Information Sciences 220 (2013) 425–441

Contents lists available at SciVerse ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

A hybrid method of fuzzy simulation and genetic algorithm to optimize constrained inventory control systems with stochastic replenishments and fuzzy demand

Ata Allah Taleizadeh [a,b], Seyed Taghi Akhavan Niaki [c,*], Mir-Bahador Aryanezhad [d], Nima Shafii [e]

[a] *Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran*
[b] *Department of Industrial Management, Raja University, Qazvin, Iran*
[c] *Sharif University of Technology, P.O. Box 11155-9414 Azadi Ave., Teharn, Iran*
[d] *Iran University of Science and Technology, Tehran, Iran*
[e] *Faculty of Engineering of the University of Porto, Rua Dr. Roberto Frias, Porto, Portugal*

ARTICLE INFO

Article history:
Received 18 January 2009
Received in revised form 12 March 2012
Accepted 19 July 2012
Available online 7 August 2012

Keywords:
Multiproduct inventory control
Partial back-ordering
Stochastic replenishment
Integer nonlinear programming
Fuzzy simulation
Genetic algorithm

ABSTRACT

Multi-periodic inventory control problems are mainly studied by employing one of two assumptions. First, the continuous review, where depending on the inventory level, orders can happen at any time, and next the periodic review, where orders can only be placed at the beginning of each period. In this paper, we relax these assumptions and assume the times between two replenishments are independent random variables. For the problem at hand, the decision variables (the maximum inventory of several products) are of integer-type and there is a single space-constraint. While demands are treated as fuzzy numbers, a combination of back-order and lost-sales is considered for the shortages. We demonstrate the model of this problem is of an integer-nonlinear-programming type. A hybrid method of fuzzy simulation (FS) and genetic algorithm (GA) is proposed to solve this problem. The performance of the proposed method is then compared with the performance of an existing hybrid FS and simulated annealing (SA) algorithm through three numerical examples containing different numbers of products. Furthermore, the applicability of the proposed methodology along with a sensitivity analysis on its parameters is shown by numerical examples. The comparison results show that, at least for the numerical examples under consideration, the hybrid method of FS and GA shows better performance than the hybrid method of FS and SA.

1. Introduction and literature review

The continuous review and the periodic review are the main applied policies in multi-periodic inventory control models. However, the underlying assumptions of these models restrict their proper use in real-world environments. In continuous review policy, one has the freedom to act anytime and place orders based upon the available inventory level. While in the periodic review policy, the user is allowed to place orders only in specific and predetermined times.

Two of the widely employed periodic review policies are the so-called (R,T) and (R,nT) policies. In the first one, at fixed predetermined intervals, T, the inventory is reviewed and an order is placed accordingly. The order quantity is determined by

* Corresponding author. Tel.: +98 21 66165740; fax: +98 21 66022702.
E-mail addresses: Taleizadeh@iust.ac.ir, Taleizadeh@raja.ac.ir (A.A. Taleizadeh), Niaki@Sharif.edu (S.T.A. Niaki), Mirarya@iust.ac.ir (M.-B. Aryanezhad).

0020-0255/$ - see front matter
http://dx.doi.org/10.1016/j.ins.2012.07.027

subtracting the on hand inventory from a predetermined value R. If this policy is used in an n-echelon inventory system, it is called (R, nT) policy. Further, the economic order quantity (EOQ) model along with the (r, Q) policy are the two other major periodic review inventory systems, where in the former the purchaser desires to determine the optimal quantity of the order while in the latter the optimal values of the reorder point and order quantity are sought.

There is substantial research reported in the literature in the area of multi-periodic inventory control. Some of these works are summarized in Tables 1 and 2. Table 1 (except for Taleizadeh et al. [39]) shows the main research efforts in the stochastic environment dealing with (R, T) and (r, Q) systems in which demand and lead-time are considered stochastic variables. The main constraints shown in these works are service level [30,39], order quantity [3], and joint order [13,33]. Although the decision variables such as order quantity, inventory level, reorder point and period length are similar, various assumptions have been made and different models and procedures have been proposed. For example Chiang [6] and Bylka [3] considered emergency orders, Feng and Rao [14] assumed non-zero lead-time, Eynan and Kropp [13] employed variable shortage cost, Mohebbi [29] used a discrete-pattern demand, and Qu et al. [33] considered an integrated inventory transportation system.

Table 2 shows the main research efforts in the fuzzy environment performed on EOQ and economic production quantity (EPQ) models. In this category, the main constraints are budget [8,41], space [28,34,41], and service level [41]. While the decision variables are similar to the ones in the stochastic environment, the demand and inventory costs are considered fuzzy variables. In some of these research undertakings such as [19,26,34] production rate, price, and deterioration rate are considered fuzzy variables as well.

A careful observation of the works listed in Tables 1 and 2 reveals that while separate emphasis has been devoted to the stochastic nature of demand and lead-time, some real-world constraints of the systems have not been investigated simultaneously. For example, no work is reported where both demand and lead-time are probabilistic. Furthermore, some constraints have been partially studied, the decision variables have been considered integer, and constraints such as budget and space have not been investigated.

In addition, many researchers have successfully used meta-heuristic methods to solve complicated optimization problems in different fields of science and engineering. Some of these meta-heuristic algorithms are: fuzzy simulation [41,44], genetic algorithms [2], harmony search [23,17,40–43], simulating annealing [1,39,44], ant colony optimization [10], neural networks [15], threshold accepting [11], Tabu search [20], and evolutionary algorithm [22,41].

This paper first extends the periodic review works in both stochastic and fuzzy environments such that the replenishment intervals become random and demands assume a fuzzy nature. Then, it presents a hybrid algorithm to solve the problem. To be more specific, the extended model assumes a stochastic replenishment, i.e., stochastic period length, multiple products that are stored in a single capacity-constrained warehouse, fuzzy customer demand, and integer decision variables. Since the time between two replenishments includes the time required to order, the time needed to provide or produce items and the transportation time, the stochastic replenishment assumption is closer to reality than the usual assumption of a deterministic period length. Furthermore, there are many situations in practice where the customer demand is fuzzy, especially in manufacturing where due to the machine breakdowns, shortage of raw materials, and fluctuating rate of nonconforming production, and so on, the demand for on hand inventories of parts and subassemblies can be considered fuzzy. The hybrid algorithm consists of a GA for inventory control optimization and a method for fuzzy simulation to evaluate different solutions in the genetic optimization process.

The models developed in this research are useful for companies and manufacturers who are faced with uncertain demands that do not follow a stochastic pattern. In other words, manufacturers who are unable to assume certain probability distributions for the uncertain demands of their products can use fuzzy set theory to find suitable patterns. Additionally, the proposed method is beneficial in situations where due to some limitations on the production capacity, the supply of the raw material, and the like, the period length may be uncertain and the goods may not be delivered on time. As an example, when demand increases and production capacity is limited, in case of breakdowns or late receipts of imported raw materials (when delayed at customs) the lead-time and hence the cycle length increases. Another example involves sale representatives that randomly visit retailers offering them product replenishments. The stochastic nature of these factors causes the period length to be stochastic.

The rest of this paper is organized as follows. In Section 2, the problem along with its assumptions is defined. The problem formulation comes in Section 3 after the parameters and the variables are defined. In this section, the single product problem is first modeled and then it is extended to a multiproduct formulation. In the fourth section of the paper, a hybrid algorithm is proposed to solve and analyze the problem at hand under special conditions. By incorporating a numerical example, the solution method is investigated in Section 5. Section 6 contains a sensitivity analysis, and finally the conclusion and recommendations for future research come in Section 7.

