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# The Geometric Law of Motion

Prospects for a perspective of motion. A vision from planetary orbits to electronic orbitals

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**ABSTRACT**

In a previous article, related to the discussion on the centrifugal force, we presented the *Geometric Law of Motion*[1] which has naturally emerged from expressing motion based on the fundamental entities *position*, *mass* and *velocity*, and the proposal that the *torque equation* be generalised to the inner product operation. This automatically leads to a new equation describing the radial energy of the system. Motion would therefore take place in a kind of continuous exchange of energy between the angular and the radial dimensions of the space, relative to an observer. This article is solely dedicated to such a law, which we now present with more detail and with some extra remarks, in a bid to also place it alongside previous proposals and, in particular, the Quantum Mechanical description. We will also try to travel from the *infinitely large world* into the *infinitely small world*, while never abandoning the *Geometric Law of Motion*.

## 1. **Introduction**

By looking at things in our day-by-day experience of life we frequently notice that the same thing can exhibit completely different properties, as if by magic. More frequently than not, this magic comes from the way things project themselves into our eyes and senses, and interact with the world. I can perfectly show a circle to a friend and tell him it is a line, and I am sure he will see it as a line. I can perfectly cut a piece of wood with a piece of paper; I may perfectly deposit a little stick of wood over a handkerchief, then obviously break it, and then "obviously" resurrect it. This is the basis of what is more commonly known as illusionism. In serious science, however, it is commonly considered and agreed that there must be no illusionism. But in fact there is, and there will always be an illusion in which the scientist plays the part of the audience instead of the part of the *performer*. Thus, the destiny of the scientist will always be to try to unveil the tricks of the *performer* in order to be able to understand the reality as it really is.

Similarly, when in Physics one looks at the several laws of motion, for example, the conservation of momentum, the conservation of angular momentum, the conservation of energy, and several more conservations, an obvious question arises to the curious mind: if the thing, the universe, the motion, is in fact a single thing, why is there the need for so many laws to describe it? Could there be a single law to include all those several properties of motion and simply look at them as different projections onto our 3D space plus time reality[2]? We like to think on the *Geometric Law of Motion* as a kind of probable seed for such an achievement and, in particular, for a renewed interest for the study of non-recommended effects, like the *centrifugal* effect, for example.

[1] J. Manuel Feliz-Teixeira, "*In Defence of the Centrifugal Force and the Geometric Law of Motion*", first published at http://www.fe.up.pt/~feliz, and YouTube, June 2011

[2] At this moment is seems quiet obvious that reality is not only a 3D+1 reality: I am in the *space-time* and I am *thinking*, so, at least one more dimension is needed to "*thinking*".

## 2. The Geometric Law of Motion

In order to better perceive this "law" it helps to start by considering motion as being described based on the following three fundamental entities[3]: *mass (m), position* ($\underline{\mathbf{r}}$) and *velocity* ($\underline{\mathbf{v}}$). All will be derived from them. We believe this is preferable than using *position* and *momentum* ($\underline{\mathbf{p}}=m.\underline{\mathbf{v}}$), since it obviously separates mass, which we like to see as an *inertial property*, from velocity, which is a *cinematic entity*. We have the feeling that motion is in fact a sort of a geometric thing, which is maintained by the power of the *mass* involved in it. Large masses keep such a geometry more stable and able to resist changes, while small masses are obviously more affected by external interactions. This would perhaps explain why the microscopic world is much more reactive and always changing than the macroscopic world. Molecular and atomic world versus planetary and cosmological world, is an example.

