \frac{1}{2k}\psi_{xx} - \frac{k_0(n_e^3 r_{33} E_s)}{2}\psi = 0, \tag{2}$$

where $\phi_z = \partial\phi/\partial z$, etc. Moreover, under strong bias and for relatively broad beam configurations, the steady-state space charge electric field is approximately given by[5,6]

$$E_s(x,z)=E_0\frac{I_d+I_\infty}{I_d+I(x,z)},\qquad (3)$$

where $I=I(x,z)$ is total power density of the two optical beams, I_d is the so-called dark irradiance[14] and I_∞ represents the total power density the soliton pair attains away from the center of the PR crystal, i.e., $I_\infty=I(x\rightarrow\pm\infty)$. E_0 is the value of the space-charge electric field[5,6] also at $x\rightarrow\pm\infty$. If the spatial extent of the optical waves involved is much less than the x-width W of the PR crystal, then under a constant voltage bias V, E_0 is approximately given by[6] $\pm V/W$. For the two mutually incoherent optical beams, the total optical power density, I, can then be obtained by summing the two Poynting fluxes,[15] i.e., $I=(n_e/2\eta_0)(|\phi|^2+|\psi|^2)$. Moreover, for convenience, let us adopt the following dimensionless variables and coordinates, i.e., let $\xi=z/(kx_0^2)$, $s=x/x_0$, $\phi=(2\eta_0 I_d/n_e)^{1/2}U$, and $\psi=(2\eta_0 I_d/n_e)^{1/2}V$. x_0 is an arbitrary spatial width and the power densities of the optical beams have been scaled with respect to the dark irradiance I_d. By employing these latter transformations and by substituting Eq. (3) into Eqs. (1) and (2), the normalized planar envelopes U and V are found to satisfy:

$$iU_\xi+\frac{U_{ss}}{2}-\beta(1+\rho)\frac{U}{1+|U|^2+|V|^2}=0,\qquad (4)$$

$$iV_\xi+\frac{V_{ss}}{2}-\beta(1+\rho)\frac{V}{1+|U|^2+|V|^2}=0,\qquad (5)$$

where $\rho=I_\infty/I_d$ and $\beta=(k_0x_0)^2n_e^4r_{33}E_0/2$. In what follows, we will discuss the possible soliton pair solutions of Eqs. (4) and (5). Their characteristics and stability properties will also be considered.

Let us first consider bright-bright soliton pairs. In this case, where bright optical beams are involved in both components, the intensity is expected to vanish at infinity ($s\rightarrow\pm\infty$) and thus $I_\infty=\rho=0$. Soliton solutions can then be readily obtained by expressing the normalized envelopes U and V in the following way: $U=r^{1/2}y(s)\cos\theta\exp(i\mu\xi)$ and $V=r^{1/2}y(s)\sin\theta\exp(i\mu\xi)$. μ represents a nonlinear shift of the propagation constant, $y(s)$ is a normalized real function bounded between $0\leq y(s)\leq 1$, and the parameter θ is an arbitrary projection angle. Direct substitution of these forms of U and V in Eqs. (4) and (5) leads to the following differential equation

$$\frac{d^2y}{ds^2}-2\mu y-\frac{2\beta}{1+ry^2}y=0,\qquad (6)$$

which is known to allow bright solitons when β or E_0 are positive quantities[6] and $\mu=-(\beta/r)\ln(1+r)$. The bright solutions of Eq. (6) have been discussed in great detail in Refs. 5 and 6, where it was shown that they depend only on two parameters, namely E_0 and their normalized peak intensity r. In this case, the soliton pair components can be considered as the θ projections of the fundamental bright soliton envelope. As an example, let us consider a SBN PR crystal with the following parameters[16] $n_e=2.33$ and $r_{33}=237$ pm/V at a wavelength $\lambda_0=0.5$ μm. If we allow the arbitrary spatial scale x_0 be 25 μm and look for self-trapped beams at $E_0=1$ kV/cm, we find that $\beta=34.5$. The total peak intensity of the pair r, is assumed to be 10. Figure 1(a) depicts the normalized intensity profiles of the soliton pair when $r=10$ and the arbitrary parameter $\theta=30°$. In this case, the intensity FWHM of the two components is found to be 12.7 μm.

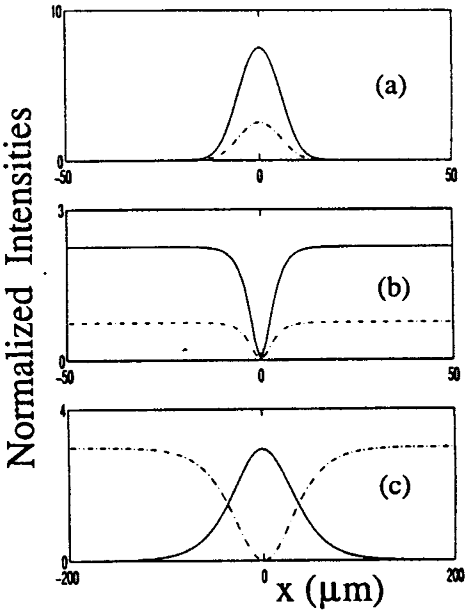

FIG. 1. Soliton components, $|U|^2$ (solid curve) and $|V|^2$ (dash–dot curve), for a (a) bright-bright pair when $r=10$ and $\theta=30°$, (b) dark-dark pair when $\rho=3$ and $\theta=30°$, and a (c) bright-dark pair when $\rho=3$ and $\delta=-0.01$.