2. Problem definition and modeling

Consider a periodic-review inventory control model for one provider in which the period lengths are stochastic in nature, i.e., the times between two replenishments are independent random variables following either Uniform or Exponential probability distribution. Triangular fuzzy variables are used to model the demands of several products, and the partial back-ordering policy is employed for shortages, i.e., a fraction of unsatisfied demands is lost and the rest is back-ordered.

Table 1
Literature review in stochastic environment.

Author	Periodic review	Continuous review	Multi products	Constraint	Discount	Fuzzy environment	Stochastic environment	Partial back-ordering	Lost sale or back-order	Solution method	Decision variable	Other considerations
Chiang [6]	(R,T)				*		Demand		B	Dynamic programming	Inventory level	Emergency order
Mohebbi and Posner [30]		(r,Q)					Demand and lead time		L	Heuristic	Order quantity	Multiple replenishment
Ouyang and Chang [32]	(R,T)			Service level			Demand			Heuristic	Review period	
Feng and Rao [14]	(R,nT)	(r,Q)					Demand		B	Heuristic	Order quantity, reorder point, inventory level	Non-zero lead time
Bylka [3]	(R,T)			Order and space			Demand	*		Heuristic and Markov process	Regular and emergency order quantity	Emergency order and two suppliers
Chiang et al. [7]	(R,T)				*		Demand		B and L	Dynamic programming	Inventory level and order quantity	Two models with back ordering and lost sale
Eynan and Kropp [13]	(EOQ)		*	Joint order			Demand		B	Heuristic	Period length	Variable shortage cost
Mohebbi [29]		(r,Q)					Demand		L	Software (MATLAB)	Reorder point and order quantity	Discrete demand and unreliable supplier
Qu et al. [33]	(R,T)		*	Joint order					B	Heuristic	Rout, order quantity and period length	Integrated inventory-transportation system
Taleizadeh et al. [39]	(R,T)		*	Service level and space			Period length	*		Simulated annealing	Inventory level	

Table 2
Literature review in fuzzy environment.

Author	Periodic review	Continuous review	Multiproduct	Constraint	Fuzzy environment	Lost sale or back-order	Solution method	Decision variable	Other considerations
Chang et al. [4]		(r,Q)			Demand	B and L	Difuzzification and heuristic	Lead time and order quantity	Variable lead-time
Das et al. [8]	(EOQ)		*	Budget and space	Demand	B	Difuzzification and heuristic	Order and shortage quantities	Time varying demand and production rate
Hsieh [19]	(EPQ)				Demand, production rate, inventory costs		Heuristic and Lagrangian	Production quantity	All of the parameters are fuzzy
Liu [25]	(EOQ)			Batch order	Demand, inventory costs		Possibility theory and geometric programming	Order quantity	
Maiti and Maiti [26]		(r,Q)	*		Demand and price		Fuzzy simulation and genetic algorithm	Reorder point, order quantity, selling price, frequency of advertisements	Two storages, advertisement, single and multi objective
Mandal and Roy [28]	(EOQ)		*	Space	Constraint goal, inventory costs		Fuzzy geometric programming	Order quantity	Multi objective
Roy et al. [34]		(r,Q)		Space	Deterioration rate		Fuzzy simulation and genetic algorithm	Order quantity	Stochastic period length, two storage facilities, time varying demand
Taleizadeh et al. [41]	(R,T)		*	Budget, space and service level	Inventory costs		Fuzzy simulation and genetic algorithm	Inventory level	Partial back ordering and incremental discount, stochastic period length
Yao et al. [46]	(EOQ)				Demand		Heuristic	Order quantity	
Chen et al. [5]	(EOQ)				Demand, inventory costs	B	Function principle	Order and shortage quantities	

Moreover, when demands are higher than replenishment levels, the back-ordered quantities in the previous cycle are carried out to the next cycle. In situations in which demands are lower than replenishment levels, extra inventories are carried out to the following cycle and are assumed to have an insignificant impact on the independence of the two periods.

Assuming all the produced items are sold, the costs associated with the inventory control system consists of holding, back-order, lost-sale, and purchase costs. Furthermore, the warehouse space is considered a constraint and the decision variables are integer. We need to identify the maximum inventory levels in each cycle such that the expected profit is maximized.

For the problem at hand, since the times between two replenishments are independent random variables, in order to maximize the expected profit over the planning horizon one needs to consider only one period. Moreover, since the costs associated with the inventory control system are holding and shortage (back-order and lost-sale), we need to calculate the expected inventory level and the expected required storage space in each period. Before doing this, parameters and variables of the model are defined based on the works of Taleizadeh et al. [39,41,42]).

2.1. Defining parameters and variables of the model

For $i = 1,2,\ldots,n$, define the parameters and the variables of the model as

R_i	Maximum inventory level of the ith product
T_i	A random variable denoting the time between two replenishments (cycle length) of the ith product
h_i	Holding cost per unit inventory of the ith product in each period
π_i	Back-order cost per unit demand of the ith product
W_i	Purchasing cost per unit of the ith product
P_i	Selling price per unit of the ith product
D_i	Constant demand rate of the ith product
β_i	Percentage of unsatisfied demands of the ith product that is back-ordered
I_i	Expected ith product inventory multiplied by the cycle time
L_i	Expected ith product lost-sale in each cycle
B_i	Expected ith product back-order in each cycle
Q_i	Expected order size of the ith product in each cycle
f_i	Required warehouse space per unit of the ith product
F	Total available warehouse space
C_h	Expected holding cost per cycle
C_b	Expected shortage cost in back-order state
C_l	Expected shortage cost in lost-sale state
C_p	Expected purchase cost
r	Expected revenue obtained from sales
Z	Expected profit obtained in each cycle

The pictorial representation of the single-product problem is given in Section 2.2. In Section 2.3, we first consider a single-product problem, and then, the formulation to the multi-product modeling is extended in Section 2.4.

2.2. Inventory diagram

According to Ertogal and Rahim [12] and considering the times between replenishments stochastic variables, two cases may occur. In the first case the time between replenishments is less than the time required for the inventory level to reach zero (see Fig. 1), and in the second, it is greater (see Fig. 2) [39,41,42]. Fig. 3 depicts the shortages in both cases. In the above figures, t_{D_i} denotes the time at which the inventory of the ith product reaches zero.

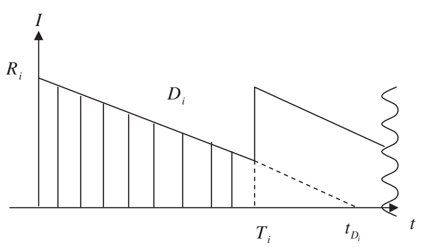

Fig. 1. Presenting the inventory cycle when $T_{Min} \leqslant T \leqslant t_{D_i}$.

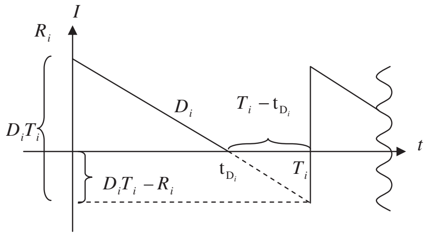

Fig. 2. Presenting the inventory cycle when $t_D < T \leqslant T_{Max_i}$.

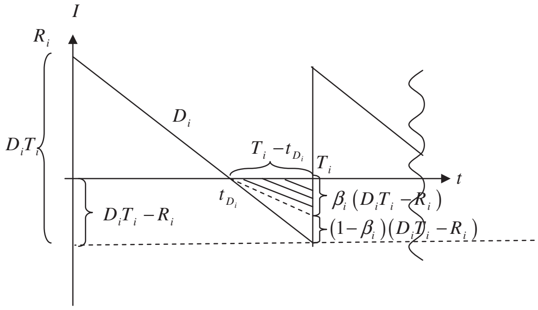

Fig. 3. Presenting shortages in two cases of compact back order and lost sales.