Then, we considered that motion must always be described relative to an observer, who we imagine is located at the center of the coordinates system. Thus, any law governing change in motion will have to explicitly contain the *position* vector. And finally, although we consider *space* as an homogeneous three-dimensional entity, in the presence of motion it tends to transform into an abstract two dimensional entity, being governed by certain tendencies for movement, or for *action*: the *radial dimension* (that we also call parallel // to the observer) which tells us how far or near the body is from the observer and how it approaches or moves away form it; and the *angular dimension* (perpendicular ⊥ to the observer) which represents the tendency for the body rotating around the observer. Since these dimensions are obviously interconnected, it is easy to understand motion in terms of an exchange between radial and angular dimensions, even if these are ruled by apparently different equations. We know, anyhow, that the equation governing the angular motion is the well known "torque equation":

$$\underline{\mathbf{r}} \times \underline{\mathbf{F}} = d\{\underline{\mathbf{r}} \times m.\underline{\mathbf{v}}\}/dt \qquad (1)$$

Torque is what changes the angular momentum vector ($\underline{\mathbf{L}} = \underline{\mathbf{r}} \times m.\underline{\mathbf{v}}$). However, the law governing the radial dimension is usually based on the Newtonian concept of force, given by $\underline{\mathbf{F}} = d\{m.\underline{\mathbf{v}}\}/dt$, together with some other concepts like the conservation of mechanical energy, and, in certain cases, also the consideration of Huygens centrifugal force as a sort of "fictitious" force[4]. In this way, the radial and the angular dimensions are in fact treated as being separate, and from a perspective which is dispersed for several different laws that must hold. Our proposal is to try express these two aspects of motion simply as two different projections of the same general law: one into the *angular* dimension, and the other into the *radial* dimension. By this assumption, it means that both motions would have to be ruled by equations containing not only the same fundamental entities, but also the same "logic" for interconnecting these entities. Driven by the idea that a projection into the radial dimension should naturally be associated to an *inner product* between vectors, it seemed reasonable to propose precisely the same "torque equation" for describing the radial law, but with the *cross product* replaced by the *inner product*. These two laws could therefore be written as the conjunction of two similar equations expressing a single law of motion:

$$\begin{cases} \underline{\mathbf{r}} \times \underline{\mathbf{F}} = d\{\underline{\mathbf{r}} \times m.\underline{\mathbf{v}}\}/dt & \text{- angular law} \qquad (2) \\ \underline{\mathbf{r}} \,.\, \underline{\mathbf{F}} = d\{\underline{\mathbf{r}} \,.\, m.\underline{\mathbf{v}}\}/dt & \text{- radial law} \qquad (3) \end{cases}$$

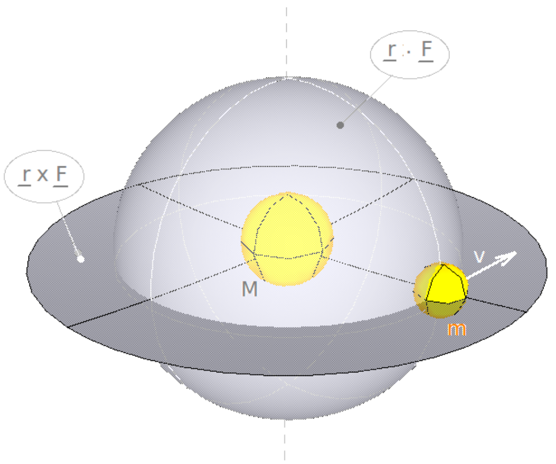



**Fig. 1** The intrinsic geometry of motion, based on what we have expressed in the previous thoughts: a spherical surface (radial dimension) driven by the term $\underline{\mathbf{r}} \,.\, \underline{\mathbf{F}}$, crossed by a rotating plan (angular dimension) dominated by the term $\underline{\mathbf{r}} \times \underline{\mathbf{F}}$. These two dimensions are, however, interconnected by the centrifugal force and the angular momentum mechanism, as we will see.

[3] As it is our practice, here we represent vectors as bold underlined letters, to substitute the usual arrow. So, vector $v = \underline{\mathbf{v}}$.

[4] J. Manuel Feliz-Teixeira, "*Apparently Deriving Fictitious Forces*", first published at http://www.fe.up.pt/~feliz, and YouTube, June 2011

This is a possible representation of the *Geometric Law of Motion*.

