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HEAT CONDUCTION IN THE HOLLOW SPHERE WITH A POWER-LAW VARIATION OF THE EXTERNAL HEAT TRANSFER COEFFICIENT

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ABSTRACT

The conduction phenomenon in an insulated sphere is re-worked through a dimensionless approach, where the heat transfer coefficient dependence on the external radius and on the surface temperature, as in the case of forced and free convection, is taken into account. Assuming a power law variation of the convection coefficient [1, 2], and using the results of Sparrow [3], equations and graphs for the most important dimensionless parameters are presented. The developed equations show, for example, that as the insulation thickness increases the heat transfer rate tends to a positive value, independent of the considered case: constant convection coefficient, forced or free convection.

Introduction

The study of radial heat conduction in the hollow sphere, or in the insulation of spherical bodies, may be improved with the use of a dimensionless approach, allowing the graphical representation of the phenomenon, through the most important governing parameters. Natural convection, assuming a power-law variation of the external convection coefficient, is analyzed, whereas forced convection and the constant convection coefficient cases (eg. Incropera and DeWitt [4]) are treated as particular situations.

Sparrow [3] used this kind of variation of the convection coefficient

$$h_o \propto r_o^{-p} |T_o - T_\infty|^n \tag{1}$$

in the study of the critical insulation radius, with $p \geq 0$ and $n \geq 0$, to obtain the following implicit equation

$$r_{o,crit} = \frac{2-p}{1+n} \frac{k}{h_{o,crit}} \tag{2}$$

Balmer [5] used the correlation developed by Yuge [6] for free convection around spheres, in the development of an equation for the critical insulation radius; similarly to equation (2) it is an implicit equation, since it uses the unknown insulation surface temperature. For the case of constant heat transfer coefficient, Russo [7] presented an equation for the minimum amount of insulation necessary to minimize the heat loss.

The following development extends these results, under the assumption of a power law variation of the external convection coefficient. Explicit solutions for the critical insulation radius, but also for the heat transfer rate and the temperature distribution are presented.

Convection Over Spheres

When far enough of the limiting case of pure conduction heat transfer, forced and free convection around spheres can be modeled using a power law. In the range of Reynolds number from 17 to 70000, McAdams [1] recommends the following equation

$$Nu_D = 0.37(Re_D)^{0.6} \tag{3}$$

In the case of free convection, an equation indicated by Schlichting [2] is

$$Nu_D = 0.429(Gr_D)^{0.25} \tag{4}$$

where Nu_D is the Nusselt number, Re_D the Reynolds number and Gr_D the Grashof number, all based in the outer diameter, D.

Typical general expressions [4, 8] for forced and natural convection around spheres are of the form

$$Nu_D = 2 + \alpha(N)^{\beta} Pr^{\gamma} \tag{5}$$

where $N = Re_D$ for forced convection and $N = Gr_D$ for free convection. This kind of equation combines the cases of pure conduction heat transfer ($N = 0$, $Nu_D = 2$) with a power law, characteristic of greater Reynolds or Grashof numbers. As examples, for the case of forced convection, we have the correlation of Ranz and Marshall [9]

$$Nu_D = 2 + 0.6 Re_D^{1/2} Pr^{1/3} \tag{6}$$

and in the case of free convection the equations of Yuge [6] ($Pr \approx 1$, $1 < Ra_D < 10^5$)

$$Nu_D = 2 + 0.43\, Ra_D^{\ 1/4} \tag{7}$$

and Churchill [10] ($Pr \geq 0.7$, $Ra_D \leq 10^{11}$)

$$Nu_D = 2 + 0.589 \Big/ \left(1 + (0.469/\mathrm{Pr})^{9/16}\right)^{4/9} Ra_D^{\ 1/4} \tag{8}$$

All the referred equations, when far enough of the pure conduction limit, can be written as power law equations. In the case of forced convection

$$Nu_D = B(Re_D)^m Pr^{1/3} \tag{9}$$

and in the free convection case

$$Nu_D = C(Ra_D)^n \tag{10}$$

where Ra_D is the Rayleigh number. Constants B, m, C and n can be found in the literature.