2.3. Single product model with constant demand

In this section, we first model a single-product inventory problem with constant demand where stochastic replenishments, back-orders, and lost-sales are allowed. Then, the model is extended in Section 2.4 to contain several products with fuzzy demands.

2.3.1. Calculating the costs and the profit

In order to calculate the expected profit in each cycle, we need to evaluate all the terms in the following equation [12]:

$$Z = r - C_p - C_h - C_b - C_l = PQ - WQ - hI - \pi B - (P - W)L \tag{1}$$

Based on Fig. 3, L, B, I, and Q are evaluated by the following equations [39,41,42]:

$$L = (1 - \beta)\int_{t_D}^{T_{\text{Max}}} (DT - R) f_T(t)dt; \quad t_D < T \leqslant T_{\text{Max}} \tag{2}$$

$$B = \beta \int_{t_D}^{T_{\text{Max}}} (DT - R) f_T(t)dt; \quad t_D < T \leqslant T_{\text{Max}} \tag{3}$$

$$I = \int_{T_{\text{Min}}}^{t_D} \left(RT - \frac{DT^2}{2}\right) f_T(t)dt + \int_{t_D}^{T_{\text{Max}}} \frac{R^2}{2D} f_T(t)dt \tag{4}$$

$$Q = \int_{T_{\text{Min}}}^{t_D} DTf_T(t)dt + \int_{t_D}^{T_{\text{Max}}} (R + \beta(DT - R)) f_T(t)dt \tag{5}$$

2.3.2. Presenting the constraints

Since the total available warehouse space is F, the space required for each unit of product is f, and the upper limit for inventory is R, the space constraint will be

$$fR \leqslant F \tag{6}$$

In short, the complete mathematical model of the single-product inventory control problem with stochastic replenishments, back-orders, lost-sales, and constant demand is

$$
\begin{aligned}
\text{Max} \quad & Z=(P-W)\left[\int_{T_{\text{Min}}}^{\frac{R}{D}}(DT)f_T(t)dt+\int_{\frac{R}{D}}^{T_{\text{Max}}}(R+\beta(DT-R))f_T(t)dt\right] \\
& \quad -h\left[\int_{T_{\text{Min}}}^{\frac{R}{D}}\left(RT-\frac{DT^2}{2}\right)f_T(t)dt+\int_{\frac{R}{D}}^{T_{\text{Max}}}\frac{R^2}{2D}f_T(t)dt\right] \\
& \quad -\pi\beta\left[\int_{\frac{R}{D}}^{T_{\text{Max}}}(DT-R)f_T(t)dt\right]-(P-W)(1-\beta)\left[\int_{\frac{R}{D}}^{T_{\text{Max}}}(DT-R)f_T(t)dt\right] \\
\text{s.t.:} \quad & fR\leqslant F \\
& R\geqslant 0,\ \text{and Integer}
\end{aligned} \tag{7}
$$

2.4. Multiproduct model with fuzzy demand

In the extending phase of the single-product model of Section 2.3 to the multiple product formulation of this section, the demands are assumed fuzzy and in addition to the two cases of back-order and lost-sales, their combination is considered as well.

Let $\widetilde{D}_i$ denote the fuzzy demand for the ith product. Then, an extension of (7) to include n products easily results in the multiproduct model as

$$
\begin{aligned}
\text{Max} \quad & Z(R_i,\widetilde{D}_i)=\sum_{i=1}^{n}[(P_i-W_i)Q_i-h_iI_i-\pi_iB_i-(P_i-W_i)L_i] \\
& =\sum_{i=1}^{n}\left\{(P_i-W_i)\left[\int_{T_{\text{Min}_i}}^{\frac{R_i}{\widetilde{D}_i}}(\widetilde{D}_iT_i)f_{T_i}(t_i)dt+\int_{\frac{R_i}{\widetilde{D}_i}}^{T_{\text{Max}_i}}(R_i+\beta_i(\widetilde{D}_iT_i-R_i))f_{T_i}(t_i)dt_i\right]\right\} \\
& \quad -\sum_{i=1}^{n}h_i\left[\int_{T_{\text{Min}_i}}^{\frac{R_i}{\widetilde{D}_i}}\left(R_iT_i-\frac{\widetilde{D}_iT_i^2}{2}\right)f_{T_i}(t_i)dt_i+\int_{\frac{R_i}{\widetilde{D}_i}}^{T_{\text{Max}_i}}\frac{R_i^2}{2\widetilde{D}_i}f_{T_i}(t_i)dt_i\right] \\
& \quad -\sum_{i=1}^{n}\pi_i\beta_i\left[\int_{\frac{R_i}{\widetilde{D}_i}}^{T_{\text{Max}_i}}(\widetilde{D}_iT_i-R_i)f_{T_i}(t_i)dt_i\right]-\sum_{i=1}^{n}(P_i-W_i)(1-\beta_i)\left[\int_{\frac{R_i}{\widetilde{D}_i}}^{T_{\text{Max}_i}}(\widetilde{D}_iT_i-R_i)f_{T_i}(t_i)dt_i\right] \\
\text{s.t.:} \quad & \sum_{i=1}^{n}f_iR_i\leqslant F \\
& R_i\geqslant 0,\ \text{and Integer}\quad \forall i=1,2,\ldots,n
\end{aligned} \tag{8}
$$

In what follows, two probability density functions for T_i are assumed and hence two models are developed. In the first model, T_i follows a uniform distribution, where the demands may occur in a finite and specific range (within an upper and a lower bound). In the second model, T_i follows an exponential distribution, where the demands may increase sharply. This model is suitable for seasonal or new products.

2.4.1. T_i follows a uniform distribution

In this case T_i follows a uniform distribution in the interval $[T_{\text{Min}_i},T_{\text{Max}_i}]$, i.e., $T_i\sim U[T_{\text{Min}_i},T_{\text{Max}_i}]$ and $f_{T_i}(t_i)=\frac{1}{T_{\text{Max}_i}-T_{\text{Min}_i}}$. Accordingly, (8) is changed to

$$
\begin{aligned}
\text{Max} \quad & Z(R_i,\widetilde{D}_i)=\sum_{i=1}^{n}\left[\frac{h_i}{6\widetilde{D}_i^2(T_{\text{Max}_i}-T_{\text{Min}_i})}\right]R_i^3-\sum_{i=1}^{n}\left[\frac{2(P_i-W_i)(1-\beta_i)+\pi_i\beta_i+h_iT_{\text{Max}_i}}{2\widetilde{D}_i(T_{\text{Max}_i}-T_{\text{Min}_i})}\right]R_i^2 \\
& \quad +\sum_{i=1}^{n}\left[\frac{4(P_i-W_i)(1-\beta_i)T_{\text{Max}_i}+h_iT_{\text{Min}_i}^2+2\pi_i\beta_iT_{\text{Max}_i}}{2(T_{\text{Max}_i}-T_{\text{Min}_i})}\right]R_i \\
& \quad +\sum_{i=1}^{n}\left[\frac{-h_iT_{\text{Min}_i}^3\widetilde{D}_i+3(P_i-W_i)(\beta_iT_{\text{Max}_i}^2-T_{\text{Min}_i}^2)\widetilde{D}_i-3T_{\text{Max}_i}^2\widetilde{D}_i(\pi_i\beta_i+(P_i-W_i)(1-\beta_i))}{6(T_{\text{Max}_i}-T_{\text{Min}_i})}\right] \\
\text{s.t.:} \quad & \sum_{i=1}^{n}f_iR_i\leqslant F \\
& R_i\geqslant 0,\ \text{Integer},\quad \forall i=1,2,\ldots,n
\end{aligned} \tag{9}
$$