This happens to be a coupled system. Notice that the sphere shrinks or expands as mass *m* approaches or moves away from the center of rotation. But this also tends to be accomplished by an increase or a decrease in the speed of rotation depending on the case. What drives the changes in the perpendicular (angular) dimension is $\underline{r} \times \underline{F}$. What drives the changes in the parallel (radial) dimension is $\underline{r} \cdot \underline{F}$. However, the two dimensions are obviously not independent, but interconnected by *radial* and *angular* forces which transfer angular dimension into radial dimension and vice-versa. In fact, if we develop the derivatives, we will get the more explicit expression (obviously, we consider $\underline{a} = d\underline{v}/dt$):

$$\begin{cases} \underline{r} \times \underline{F} = m\,.[d\underline{r}/dt] \times \underline{v} + [dm/dt].\underline{r} \times \underline{v} + m.\underline{r} \times \underline{a} \\ \underline{r}\,.\,\underline{F} = m\,.[d\underline{r}/dt]\,.\,\underline{v} + [dm/dt].\underline{r}\,.\,\underline{v} + m.\,\underline{r}\,.\,\underline{a} \end{cases} \quad (4)$$

Where, from several forces that can already be identified in terms of the associated energy, we have:

Coriolis energy = $m\,.[d\underline{r}/dt] \times \underline{v}$
Euler energy = $m\,.\,\underline{r} \times \underline{a}$
Centrifugal energy = $m\,.\,[d\underline{r}/dt]\,.\,\underline{v}$
Potential energy, radial = $m\,.\,\underline{r}\,.\,\underline{a}$
Energy due to variations of mass = $[dm/dt].\underline{r} \times \underline{v}$
Energy due to variations of mass = $[dm/dt].\underline{r}\,.\,\underline{v}$

From this, let us try to imagine, for example, what happens when a space station or satellite is suddenly loaded with additional mass. From Newton's point of view, the orbit does not depend on the mass orbiting (in first approximation, of course), only on the orbiting velocity, therefore the sudden appearance of new mass inside the space station would change nothing in its motion. But at the same time, a force in the direction of motion is also predicted by Newton[5] due to a change in mass, with intensity $[dm/dt].\boldsymbol{v}$. And this force should therefore accelerate the body...

From our equations, the torque $[dm/dt].\underline{r}\times\underline{v}$ will try to increase the angular momentum of the system. However, if the system has already a strong angular momentum, it will try to keep it by slightly declining its orbit and decreasing its speed. Apparently this will not *directly* affect the radial energy, since in the case of a presumed circular orbit we have $\underline{r}\,.\,\underline{v} = 0$, but only indirectly, due to such a variation of speed.

For simplicity, however, here we consider that no mass variations are allowed, as we are dealing with solid objects. Thus, our law can be simplified to:

$$\underline{\mathcal{M}} = \begin{cases} \underline{r} \times \underline{F} = m\,.[d\underline{r}/dt] \times \underline{v} + m.\underline{r} \times \underline{a} \\ \underline{r}\,.\,\underline{F} = m\,.[d\underline{r}/dt]\,.\,\underline{v} + m.\,\underline{r}\,.\,\underline{a} \end{cases} \quad (5)$$

Notice that $\underline{\mathcal{M}}$ can be seen as a kind of an abstract vector responsible for "modifying" the state of the system, the reason why we called it "*modifier*"[6]. It has an angular (perpendicular) component, and a radial (parallel) component, therefore we can also write it like this:

$$\underline{\mathcal{M}} = (\mathcal{M}_{\perp}\,,\,\mathcal{M}_{//}) = (\underline{r} \times \underline{F}\,,\ \underline{r}\,.\,\underline{F}) \quad (6)$$

If this state modifier is $(\underline{0}, 0)$ the system will be in a stationary state. Something can change internally, but only as a sort of an internal redistribution of movement, or energy, and not more than that; like when a planet describes a stationary elliptical orbit while passing by several and repeated transferences of energy along its trajectory, for example. Probably, the most stationary orbit is in fact the circular orbit[7].