Heat Conduction Under Free External Convection

A sphere of external radius r_i, covered with an external insulation layer of thickness $e = r_o - r_i$ and thermal conductivity k, is losing heat to a surrounding fluid, in free convection regime. The analysis presented in this work is based in the following choice of dimensionless parameters

$$Bi = \frac{h_i r_i}{k}, \quad r^* = \frac{r}{r_i}, \quad r_o^* = \frac{r_o}{r_i}, \quad T^* = \frac{T - T_\infty}{T_i - T_\infty}, \quad q^* = \frac{q}{q_i} \quad \text{and} \quad h^* = \frac{h}{h_i} \tag{11}$$

The subscript 'i' stands for the inner insulation surface, 'o' for the outer insulation surface and '∞' for the surrounding fluid. Bi is the characteristic Biot number, based on h_i, the convection coefficient in the absence of insulation. The dimensionless radial coordinate is r^*, T^* is the dimensionless temperature difference, q^* is the dimensionless heat transfer rate and the dimensionless convection coefficient is h^*.

In the case of natural convection, equation (10) may be written in a dimensionless form, similar to equation (1), as

$$h_o^* = (r_o^*)^{m-1}(T_o^*)^n \tag{12}$$

Forced convection and the constant heat transfer coefficient case may be treated as specific examples of this more general one. In the free convection case $m = 3n$ and under forced convection $n = 0$. In the case of constant convection coefficient, $m = 1$ and $n = 0$. In the following development, m, n and fluid properties, calculated at an average film temperature, are considered constants.

Temperature Distribution

For steady-state conditions, no internal heat sources, and constant properties for the insulating material, the conduction equation in spherical coordinates reduces to

$$\frac{d^2}{dr^{*2}}(r^*T^*)=0 \tag{13}$$

The boundary conditions – known internal temperature and prescribed external heat transfer coefficient – can be written as

$$T^*(r_i^*)=1 \quad \text{and} \quad \left.\frac{dT^*}{dr^*}\right|_{\bar{r}=\bar{r}_o}=-Bih_o^*T_o^* \tag{14}$$

Integration of equation (13) under these conditions, originates the following temperature distribution across the insulation

$$T^*=1-BiT_o^*h_o^*(r_o^*)^2\left(1-\frac{1}{r^*}\right)=1-(1-T_o^*)\frac{1-1/r^*}{1-1/r_o^*} \tag{15}$$

The external surface temperature can be obtained introducing equation (12) into equation (15)

$$T_o^*=1-Bi(r_o^*)^{1+m}(T_o^*)^{1+n}\left(1-\frac{1}{r_o^*}\right) \tag{16}$$

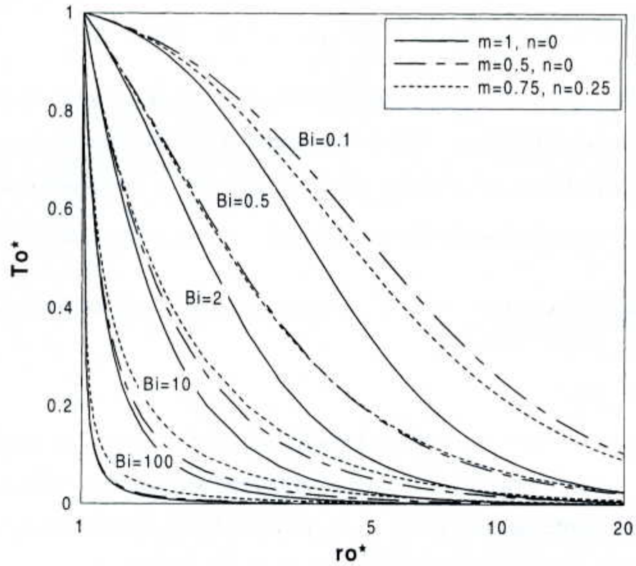

FIG. 1
Variation of the dimensionless surface temperature of the insulation layer, as a function of the Biot number, for for m=1, n=0, m=0.5, n=0 and m=0.75, n=0.25.