ABSTRACT

This investigation explores both the theoretical and empirical modelling of the dynamic performance of floating slab track systems used for reducing groundborne noise and vibration from rail transit systems. Accurately predicting the performance of floating slab track systems is important to the success of a new rail project. In general it has been observed that analytical and numerical models tend to over-predict the performance. An empirical approach to characterizing the dynamic performance of floating slab track is explored. The results of this approach are compared to a theoretical model and the measured data.

INTRODUCTION

Floating slab track (FST) systems have been in use for over forty years on rail transit systems around the world. They are implemented to control ground-borne noise and vibration that would otherwise cause negative environmental impacts. Several design types have been developed and implemented. Some authors also refer to the FST as a mass-spring-system (MSS). The general details of the various types of FST are briefly discussed and compared. Fig. 1 shows an example of an FST being constructed for an at-grade track installation for a new light rail transit system in Newark, New Jersey.

Simple analytical models such as single degree-of-freedom (SDOF) spring-mass models and even more detailed numerical models (e.g., finite element method) have been used to predict the dynamic performance of new FST designs. We explore the ability of these various models to replicate the actual performance of an FST reliably. The results predicted by theoretical models are compared to field measurements for actual FST installations.
It was first noted by Wilson (1977) that SDOF models tend to over-emphasize the amplification seen at the natural frequency of the FST, and that SDOF models over-predict the performance of an FST above its natural frequency. In the current investigation, a similar trend of over-prediction not only by SDOF models, but also from more sophisticated numerical models (e.g., FEA) was observed. The shortcomings of these models to accurately predict the performance of floating slab track systems are discussed and possible explanations for why are explored.

Wilson (1977) discussed the fact that measured data often indicated amplification at the natural frequency of the FST of only 2 to 3 decibels (dB) and not the 8 to 14 dB indicated by an SDOF subjected to steady-state excitation. Wilson (1977) attributed this to the fact that the dynamic excitation of the FST is better represented instead by a combination of random impact and periodic forces (due to the interaction between multiple wheels and rail) moving with the train. A similar argument could be applied to the amount of vibration reduction possible at frequencies greater than the natural frequency. Wettschureck, et al. (1999) presents measured data that supports these observations as well and confirms that even a two degree-of-freedom mass/spring model overemphasizes amplification at resonance and overestimates the amount of reduction at higher frequencies.

In fact, it would appear that at frequencies greater than one octave above the natural frequency, the FST appears to behave as a highly damped system. The empirical model explored in this investigation is based on an iterative process of matching measured data. The empirical model also provides a means to examine possible reasons why the analytical and numerical models used to date appear to fall short of accurately predicting performance.

Various forms of FST have been implemented. Fig. 1 illustrates a hybrid construction technique with precast concrete slabs shown being lowered in place. Once in place a second pour of concrete is added on top. Fig. 2 illustrates a so called pre-cast design (sometimes referred to as a discontinuous FST) with concrete slabs that support four rail fasteners each. The FST shown in Fig. 2 consists of concrete slabs weighing 2,500 kg each and supported vertically on four elastomeric spring elements (in this case support pads made of natural rubber). Another form of construction is to cast the slabs in place using metal forms up to 20 m long. The latter construction technique is sometimes referred to as a continuous FST design.
It is conventional to quantify the vibration reduction performance of an FST as the negative of insertion loss in decibels (dB). This is called insertion gain, which is defined as the vibration level (in decibels) at the invert (concrete base on which the spring elements of the FST are supported) of one rail support system relative to another. The other rail support system is typically a rail fastener. Hence, insertion gain = $L_v(FST) - L_v(Ref)$.

For the purpose of the theoretical modelling in this investigation, we have chosen to use a rigid invert as the reference system, which most investigators do. In general however, the rail support system reference is an elastomeric type rail fastener which typically has some compliance inherent in its construction. The resilient fastener produces less vibration than a rigid invert, but this is the subject of another discussion. For simplicity, we use a rigid invert as the reference. In so doing, we are essentially looking at the transmissibility of forces from the FST to the invert.

**SINGLE DEGREE-OF-FREEDOM MODEL**

Many investigators have chosen to represent an FST with a simple SDOF system, which consists of a mass, spring and damper, as shown in Fig. 3. Some have recognized the importance of including the unsprung mass of the transit vehicle’s bogie (or truck) along with the slab mass. Damping is usually assumed to be between 5 and 10% of critical based on the physical properties of the springs, which are typically an elastomeric material (e.g., natural rubber) or more recently steel springs.

![Fig. 3 SDOF Model of FST](image)

It is clear from measured data that the SDOF model reasonably captures the natural frequency of the FST. It is also clear that the SDOF model does not produces a realistic picture of vibration reduction performance at least not as most investigators have employed this model. It is common to refer to an FST by its natural frequency and the insertion gain curve is commonly displayed in terms of its 1/3-octave band spectrum.

Fig. 4 shows the IG curve for a SDOF model of a 12 Hz FST. The SDOF model in this case has 8% critical damping or $\zeta = (c/c_o) = 0.08$. The SDOF model results are compared to the measured data by Wolfe (1995) for a cast-in-place FST. This comparison illustrates the two main limitations of the SDOF model when a realistic amount of physical damping is employed (i.e., $\zeta<10\%$). The SDOF model tends to overemphasize amplification at resonance and overestimate vibration reduction above resonance at least at higher frequencies.