Similarly, dark-dark pairs can also be analyzed. Such dark beams are known to exhibit an antisymmetric field profile (with respect to x) and, moreover, they are embedded in a constant intensity background I_∞, that is I_∞ and ρ are now finite quantities. The envelopes U and V are expressed again as $U=\rho^{1/2}y(s)\cos\theta\exp(i\mu\xi)$ and $V=\rho^{1/2}y(s)\sin\theta\exp(i\mu\xi)$ where $|y(s)|\leq 1$. Therefore, from Eqs. (4) and (5) dark-type pairs should satisfy

$$\frac{d^2y}{ds^2}-2\mu y-2\beta(1+\rho)\frac{y}{1+\rho y^2}=0,\qquad (7)$$

which can be numerically solved provided that the bias voltage is negative,[6] i.e., β or $E_0<0$ and $\mu=-\beta$. As in the previous case, Eq. (7) is known to describe single beam dark solitons[5,6] with peak background intensity ρ. The pair components can then be simply obtained through a θ projection. Such a solitary-wave pair solution at $\rho=3$ and $E_0=-1$ kV/cm in a SBN crystal is shown in Fig. 1(b). The intensity FWHM of the two dark beams is found to be 6.5 μm. It is also interesting to note that one could also anticipate gray-gray pair solutions when $E_0<0$, provided their phase[6] varies like $\exp[i\mu\xi+i\int^s J ds'/y^2(s')]$, where J is an appropriate constant.

Finally, bright-dark soliton pairs are also possible. To find a solution of this sort, the normalized envelopes U and V are expressed in the following way, $U=r^{1/2}f(s)\exp(i\mu\xi)$ and $V=\rho^{1/2}g(s)\exp(i\nu\xi)$ where $f(s)$ corresponds to a bright beam envelope and $g(s)$ to a dark one. The positive variables r and ρ represent the ratios of their maximum power density with respect to the dark irradiance I_d. By substituting these forms of U and V in Eqs. (4) and (5) we find that

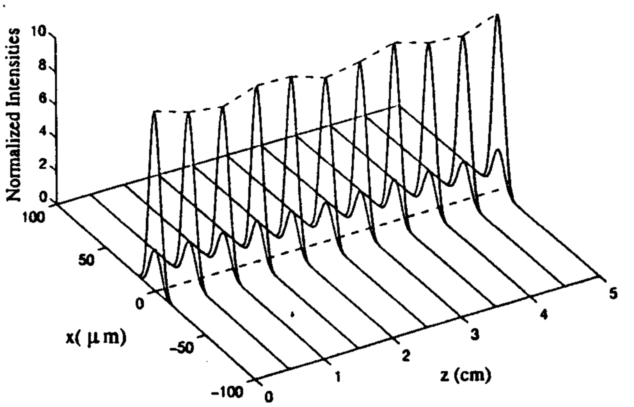

FIG. 2. Stable propagation of a ($r=10$, $\theta=30°$) bright-bright soliton pair when its larger intensity component is perturbed by 20% at the input.

$$\frac{d^2f}{ds^2}=2\left[\mu+\frac{\beta(1+\rho)}{1+rf^2+\rho g^2}\right]f, \tag{8}$$

$$\frac{d^2g}{ds^2}=2\left[\nu+\frac{\beta(1+\rho)}{1+rf^2+\rho g^2}\right]g. \tag{9}$$

We now look for particular solutions that also satisfy the condition $f^2+g^2=1$. Using appropriate boundary conditions, μ and ν can be readily found[8] and are given by $\mu=-(\beta/\delta)\ln(1+\delta)$ and $\nu=-\beta$ where $\delta\equiv(r-\rho)/(1+\rho)$. Equations (8) and (9) can then be solved perturbatively provided that $|\delta|\ll 1$, that is when the peak intensities of the two beams are approximately equal. In this case, an approximate soliton pair solution can be obtained and is given by[8]

$$U=r^{1/2}\,\text{sech}[(\beta\delta)^{1/2}s]\exp[-i\beta(1-\delta/2)\xi], \tag{10}$$

$$V=\rho^{1/2}\tanh[(\beta\delta)^{1/2}s]\exp(-i\beta\xi). \tag{11}$$

The above equations clearly show that these solutions are possible only when the product $(\beta\delta)$ is a positive quantity. The soliton pair obtained at $\rho=3$, $\delta=-0.01$ and $E_0=-1$ kV/cm in SBN is shown in Fig. 1(c). In this case, the peak intensity of the bright beam r, is slightly lower than that of the dark background intensity. The intensity FWHM of the pair components is found to be 75 μm.

The stability properties of these soliton pairs will now be discussed. In particular, their stability is investigated here using numerical techniques such as beam propagation methods. In general, we have found that bright-bright and dark-dark pairs are stable against small perturbations in amplitude and width up to distances of several centimeters. For example, let us consider a bright-bright pair obtained earlier at $r=10$ and $\theta=30°$. Figure 2 depicts the evolution of the pair components when the high intensity beam has been perturbed by 20% in its amplitude. Evidently, the pair exhibits robustness, does not break up, and instead oscillates around the soliton solution. Similarly, a stability study of the bright-dark pair reveals that they are stable only when β (or the applied bias) is negative. On the other hand, in the range of positive β's, the pair was found to be unstable as a result of modulational instability in the infinite tail regions.

In conclusion, we have found that a new kind of incoherently coupled soliton pairs is possible in photorefractive crystals under steady-state conditions. These soliton states can be established provided that their carrier beams share the same polarization, wavelength, and are mutually incoherent. Such soliton pairs can exist in bright-bright, dark-dark, as well as in bright-dark configurations. Their characteristics and stability properties were also discussed.

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