If we use Geometric Algebra, however, all this can be described as the *geometric product* of the vector $\underline{r}$ by the vector $\underline{F}$, which is simply written as[8]:

$$\underline{\mathcal{M}} = \underline{r}\,\underline{F} = \underline{r}\,.\,\underline{F} + \underline{r} \wedge \underline{F} \quad (7)$$

where the cross product has been replaced by the outer product only for convenience. Those expert in Geometric Algebra consider this a single entity which they call a "*spinor*": the conjunction of a *scalar* and a *bivector*, which is a n-dimensinal *rotator* of vectors; in this case a two dimensional rotator. So, we may now express this equation in terms of our

[5] By Newton we may write: $F = d\{m.v\}/dt = m.dv/dt + [dm/dt].v$

[6] This is not a real vector, since the first component is a *vector* while the second is a *scalar*. In Geometric Algebra it is called a *spinor*.

[7] It is funny that Galileo had this same perception, when defending that perfect orbits would be circular.

[8] You may learn more about the geometric product in:
Jaap Suter, (March 12, 2003 ), "*Geometric Algebra Primer* ":
http://www.jaapsuter.com/2003/03/12/geometric-algebra/

fundamental "entities" as a true *Geometric Law of Motion*:

$$\underline{\mathcal{M}} = \underline{r}\,\underline{F} = d\{\underline{r}\,.\,m.\underline{v}\}/dt \;+\; d\{\underline{r}\wedge m.\underline{v}\}/dt$$
$$\underline{\mathcal{M}} = \underline{r}\,\underline{F} = d\{\underline{r}\,.\,m.\underline{v} \;+\; \underline{r}\wedge m.\underline{v}\}/dt \qquad (8)$$

Or, if we want it in a more compact form, using only the *geometric product* of vectors:

$$\underline{\mathcal{M}} = \underline{r}\,\underline{F} = d\{\underline{r}\,m.\underline{v}\}/dt \qquad (9)$$

## 3. Possible interpretations

Since $\underline{\mathcal{M}}$ means change, and systems in general seem not to have a special interest in changing unless their feel in danger or forced to, as if good life would mean a sort of a dream of tranquillity, it seems reasonable to imagine that all motion advances in a way that $\underline{\mathcal{M}}$ can be kept a minimum. It is interesting to notice, at this point, that Euler also used the concept of *action*[9] and stated that the path or trajectory of a system would be in such a way that *action* is minimized. It could be called the "laziness law", but in fact it is more than that: it is a very intelligent law for spending the minimal amount of energy in a process. People in general should learn much more about planets and particles, instead of wasting so many resources and their precious time in producing "noise" and "agitation" for nothing. All things are, of course, interconnected. But it is also interesting to notice that *action* has been defined by Euler as $dS=m.\underline{v}.d\underline{r}$, as a measure of the projection of the velocity into the direction of motion, precisely what our term ($\underline{r}.m.\underline{v}$) means, in the radial equation. On the other hand, Lagrange and Hamilton used *action* based on the time parameter ($dS=\mathcal{L}.dt$), thus related to the energy in the form of the Lagrangian ($\mathcal{L}$). So, knowing this, our $\underline{\mathcal{M}}$ seems also a kind of *generalised Lagrangian* which includes both *angular* and *radial* dimensions. Therefore we have:

$$\underline{\mathcal{M}} = d\{\underline{r}\,m.\underline{v}\}/dt$$

And we can possibly write it as:

$$\underline{\mathcal{M}}\,dt = m.\underline{v}\,d\underline{r} = dS \;\Rightarrow\; dS = \underline{\mathcal{M}}\,dt \qquad (10)$$

[9] Euler defined action as $dS = m.\underline{v}.d\underline{r}$, while later Lagrange and Hamilton used it in the form $S = \int L.dt$ being $L$ the Lagrangian of the system = (Kinetic energy – Potential energy).