When 40% critical damping is used in the SDOF model, we find that there is a better match between the SDOF model and the measured data as shown in Fig. 4. There is only 4 dB of amplification at resonance although the peak is much broader. At higher frequencies (greater than one octave above resonance) there is reasonably good agreement. With this much
damping though, the SDOF model does not capture the sharp drop just above resonance. This comparison suggests that an FST effectively behaves as a highly damped system, which appears to confirm the explanation put forth by Wilson (1977) that random moving loads are akin to effective damping.

Another explanation for the over-prediction at higher frequencies could be that a “true” insertion gain should compare the FST transmitted vibration to the vibration transmitted by a rail fastening system with some degree of resilience rather than a rigid invert. Typically the rail fasteners provide some reduction in vibration, but only at higher frequencies (somewhat above 30 to 50 Hz depending on the properties of the fastener).

**EMPIRICAL MODEL**

In this study, the author’s review of measurement data for several different FST led to observations of certain aspects of the insertion gain curves. As Wilson (1977) observed, the data appear to exhibit only minor amplification at the natural frequency of the FST (i.e., resonance). Similar to Wilson (1977), it was observed that the curve reaches a plateau at higher frequencies and does not seem to continue to increase in the same manner as predicted by the simple SDOF model when using realistic values for physical damping.

In practice it has often been assumed that as the natural frequency of the FST decreases the curve simply shifts to the left while maintaining the same shape. A review of the data from several measurements instead seems to indicate that as the natural frequency of the FST decreases the amount of insertion gain at higher frequencies tends to increase more than the increase due to a shift in the curve.

Using the data from several FST measurements, it was possible to construct an empirical model consisting of a family of curves that follow the trend in the data. The empirical modelling was done by matching data and not through a more standard technique such as regression analysis. Although the latter approach might be possible, it would be necessary to have at least three sets of data for each FST natural frequency.
The empirical modelling for this investigation was accomplished by making certain adjustments to a family of curves in such a manner to reasonably match the data. A curve for each FST with natural frequencies at the seven 1/3-octave bands between 4 and 16 Hz were constructed. The data were measured for three FST natural frequencies, which include 5, 7, and 12 Hz. The 5 Hz FST performance was obtained from data found in Jaquet and Loewenstein (2010). The 7 Hz FST performance was obtained from data found in Jaquet and Garburg (2007). Data presented by Wolfe (1995) was used for the 12 Hz FST performance. It should be mentioned that both the 5 and 7 FST referenced employ steel springs.

Seven insertion gain curves, as shown in Fig. 5, were generated over the frequency range of 2 to 500 Hz. The starting point was a smoothed curve for a 16 Hz FST. The measured data for a 16 Hz FST is presented by Nelson (1996). Various small adjustments were made to the smoothed curve to arrive at a family of curves one for each FST natural frequency. Two adjustments were applied to the curve for the 16 Hz FST to arrive at a curve reasonably matching the measured data. An additional four adjustments were applied to the curves of the other six FST natural frequencies.

For example, in matching the data for the three measured FST, 1.25 dB was added at 500 Hz (and all lower frequencies) to the insertion gain of the next lower natural frequency FST. This results in a 4 Hz FST having 7.5 dB more insertion gain at 500 Hz than a 16 Hz FST. In absolute terms this means that for a 16 Hz FST insertion gain = 25 dB at 500 Hz, and for a 4 Hz FST insertion gain = 32.5 dB at 500 Hz.

The curves indicated in Fig. 5 are intended to be a generic family of curves representative of the performance of FST with natural frequencies in the range of 4 to 16 Hz. One of the main features of the proposed model is that the insertion gain for each FST with a lower natural frequency is equal to or greater than the insertion gain for FST with higher natural frequencies. In a sense the curves are monotonically increasing with decreasing natural frequency.

We now compare the curves to measured data. Fig. 6 shows a comparison of the insertion gain predicted by the proposed empirical model and the measured data for a 12 Hz FST.
can be seen that the empirical curve replicates the measured data better than either of the two cases using an SDOF as shown in Fig. 4. This is not surprising since the curves are based on data matching.

We then compare the model data with the measured data for a 5 Hz FST in Fig. 7. Although the match is not as good, in particular over the range of 16 to 25 Hz, the rest of the model matches the measured data well. This would be expected in trying to represent all of the data with a family of curves.
POSSIBLE IMPROVEMENTS TO THEORETICAL MODELS

Wilson (2011) has suggested that the soil conditions supporting the transit structure (subway tunnel), could possibly affect the performance of an FST. An example of this can be seen in the data presented by Nelson (1996) for two different FST with a natural frequency of 16 Hz. It is possible that a two degree-of-freedom model could be used to investigate this hypothesis. Certainly using the performance characteristics of a rail fastener as the reference instead of assuming a rigid invert would be useful and should produce more reliable normalization of the insertion gain data.

RESULTS AND CONCLUSIONS

An empirical approach to modelling the performance of FST for rail transit has been used to develop a family of insertion gain curves that replicate the measured data for FST with natural frequencies between 4 and 16 Hz. In this investigation possible explanations for why theoretical models are not sufficiently accurate have been explored and possible improvements that could be investigated were presented.

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REFERENCES


Wilson, G., personal communication, 2011.