2.4.2. T_i follows an exponential distribution

If T_i follows an exponential distribution with parameter λ_i, then the probability density function of T_i is $f_{T_i}(t_i) = \lambda_i e^{-\lambda_i T_i}$. In this case, the model is derived as

$$\begin{aligned}
&\text{Max} \quad Z(R_i, \widetilde{D}_i) = \sum_{i=1}^{n} \left\{ \frac{1}{\lambda_i}[2\widetilde{D}_i(1-\beta_i)(W_i - P_i) - \pi_i\beta_i\widetilde{D}_i]e^{-\left(\frac{R_i}{\widetilde{D}_i}\right)\lambda_i} + \frac{1}{\lambda_i}[\widetilde{D}_i(P_i - W_i) - h_iR_i] + \frac{h_i\widetilde{D}_i}{\lambda_i^2}\left(1 - e^{-\left(\frac{R_i}{\widetilde{D}_i}\right)\lambda_i}\right) \right\} \\
&\text{s.t.:} \quad \sum_{i=1}^{n} f_iR_i \leqslant F \qquad (10) \\
&R_i \geqslant 0, \ \text{and Integer} \quad \forall i = 1, 2, \ldots, n
\end{aligned}$$

In the next section, a hybrid intelligent algorithm is introduced to find near optimum solutions of the formulated problems in (9) and (10).

3. A hybrid intelligent algorithm

Since analytical solutions (if any) of the integer-nonlinear models in (9) and (10) are hard to obtain [16], a hybrid intelligent algorithm of fuzzy simulation and genetic algorithm is developed in this section. Some related research that have employed the fuzzy simulation approach along with other meta-heuristic algorithms include [20,37,41,44]. In the next subsection, a brief background in fuzzy simulation is given.

3.1. Some definitions in fuzzy environment

In this paper, we adopt the concepts of the credibility theory including possibility, necessity, credibility of fuzzy events, and the expected value of a fuzzy variable as defined in [21,24,47–51].

Definition 1. Let ξ be a fuzzy variable with the membership function $\mu(x)$. Then the possibility, necessity, and credibility measures of the fuzzy event $\xi \geqslant r$ can be represented, respectively, by

$$Pos\{\xi \geqslant r\} = \sup_{u \geqslant r} \mu(u) \qquad (11)$$

$$Nec\{\xi \geqslant r\} = 1 - \sup_{u < r} \mu(u) \qquad (12)$$

$$Cr\{\xi \geqslant r\} = \frac{1}{2}[Pos\{\xi \geqslant r\} + Nec\{\xi \geqslant r\}] \qquad (13)$$

Definition 2. The expected value of a fuzzy variable is defined as

$$E[\xi] = \int_0^{\infty} Cr\{\xi \geqslant r\}dr - \int_{-\infty}^{0} Cr\{\xi \leqslant r\}dr \qquad (14)$$

Definition 3. The optimistic function of α is defined as

$$\xi_{\sup}(\alpha) = \sup[r | Cr\{\xi \geqslant r\} \geqslant \alpha], \quad \alpha \in (0, 1] \qquad (15)$$

Definition 4. If $\tilde{\xi} = (a, b, c)$ is a triangular fuzzy number with center b, left width $a > 0$, and right width $c > 0$, then its membership function has the following form

$$\mu(r) = \begin{cases} \frac{r-(b-a)}{a}; & b - a \leqslant r \leqslant b \\ \frac{(b+c)-r}{c}; & b \leqslant r \leqslant b + c \\ 0: & \text{elsewhere} \end{cases} \qquad (16)$$

Definition 5. For the fuzzy variable described in Definition 4, the credibility of the event $Cr\{\xi \leqslant r\}$ is defined based on the definition in (13) as

$$\mu(r)=\begin{cases}0; & r\leqslant b-a\\ \frac{r-(b-a)}{2a}; & b-a\leqslant r\leqslant b\\ \frac{r-(b-c)}{c}; & b\leqslant r\leqslant b+c\\ 1; & \text{elsewhere}\end{cases} \tag{17}$$

In this research, the triangular fuzzy variable is used to model the fuzzy demand.

3.2. Fuzzy simulation

A fuzzy simulation technique is employed to estimate the fuzzy demands. Denoting $\tilde{\xi}_i$ by $\widetilde{D}_i = (\widetilde{D}_1, \widetilde{D}_2, \ldots, \widetilde{D}_n)$, μ as the membership function of $\widetilde{D}$, and μ_i as the membership functions of $\widetilde{D}_i$, we randomly generate D_i^k from the α-level sets of fuzzy variables $\widetilde{D}_i$, $i = 1, 2, \ldots, n$ and $k = 1, 2, \ldots, K$ as $D^k = \left(D_1^k, D_2^k, \cdots, D_n^k\right)$ and $\mu(D^k) = \mu_1(D_1^k)\wedge\mu_2(D_2^k)\wedge,\ldots,\wedge\mu_n(D_n^k)$, where α is a sufficiently small positive number. Then, the expected value of the fuzzy variable is

$$E[Z(R,\widetilde{D})] = \int_0^{+\infty} Cr\{Z(R,\widetilde{D}) \geqslant r\}dr - \int_{-\infty}^{0} Cr\{Z(R,\widetilde{D}) \leqslant r\}dr \tag{18}$$

Provided that O is sufficiently large, for any number $r \geqslant 0$, $Cr\{Z(R,D_k) \geqslant r\}$ can be estimated by

$$Cr\{Z(R,D_k) \geqslant r\} = \frac{1}{2}\left(\underset{k=1,2,\ldots,O}{\text{Max}}\{\mu_k|Z(R,D_k) \geqslant r\} + 1 - \underset{k=1,2,\ldots,O}{\text{Max}}\{\mu_k|Z(R,D_k) < r\}\right) \tag{19}$$

And for any number $r < 0$, $Cr\{Z(R, D_k) \leqslant r\}$ can be estimated by

$$Cr\{Z(R,D_k) \leqslant r\} = \frac{1}{2}\left(\underset{k=1,2,\ldots,O}{\text{Max}}\{\mu_k|Z(R,D_k) \leqslant r\} + 1 - \underset{k=1,2,\ldots,O}{\text{Max}}\{\mu_k|Z(R,D_k) > r\}\right) \tag{20}$$

The procedure of estimating $Z(R,\widetilde{D})$ in (19) and (20) is shown in the following algorithm:

Algorithm 1. Estimating $Z(R,\widetilde{D})$

1.	Set $E = 0$ and initialize K and O.
2.	Randomly generate D_i^k from α -level sets of fuzzy variables $\widetilde{D}_i$, and set $D^k = (D_1^k, D_2^k, \cdots, D_n^k)$
3.	Set $a = Z(R, D_1) \wedge Z(R, D_2) \wedge \cdots \wedge Z(R, D_O)$, $b = Z(R,D_1) \vee Z(R,D_2) \vee \cdots \vee Z(R,D_O)$.
4.	Randomly generate r from Uniform $[a, b]$.
5.	If $r \geqslant 0$, then $E \leftarrow E + Cr\{Z(R,\widetilde{D}) \geq r\}$. Otherwise, $E \leftarrow E - Cr\{Z(R,\widetilde{D}) \leq r\}$.
6.	Repeat 4 and 5 for O times.
7.	Calculate $E(Z(R,\widetilde{D})) = a \vee 0 + b \wedge 0 + E \times \frac{b-a}{O}$.