Could we from this say that $\underline{\mathcal{M}}$ is equivalent to a generalised Lagrangian representing not only the energy component, as the normal Lagrangian does, but also the angular momentum component, and thus be even seen as a kind of seed for some sort of quantum states? In effect, from this it seems natural the idea of representing the states of motion of a system based on *energy* and the *angular momentum*, precisely as it is commonly considered in Quantum Mechanics. This is very interesting. As long as we also include in the *Geometric Law of Motion* both the *spin-orbit* interaction and the *magnetic moment interaction*, then maybe also the states of *spin* and *magnetic moment* could be part of this new description of motion; just like the quantum numbers *(n, l,* $m_s$*, s)* do in the universe of the infinitely small. Perhaps we will return to this issues in a future work. For now, anyhow, it seems we have something in our hands which includes and relates *linear action* and *angular action*, if we can say this way, where *angular action* is in fact the same as *angular momentum*.

## 4. Radial and angular exchange of motion

In our standard way of thinking it is expected that a displacement of a body in a certain direction will be related to, if that will be the case, a force or an acceleration in that same direction, but not in the perpendicular direction. In fact, that would happen in processes of stress or deformation of materials[10], and in friction forces, for example, but these are structures highly bounded to internal forces in all directions. An interesting thing introduced by the *Geometric Law of Motion* is the possibility for a displacement be related to a force perpendicular to it. A radial displacement produces an angular force (Coriolis); and an angular displacement produces a radial force (centrifugal). Thus, the two dimensions are obviously coupled, therefore they may exchange energy and momentum between them.

Let us try to rewrite our equations in a form so that these forces can easily be identified. Starting by:

$$\underline{\mathcal{M}} = \begin{cases} \underline{r} \times \underline{F} = m\,.[d\underline{r}/dt] \times \underline{v} \;+ m.\underline{r} \times \underline{a} \\ \underline{r}\,.\,\underline{F} \;= m\,.[d\underline{r}/dt]\,.\,\underline{v} \;+ m.\,\underline{r}\,.\,\underline{a} \end{cases}$$

we may represent these vectorial operations by

[10] In Mechanics of Materials, with the stress and the deformation, for example.

the proper projections onto their angular (⊥) and radial ($_{//}$) dimensions, using the angle $\beta$ between $d\underline{r}/dt$ and $\underline{v}$, and we may look at $\underline{\mathcal{M}}$ as a real vector with components $(\mathcal{M}_\perp , \mathcal{M}_{//}) = (r \,.\, F_\perp \,,\, r \,.\, F_{//})$:

$$\underline{\mathcal{M}} = \begin{cases} r \,.\, F_\perp = m.\,(\sin\beta).\,[dr/dt] \,.\, v \;+\; m \,.\, r \,.\, a_\perp \\ r \,.\, F_{//} = m.\,(\cos\beta).\,[dr/dt] \,.\, v \;+\; m \,.\, r \,.\, a_{//} \end{cases} \quad (11)$$

Notice that some more interesting things come out from this fact. For example, if we divide both equations by $r \neq 0$, we find the following coupled equations that describe the motion in terms of forces:

$$\begin{cases} F_\perp = m.\,(\sin\beta).\,[dr/dt] \,.\, v/r \;+\; m \,.\, a_\perp \\ F_{//} = m.\,(\cos\beta).\,[dr/dt] \,.\, v/r \;+\; m \,.\, a_{//} \end{cases} \quad (12)$$

In the case of a body moving in a circular orbit with a large radius it holds that $\beta = 0$, thus only a *centrifugal* force with intensity $(m.[dr/dt].v/r)$ will act in reaction to the curved restriction imposed by the orbit, while the angular velocity will be constant. But in the case of an elliptical orbit, for example, most of the time $\beta \neq 0$ and, by the angular equation, the body will also be acted upon by an *angular* force (⊥) given by $m.(\sin\beta).[dr/dt].v/r$. This force ($F_a$) will tend to increase or decrease the orbital speed, depending on the direction of $[dr/dt]_{//}$, while the centrifugal force ($F_c$) tries to keep the body away from the centre of gravitation, see (Fig. 2).

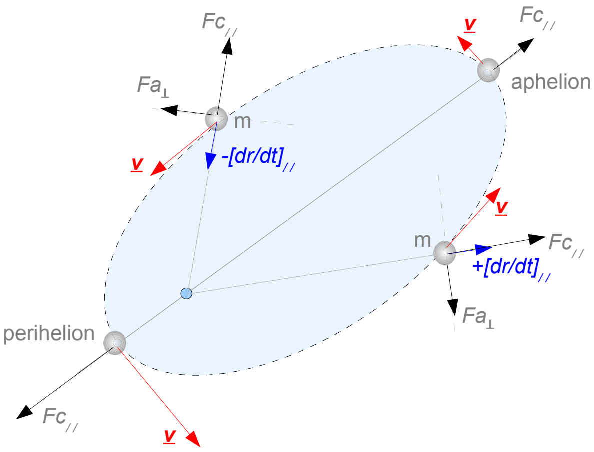



**Fig. 2** Example of an elliptical orbit showing the roles of the centrifugal force ($F_c$) and angular force ($F_a$) on some points of the trajectory. To these, one must also add the gravitational force $-m.g_{//}(r)$ which maintains the closed orbit, of course.

As we know, as long as the body crosses the *aphelion*[11], the (⊥) force starts to contribute to the increase of the orbital speed, which will achieve a maximum at the *perihelion:* precisely where this force will be null again. Then, slowly it will increase again, but now reducing the orbital speed till it reaches the next minimum, which will be at the *aphelion* again. And the process repeats eternally if no energy is lost or exchanged with the "external world". That is, while $\underline{\mathcal{M}} = (\underline{0}, 0)$, meaning that the system is in a *stationary state*. It is also interesting to perceive from this that the only parameter maintained constant during all the elliptical trajectory[12] is the *angular momentum*. And this means that we can think of a *stationary state* having a *fixed* angular momentum but *different* levels of intrinsic body energy (kinetic). If we would be in the primordials of Quantum Mechanics we would probably feel tempted to look at $\underline{L}$ as the first quantum number, instead of "energy". In effect, $\underline{L}$ is what seems to define not only the geometry of motion but also the "determination" which keeps the stationary state stationary. In this perspective, the energy seems to be a secondary issue.

Another interesting aspect resulting from this description of motion is that at very long distances from the centre of observation these *angular* and *centrifugal* forces tend to be null, the motion reduces to a rectilinear motion, and equation (12), written in its coordinates, will reduce to:

$$(F_\perp \,,\, F_{//}) = (m \,.\, a_\perp \,,\; m \,.\, a_{//})$$

Or, in a different vectorial notation:

$$\underline{F} = m \,.\, \underline{a} \quad (13)$$

This makes us look at Newton's definition of force as more appropriate to describe motion as a sequence of rectilinear contributions, where the concept of centrifugal force has obviously no place. But such a definition also implies several other definitions, as we know; one of them is the representation of kinetic energy as $(½.m.v^2)$,

[11] The position in its orbit more distanced from the center of gravitation. The opposite point is called *Perihelion*.

[12] In true, $\underline{L}$ is a constant in any "natural" trajectory, even in the simplest rectilinear motion. This means $\underline{L}$ is strongly associated with the geometry of the orbit.

something that also seems at odds with the $(m.v^2)$ term on the present approach[13].