3.3. Genetic algorithm

In the usual form of genetic algorithm (GA), described by Goldberg [18], the best solution is the winner of the genetic game and any potential solution is assumed a creature determined by different parameters. Several authors have employed GA to solve complicated inventory control problems. A selection of these works is demonstrated in Table 3.

In what follows the main characteristics of the genetic algorithm employed in this research are described.

3.3.1. Chromosomes

A chromosome, an important part of GA, is a string or trail of genes that is considered the coded figure of a possible solution (proper or improper). In this paper, the chromosomes are strings of the maximum inventory levels of the products (R_j) that are integers [39,41,42]. Therefore, integer numbers are randomly generated in the closed interval [0, 1000] to represent the genes. Moreover, infeasible chromosomes, the ones that do not satisfy the constraints of the models in (9) and (10), are not considered. For an 8-product system, the chromosome structure is given in Fig. 4.

3.3.2. Population

Each population or generation of chromosomes has the same size that is known as the population size denoted by N. Similar to Taleizadeh et al. [39,41,42], 50, 100, and 150 are chosen as different population sizes of the GA algorithm of the current research.

3.3.3. Crossover

In a crossover operation, mating pairs of chromosomes creates offspring. Crossover operates on the parents' chromosomes with the probability of P_c. If no crossover occurs, the offspring's chromosomes will be the same as their parents'

Table 3
Literature review of GA applications in inventory control.

Author	Area	Variables	Gene represents	Initialization	Mutation	Crossover	Stopping criteria	Hybrid by	Other considerations
Roy et al. [34]	Integrated production inventory system	Cycle length, maximum inventory level	Product type	Random generation	Randomly by using mutation probability	Single point	Maximum iteration number	Fuzzy logic	Fuzzy genetic algorithm is proposed
Suer et al. [38]	Capacitated lot size problem	Production quantity, human resource requirement, etc.	Product type	Random generation	Randomly by using mutation probability	Multiple chromosome	Maximum number of generation	–	Multiple chromosome crossover is proposed
Shahabudeen and Sivakumar [36]	Kanban system	Number of Kanban and extra cards	Number of Kanban and extra cards	Random generation	Order based shift mutation	Single point	Maximum number of generation	–	GA compared by simulated annealing and performed better
Roy et al. [35]	Inventory model with deterioration items	Order quantity, cycle length	Product type	Random generation	Randomly by using mutation probability	Single point	Maximum number of generation	Fuzzy simulation	Necessity and possibility theories are considered
Nachiapan and Jawahar [31]	Vendor managed inventory	Sales quantity of each buyer	Sales quantity of buyers	Random generation	Randomly by using mutation probability	Single point	Maximum number of generation	–	GA based heuristic is proposed
Taleizadeh et al. [42]	Inventory model with random period length	Maximum inventory level	Product type	Random generation	Randomly by using mutation probability	Two point crossover	Maximum number of generation	Pareto and TOPSIS selections	TOPSIS is used to rank the Pareto
Taleizadeh et al. [43]	Newsboy inventory system	Order quantity	Product type	Random generation	Randomly by using mutation probability	Two point crossover	Maximum number of generation	Goal programming	GA solved a multi objectives problem

R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8

Fig. 4. The structure of a chromosome.

R_j	120	78	130	105	78	140	86	90
R_j	105	75	130	100	66	140	91	84

M=6

R_j	120	78	130	105	78	140	91	84
R_j	105	75	130	100	66	140	86	90

Fig. 5. The single-point crossover operation.

[45]. Fig. 5 depicts a single-point crossover operation in which R_j shows the chromosome containing the maximum inventory levels of the products and the break point is chosen at $M = 6$ [39,41,42].

In this research, a single point crossover with different probabilities (P_c) of 0.6, 0.7, and 0.8 is utilized. Note that infeasible chromosomes do not move to the new population.

3.3.4. Mutation

Mutation is the second operation in GA to explore new solutions by operating on each chromosome resulted from the crossover operation, where genes are replaced with randomly selected numbers within the boundaries of the parameter [16,27]. To do this, a random number *RN* between (0, 1) is generated for each gene. If *RN* is less than a predetermined mutation probability P_m, then the mutation occurs in the gene. Otherwise, it does not. The usual value of P_m is 0.1 over the numbers of genes in a chromosome. In this research, 0.010, 0.015, and 0.020 are chosen as different values of P_m. Note that infeasible chromosomes resulted by this operation do not move to the new population [39,41,42]. Fig. 6 depicts a mutation operation in which P_m is chosen to be 0.01 [39,41,42].

3.3.5. Objective function evaluation

In a maximization problem, the more adequate the solution, the greater the objective function (fitness value) will be. Therefore, the fittest chromosomes will take part in offspring generation with a larger probability. The fuzzy simulation of Section 3.2 is used to evaluate the objective function of this research [39,41,42].

3.3.6. Selection

Selection plays a central role in GAs by determining how individuals compete for survival. Selection weeds out the bad solutions and keeps the good ones. This can be performed by proportional fitness selection that assigns a selection probability in proportion to the fitness of the given individual. The tournament selection is the most commonly used method, in which a number of randomly picked individuals are compared to each other [9]. The fittest individual is then selected to be a part of the next generation. The tournament size determines how many individuals are to be compared per selected population. Because of the randomness of the selection method, most techniques, including traditional recombination and mutation operators, cannot guarantee the survival of the current best solution. In this research, Elitism is used to provide guarantee by explicitly selecting the best individual or group of individuals. The implementation of these two techniques leads to duplicates of good individuals [9].

In this paper, we move the five best solutions to the next generation as elites. After each generation, solutions are checked for feasibility in terms of satisfying the constraint. If the constraint is satisfied, the corresponding chromosome will immigrate to the next population, otherwise the solution will be removed, and the generation will continue until a sufficient number of chromosomes are produced.

R_j	120	78	130	105	78	140	86	90
RN	0.573	0.003	0.008	0.284	0.743	0.972	0.007	0.824

R_j	120	75	140	105	78	140	80	90

Fig. 6. A sample of mutation operation.

Table 4
General data.

Product	1	2	3	4	5	6	7	8
h_i	2	2	2	2	2	2	2	2
π_i	5	5	5	5	5	5	5	5
P_i	100	100	100	100	150	150	150	150
β_i	0.5	0.9	0.9	0.5	0.5	0.9	0.9	0.5
f_i	3	3	3	3	6	6	6	6
W_i	70	70	70	70	70	70	70	70
D_i	(7,10,13)	(7,10,13)	(7,10,13)	(7,10,13)	(18,20,22)	(18,20,22)	(18,20,22)	(18,20,22)

3.3.7. Stopping criterion

The last step in a GA is to check whether the algorithm has found a solution that is good enough to meet the user's expectations. Stopping criterion is a set of conditions such that when satisfied, hints at a good solution.

In this research, since the population sizes of 50, 100, and 150 are used, it is better to stop the algorithm until a maximum number of 500 evaluations ($MN = 500$) are performed [39,41,42].

In short, the steps involved in the GA algorithm used in this research are

1. Set the parameters P_c, P_m, and N initialize the population randomly (the individuals should satisfy the constraints)
2. Evaluate the objective function for all chromosomes based on Flowchart (1)
3. Select an individual for mating pool by tournament selection method using elitisms
4. Apply crossover to each pair of chromosomes with probability P_c
5. Apply mutation to each chromosome with probability P_m
6. Replace the current population by the resulting new feasible population (before replacing the old population, the feasibility of the newly generated chromosomes is checked and reproduction will continue until a sufficient number of required chromosomes is obtained)
7. Evaluate the objective function
8. If the stopping criterion is met, stop. Otherwise, go to step 4.

In order to demonstrate and evaluate the performance of the proposed hybrid intelligent algorithm, in the next section we present three numerical examples that were originally used in Ertogal and Rahim [12]. In these examples, two cases of the uniform and the exponential distributions for the time between two replenishments are investigated. To validate the results obtained, an existing hybrid method of FS and SA [44] is employed as well.

4. Numerical examples

Consider two multiproduct inventory control problems with different numbers of products. The first one has eight products, where its general data is given in Table 4. Tables 5 and 6 show the parameters of the uniform and the exponential distributions used for the times between two replenishments, respectively. The total available warehouse space is 4800, and Table 7 shows different values of the parameters of the GA method. In this research all different combinations of the parameters of GA (P_c, P_m and N) given in Table 7 are employed and using the *Max* (*Max*) criterion the best combination of the parameters has been selected. Moreover, $K = 15$ and $O = 100$ are considered in the fuzzy simulation procedure. All runs are performed using MATLAB on a Pentium4 computer with 2.2 GHZ coreduo2 processor.

In order to show the effectiveness of the proposed hybrid method of FS and GA in solving the complicated inventory problem of this research, Taleizadeh et al.'s [44] hybrid method of FS + SA is also employed to solve the numerical examples.

Tables 8 and 9 show the best results of the two approaches. The best combinations of the GA algorithms are shown in Table 10. Furthermore, the convergence paths of the objective-function values of the FS + GA and FS + SA algorithms for uniform and exponential distributions are shown in Figs. 7–10.

The results in Tables 8 and 9 show that the hybrid FS + GA method provides a better near-optimal solution in terms of the objective-function value. Moreover, from Figs. 7 to 10, one can observe that more generations and iterations are required to reach the best result in the case of uniform compared to exponential distribution.

In the first numerical example, to compare the performances of the two hybrid methods, while the number of runs in each example is set at 25 for both methods; the sample means of the CPU times in reaching the best solution in exponential

Table 5
Data for uniform distribution.

Product	1	2	3	4	5	6	7	8
T_{Min_i}	20	20	50	50	20	20	50	50
T_{Max_i}	40	40	70	70	40	40	70	70

Table 6
Data for exponential distribution.

Product	1	2	3	4	5	6	7	8
λ_i	1/30	1/30	1/60	1/60	1/30	1/30	1/60	1/60

Table 7
The parameters of the GA method.

P_c	P_m	N
0.6	0.010	50
0.7	0.015	100
0.9	0.020	150

Table 8
The best result for R_i by FS + GA algorithm.

Distribution	Product								$Z(R,\widetilde{D})$
	1	2	3	4	5	6	7	8	
Uniform	53	70	84	56	13	88	236	291	39,400
Exponential	67	32	11	105	299	14	23	379	168,820

Table 9
The best result for R_i by FS + SA algorithm.

Distribution	Product								$Z(R,\widetilde{D})$
	1	2	3	4	5	6	7	8	
Uniform	188	3	41	109	197	51	93	268	36,366
Exponential	54	10	129	10	245	18	75	357	151,550

Table 10
The best combination of the GA parameters.

Numerical example with	P_c	P_m	N
Uniform distribution	0.6	0.01	100
Exponential distribution	0.6	0.015	100

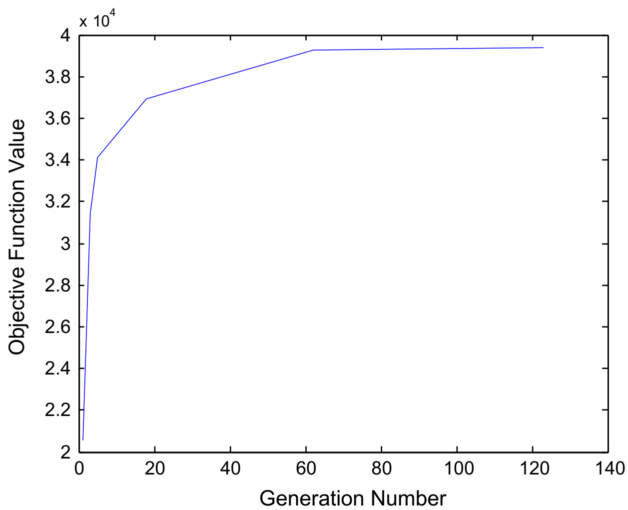

Fig. 7. The convergence paths of the best result by FS + GA in uniform example.

distribution case are 9.35 and 9.59 s for FS + GA and FS + SA methods, respectively. The corresponding sample variances are 0.25 and 0.27. In the uniform case however, the sample means are 10.59 and 11.31 with the sample variances of 0.62 and 0.65, respectively. This shows that the proposed hybrid method has better performance in terms of the CPU time to reach the best result in both distributional cases.

Similar results are obtained for the next two numerical examples containing 20 and 40 products. The summarized CPU sample means in Table 11 show that, as expected, as the number of products increases, the required CPU time to reach the best solution increases as well. Further, the proposed hybrid FS + GA provides better results in terms of the objective-function value and CPU time for both uniform and exponential cases of the two different problem sizes.

5. A sensitivity analysis

To study the effects of parameter changes on the best result obtained by the proposed method and the required CPU time, a sensitivity analysis is performed to investigate the effect of increase or decrease of the parameters, one at a time, by 20% and 40%. The parameters of the proposed method are the fuzzy demand (D_i), the parameters of the distribution of the period length (T_{Max_i} and λ_i), the crossover probability (P_c), the mutation probability (P_m), the parameters of the fuzzy simulation algorithm (K and O), the number of products (NP), and the maximum number of the population sizes (MN). Table 12 shows

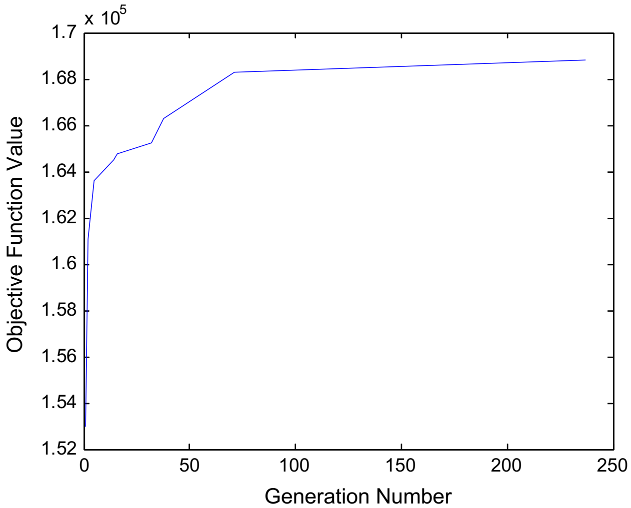

Fig. 8. The convergence paths of the best result by FS + SA in exponential example.

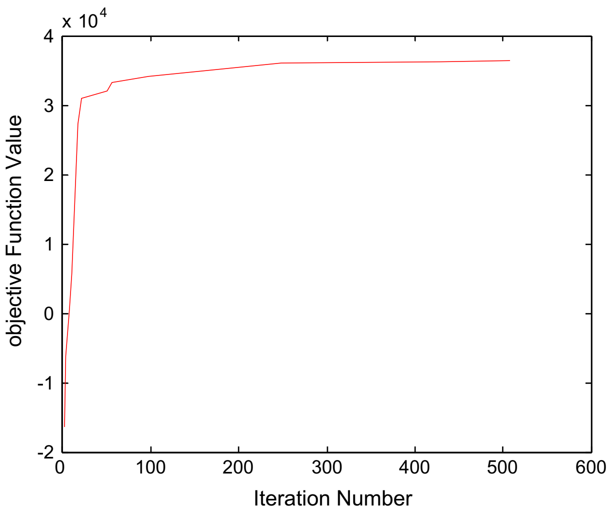

Fig. 9. The convergence path of the best result by FS + SA in the uniform example.