## 5. Some basic examples of states of motion

In our perspective, any type of motion satisfying the equation $\underline{\mathcal{M}} = (\underline{0},\ 0)$ is to be considered a *stationary state*. Otherwise, it is a *transitory state*, that can even be an *equilibrium state*. A stationary state may therefore be described by the following "energy" equations (valid for all $r$):

$$\begin{cases} \underline{\mathcal{M}}_{\perp} = m\,.[d\underline{r}/dt] \times \underline{v} + m.\underline{r} \times \underline{a} = \underline{0} \\ \underline{\mathcal{M}}_{//} = m\,.[d\underline{r}/dt]\,.\,\underline{v} + m.\,\underline{r}\,.\,\underline{a} = 0 \end{cases}$$

Or alternatively by the force equations (for $r \neq 0$):

$$\begin{cases} F_{\perp} = m.\,(\sin\beta).\,[dr/dt]\,.\,v/r + m\,.\,a_{\perp} = 0 \\ F_{//} = m.\,(\cos\beta).\,[dr/dt]\,.\,v/r + m\,.\,a_{//} = 0 \end{cases} \quad (14)$$

As an exercise on the utilization of this *Geometric Law*, let us try analyse some simple cases of motion.

**Circular uniform motion**:

Since in this case, for a sufficiently large radius, we have $\beta = 0$ and no source of angular acceleration, it must hold that $[dr/dt] = v$, and the previous equations (14) will naturally reduce to:

$$\begin{cases} F_{\perp} = 0 + 0 = 0 \\ F_{//} = m.v^2/r + m\,.\,a_{//} \end{cases} \quad (15)$$

from where we deduce that the condition for a stationary state is $(m.v^2/r = -\ m.a_{//})$. This is obviously the condition for orbiting, as we know. The centrifugal force is what compensates the centripetal force which restricts the motion to a circle. In the case of a gravitational system, the centripetal force is the gravitational force, given by $-m.g(r)$. In this case, the restrictive force is acting on the body at distance, and it is compensated by the centrifugal force, thus the orbiting body will feel as if being acted upon by absolutely no force at all, similar to the situation of free fall or of moving in a rectilinear path with a constant speed. Orbiting means tranquility, a kind of protection from the source of the "attractive" force[14]. Once the body orbits its attractive source, it is as if this source would have ceased to exist, at least while the body will be able to maintain its level of speed. And this is interesting, since in this way any laboratory in orbit can be considered an exceptional place for experiments in the absence of gravity, as in fact they are.

But there is a completely different situation of circular motion if the circular restriction is imposed not by a real force but instead by a spatial constraint, like a wall, for example, as in the curious wall of death, the Finnish fling, or the death loop by car, amongst others. Contrary to what happens during orbiting, in these cases the body feels a kind of rotation on the gravity vector acting upon it, which is simply the vectorial sum of the *normal gravity* ($g$) with the *centrifugal acceleration* induced by the circular restriction. And so, instead of feeling no forces, it feels a deformed gravitational field powerful enough to keep it from falling in the usual direction. Due to the reaction of the wall, the body feels in a situation of contact with the wall, as if it were being smashed into it. It is a state of *equilibrium*, not a *stationary state*, because there is tension. And this is precisely the opposite sensation of floating while orbiting. The force equations for these cases could generally be written as:

$$\begin{cases} F_{\perp} = 0 + 0 = 0 \\ F_{//} = m.\,v^2/r + m.\,g_{//} \\ F_z = -\,m.\,g_z \end{cases} \quad (16)$$

Notice that in the "*death loop by car*" the $z$ equation does not exist[15], while in the other two cases we have $m.g_{//} = 0$ and only $m.g_z$ exists. Next figure (Fig. 3) helps to better visualize these forces. Since in the "*death loop by car*" the plane of rotation is parallel to the gravitational field ($\underline{g}$), and no $z$ direction is relevant due to the symmetry of the case, the radial equation will be

[13] The derivation of the formula for the kinetic energy starts by assuming $Work = F.dx = [d(m.v)/dt].v.dt = v.d(mv)$. . . thus, it considers generally valid $F = d(m.v)/dt$ ... at odds with the present perspective of motion.

[14] Since there is no net acceleration, and $\underline{\mathcal{M}} = (\underline{0},\ 0)$, there will be no irradiation in the case the mass $m$ is of an electron and the attractive source is coming from a proton, for example. Emission and absorption of radiation would only occur when passing from a stationary state to another (and different) stationary state, that is, while $\underline{\mathcal{M}} \neq (\underline{0},\ 0)$.