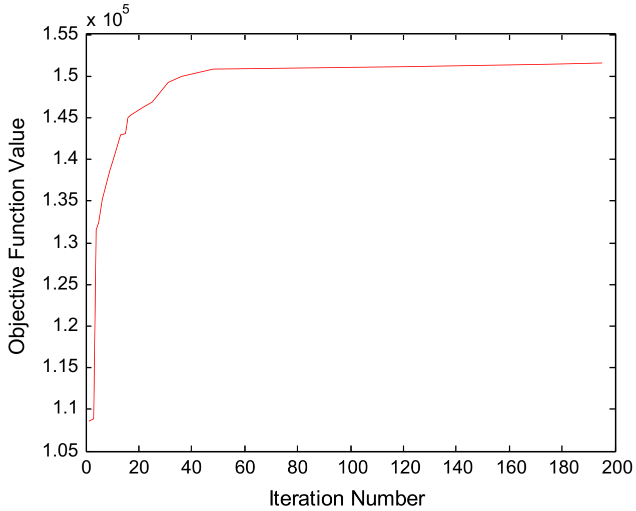

Fig. 10. The convergence path of the best result by FS + SA in the exponential example.

Table 11
The summarized results of the second and the third numerical examples.

Example	Uniform				Exponential			
	FS + GA		FS + SA		FS + GA		FS + SA	
	Objective function value	CPU time (s)	Objective function value	CPU time (s)	Objective function value	CPU time (s)	Objective function value	CPU time (s)
Second example (20 products)	106,450	31.24	93,320	34.36	373,250	28.18	332,780	29.76
Third example (40 products)	218,790	64.38	181,980	69.76	790,040	59.16	735,620	62.39

Table 12
The effects of the parameter changes on the objective-function value.

% Changes in parameters	% Changes in	
	Uniform distribution Objective function	Exponential distribution Objective function
D_i		
+40	+34.38	+29.49
+20	+19.45	+16.82
−20	−17.46	−21.37
−40	−37.39	−34.79
T_{Max_i}		
+40	+21.45	–
+20	+13.67	–
−20	−11.4	–
−40	−19.49	–
λ_i		
+40	–	+17.45
+20	–	+9.56
−20	–	−8.57
−40	–	−16.49

the results of the sensitivity analysis on the sample mean of the 25 best results obtained for the uniform and exponential distribution cases. The results in Table 12 show that there is a direct relationship between the objective-function value and the changes in D_i, T_{Max_i} and λ_i, that is, increase or decrease of these parameters cause the objective function value to increase or decrease, respectively.

The numbers in Table 13 are the sample means of the 25 required CPU times to solve the problem. The relative percentages of increase or decrease in average CPU time compared to the ones required to achieve the results of Table 5 are also

Table 13
The results of the sensitivity analysis on CPU time in FS + GA algorithm.

Parameters	Uniform		Exponential	
	Value	Difference (%)	Value	Difference (%)
NP	20	+4	20	+10
	30	+9	30	+23
	50	+18	50	+34
O	10	−7	10	−7
	50	−2	50	−2
	150	+3	150	+3
K	5	−2	5	−2
	10	−0.5	10	−0.5
	20	+1	20	+1
P_c	0.6	–	0.6	–
	0.7	+1	0.7	3
	0.9	+6	0.9	+11
P_m	0.010	–	0.010	+5
	0.015	+3	0.015	–
	0.020	+6	0.020	+8
MN	750	+9	750	+6
	1000	+12	1000	+8
	1500	+17	1500	+11

given. The results in Table 13 show that in all situations the average CPU time to solve the problem in a uniform case is larger than that of the exponential distribution. Furthermore, the fuzzy simulation parameters, *K* and *O*, do not have much impact on the CPU times. However, in both distributions, the CPU times are very sensitive to the changes in the number of decision variables. Finally, the parameters of the GA have relatively mild impact on the required CPU time.

6. Conclusion and recommendation for future research

In this paper, a stochastic replenishment multiproduct inventory model was developed. Two integer-nonlinear programming models for two cases of uniform and exponential distribution of the time between two replenishments have been proposed. A hybrid method of FS and GA was developed to solve the problem and the results were validated by both a sensitivity analysis and a comparison with an existing hybrid method of FS and SA. The comparison results showed that at least for the selected numerical examples the proposed hybrid method of FS and GA had better performance in terms of objective-function values and required CPU time to obtain the best solution.

The models developed in this research can help the practitioners who are faced with uncertain demands that do not follow a probability distribution. Moreover, the models are helpful in situations in which due to some limitations on the production capacity, the supply of the raw material, and the like, the period length may be uncertain and the suppliers may not be able to deliver the goods on time.

Some avenues for future works follow:

1. The demand or other parameters of the problem may take uncertain forms (stochastic or rough) as well.
2. Some other probability density functions rather than uniform and exponential may be considered for the time between replenishments.
3. Some other meta-heuristic algorithms such as harmony search or particle swarm may be employed to solve the problem.
4. Fuzzy discount factor or fuzzy discrete delivery orders may be considered as well.
5. Differential evolution can be considered as an effective technique to solve the problem.

Acknowledgements

The authors are thankful for constructive comments of reviewers that significantly improved the presentation of the article.

References

[1] E.H.L. Aarts, J.H.M. Korst, Simulated Annealing and Boltzmann Machine, A Stochastic Approach to Computing, John Wiley and Sons, Chichester, 1989.
[2] H. Al-Tabtabai, A.P. Alex, Using genetic algorithms to solve optimization problems in construction, Engineering, Construction and Architectural Management 6 (1999) 121–132.
[3] S. Bylka, Turnpike policies for periodic review inventory model with emergency orders, International Journal of Production Economics 93–94 (2005) 357–373.
[4] H.C. Chang, J.S.H. Yao, L.Y. Ouyang, Fuzzy mixture inventory model with variable lead time based on probabilistic fuzzy set and triangular fuzzy number, Mathematical and Computer Modeling 39 (2004) 287–304.

[5] S.H. Chen, C.C. Wang, A. Ramer, Backorder fuzzy inventory model under function principle, Information Sciences 95 (1996) 71–79.
[6] C. Chiang, Optimal replenishment for a periodic review inventory system with two supply modes, European Journal of Operational Research 149 (2003) 229–244.
[7] C. Chiang, Optimal ordering policies for periodic-review systems with replenishment cycles, European Journal of Operational Research 170 (2005) 44–56.
[8] K. Das, T.K. Roy, M. Maiti, Multi-item stochastic and fuzzy-stochastic inventory models under two restrictions, Computers and Operations Research 31 (2004) 1793–1806.
[9] S. Das, A. Abraham, A. Konar, Metaheuristic Clustering, Springer Press, Chennai, India, 2009.
[10] M. Dorigo, T. Stutzle, Ant Colony Optimization, MIT Press, Cambridge, MA, USA, 2004.
[11] G. Dueck, T. Scheuer, Threshold accepting: a general purpose algorithm appearing superior to simulated annealing, Journal of Computational Physics 90 (1990) 161–175.
[12] K. Ertogal, M.A. Rahim, Replenish-up-to inventory control policy with random replenishment interval, International Journal of Production Economics 93–94 (2005) 399–405.
[13] A. Eynan, D. Kropp, Effective and simple EOQ-like solutions for stochastic demand periodic review systems, European Journal of Operational Research 180 (2007) 1135–1143.
[14] K. Feng, U.S. Rao, Echelon-stock (R–nT) control in two stage serial stochastic inventory system, Operation Research Letters 35 (2007) 95–104.
[15] A.R. Gaiduk, Y.A. Vershinin, M.J. West, Neural networks and optimization problems, in: Proceedings of IEEE 2002 International Conference on Control Applications, vol. 1, 2002, pp. 37–41.
[16] M. Gen, Genetic Algorithm and Engineering Design, John Wiley & Sons, New York, 1997.
[17] Z.W. Geem, J.H. Kim, G.V. Loganathan, A new heuristic optimization algorithm: harmony search, Simulation 76 (2001) 60–68.
[18] D. Goldberg, Genetic Algorithms in Search, Optimization and Machine Learning, Addison-Wesley, Reading, MA, 1989.
[19] C.H. Hsieh, Optimization of fuzzy production inventory models, Information Sciences 146 (2002) 29–40.
[20] X. Ji, Z.H. Shao, Model and algorithm for bi-level SPP problem with fuzzy demands and discounts, Applied Mathematics and Computation 172 (2006) 163–174.
[21] F. Jolai, J. Razmi, N.K.M. Rostami, A fuzzy goal programming and meta heuristic algorithms for solving integrated production: distribution planning problem, Central European Journal of Operations Research 19 (2011) 547–569.
[22] M. Laumanns, L. Thiele, K. Deb, E. Zitzler, Combining convergence and diversity in evolutionary multi-objective optimization, Evolutionary Computation 10 (2002) 263–282.
[23] K.S. Lee, Z.W. Geem, A new structural optimization method based on the harmony search algorithm, Computers & Structures 82 (2004) 781–798.
[24] B. Liu, Uncertainty Theory: An Introduction to Its Axiomatic Foundations, Springer, Berlin, 2004.
[25] S.T. Liu, Fuzzy profit measures for a fuzzy economic order quantity model, Journal of Applied Mathematical Modeling 32 (2008) 2076–2086.
[26] M.K. Maiti, M. Maiti, Fuzzy inventory model with two warehouses under possibility constraint, Fuzzy Sets and Systems 157 (2006) 52–73.
[27] N. Manavizadeh, M. Rabbani, D. Moshtaghi, F. Jolai, Mixed-model assembly line balancing in the make-to-order and stochastic environment using multi-objective evolutionary algorithms, Expert Systems with Applications 39 (2012) 12026–12031.
[28] N.K. Mandal, T.K. Roy, A displayed inventory model with L–R Fuzzy number, Fuzzy Optimization and Decision Making 5 (2006) 227–243.
[29] E. Mohebbi, A replenishment model for the supply-uncertainty problem, International Journal of Production Economics 87 (2004) 25–37.
[30] E. Mohebbi, M.J.M. Posner, Multiple replenishment orders in continuous-review inventory system with lost sales, Operation Research Letters 30 (2002) 117–129.
[31] S.P. Nachiapan, N. Jawahar, A genetic algorithm for optimal operating parameters of VMI system in a two-echelon supply chain, European Journal of Operational Research 182 (2007) 1433–1452.
[32] L.Y. Ouyang, B.R. Chuang, A periodic review inventory model involving variable lead-time with a service level constraint, International Journal of System Science 31 (2000) 1209–1215.
[33] W.W. Qu, J.H. Bookbinder, P. Iyogun, An integrated inventory-transportation system with modified periodic policy for multiple products, European Journal of Operational Research 115 (1999) 254–269.
[34] A. Roy, M.K. Maiti, S. Kar, M. Maiti, Two storage inventory model with fuzzy deterioration over a random planning horizon, Mathematical and Computer Modeling 46 (2007) 1419–1433.
[35] A. Roy, S. Pal, M.K. Maiti, A production inventory model with stock dependent demand incorporating learning and inflationary effect in a random planning horizon: a fuzzy genetic algorithm with varying population size approach, Computers & Industrial Engineering 57 (2009) 1324–1335.
[36] P. Shabudeen, G.D. Sivakumar, Algorithm for the design of single-stage adaptive Kanban system, Computers & Industrial Engineering 54 (2008) 800–820.
[37] Z.H. Shao, X. Ji, Fuzzy multi-product constraint SPP problem, Applied Mathematics and Computation 180 (2006) 7–15.
[38] G.A. Suer, F. Badurdeen, N. Dissanayake, Capacitated lot sizing by using multi-chromosome crossover strategy, Journal of Intelligent Manufacturing 19 (2008) 273–282.
[39] A.A. Taleizadeh, M.B. Aryanezhad, S.T.A. Niaki, Optimizing multi-product multi-constraint inventory control systems with stochastic replenishment, Journal of Applied Sciences 8 (2008) 1228–1234.
[40] A.A. Taleizadeh, S.T.A. Niaki, V. Hosseini, The multi-product multi-constraint newsboy problem with incremental discount and batch order, Asian Journal of Applied Sciences 1 (2008) 110–122.
[41] A.A. Taleizadeh, S.T.A. Niaki, M.B. Aryanezhad, Multi-product multi-constraint inventory control systems with stochastic replenishment and discount under fuzzy purchasing price and holding costs, American Journal of Applied Sciences 6 (2009) 1–12.
[42] A.A. Taleizadeh, S.T.A. Niaki, M.B. Aryanezhad, A hybrid method of Pareto, TOPSIS and genetic algorithm to optimize multi-product multi-constraint inventory systems with random fuzzy replenishment, Mathematical and Computer Modeling 49 (2009) 1044–1057.
[43] A.A. Taleizadeh, S.T.A. Niaki, V. Hosseini, Optimizing multi-product multi-constraint bi-objective newsboy problem with discount by a hybrid method of goal programming and genetic algorithm, Engineering Optimization 41 (2009) 437–457.
[44] A.A. Taleizadeh, H. Shavandi, S. Aslani, Developing a multi product newsboy problem with a hybrid method of fuzzy simulation and simulated annealing, in: 39th International Conference on Computers and Industrial Engineering, Paris, France, 6–8 July, 2009.
[45] C.K. Ting, C.C. Liao, A memetic algorithm for extending wireless sensor network lifetime, Information Sciences 180 (2010) 4818–4833.
[46] J. Yao, S. Chang, J. Su, Fuzzy inventory without backorder for fuzzy order quantity and fuzzy total demand quantity, Computers and Operations Research 27 (2000) 935–962.
[47] J. Xu, L. Zhao, A multi-objective decision-making model with fuzzy rough coefficients and its application to the inventory problem, Information Sciences 180 (2010) 679–696.
[48] J. Xu, Y. Lio, Multi-objective decision making model under fuzzy random environment and its application to inventory problems, Information Sciences 178 (2008) 2899–2914.
[49] M.H.F. Zarandi, A. Hemmati, S. Davari, The multi-depot capacitated location-routing problem with fuzzy travel times, Expert Systems with Applications 38 (2011) 10075–10084.
[50] M.H.F. Zarandi, S. Davari, I.B. Turksen, A fuzzy simulation-embedded metaheuristic to solve a variable radius covering problem, Annual Conference of the North American Fuzzy Information Processing Society - NAFIPS, 5751914.
[51] B. Zahraie, M. Tavakolan, Stochastic time-cost-resource utilization optimization using nondominated sorting genetic algorithm and discrete fuzzy sets, Journal of Construction Engineering and Management 135 (2009) 1162–1171.