[15] The direction $z$ is assumed here pointing in the same direction as the angular momentum.

$$F_{//} = m.\ v^2/r\ +\ m.\ g_{//} \tag{17}$$

With $g_{//}$ obviously depending on the car's position: at the top ($g_{//} = -g$), at the sides ($g_{//} = 0$) and at the bottom ($g_{//} = +g$). This is the total radial action, so, in order to maintain the circular path the wall must be rigid enough to react with an opposite reaction ($\underline{R}$) and ensure it holds:

$$R_{//} = F_{//} \tag{18}$$

In that case, we may write:

$$0 = m.\ v^2/r\ +\ m.\ g_{//} - R_{//} \tag{19}$$

making the case apparently similar to a stationary state. However, it cannot be considered a stationary state since there is tension in the body. There is the reaction of the wall, and there is energy being dissipated through the wall due to it. Of course at the top position this reaction is minimal, and may even be null (the car would for an instant float) if the intensity of the centrifugal acceleration will exactly match gravity.

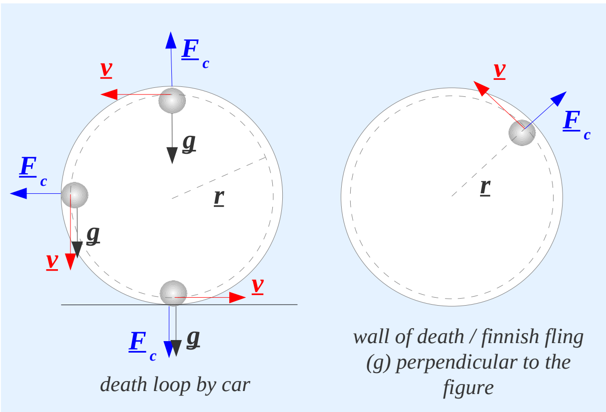



Fig. 3 Circular motion constrained by a wall and the rules of the centrifugal force (Fc) and gravity (g) at several points of the trajectory. Notice that for enough large radius the vector d$\underline{r}$/dt is at all times parallel to $\underline{v}$, thus b = 0.

In the second case (*Finnish fling*), gravity has only component in the *z* direction, no component in the plane of circular motion, so its parallel component is always null and the equations of motion may be written as:

$$\begin{cases} F_{\perp} = 0 + 0 = 0 \\ F_{//} = m.\ v^2/r \\ F_z = -\ m.\ g_z \end{cases} \tag{20}$$

Again, the system will only be in equilibrium if the wall will be rigid enough to generate a force of reaction $\underline{R} = -\ (0,\ F_{//},\ F_z)$, transforming the previous equations into something like an equilibrium state:

$$\begin{cases} 0 = 0 + 0 \\ 0 = m.\ v^2/r\ -\ R_{//} \\ 0 = -\ m.\ g_z\ +\ R_z \end{cases} \tag{21}$$

Obviously, $R_z$ is the *friction force*, which is very dependent not only on the type of wall but also on the intensity of the centrifugal force, through its *coefficient of friction*. The body will fall into the wall instead of into the normal gravitational field while it holds ($R_z > m.\ g_z$) of course, which is a very easy situation to achieve given the fact that centrifugal forces are very intense. As in the previous case, this must not be considered a stationary state. The reaction $\underline{R}$ cannot be compared to a force acting at distance, it is a force acting only on the surface of contact between the body and the wall. An electron in any of these situations would probably irradiate energy due to friction, or in the form of *phonons*.

**Rectilinear uniform motion**:

Rectilinear motion is a very interesting state of motion (Fig. 4). There are some mysteries. And the first question we could ask ourselves is: is there any angular momentum? And then we notice that not only is there angular momentum but the angular momentum is conserved along all the trajectory; like the area speed $\underline{r} \times \underline{v}$, precisely as in the circular motion case, for example. The shaded areas in the figure are equal. Thus, in what concerns angular momentum, a circle is the same as a straight-line.

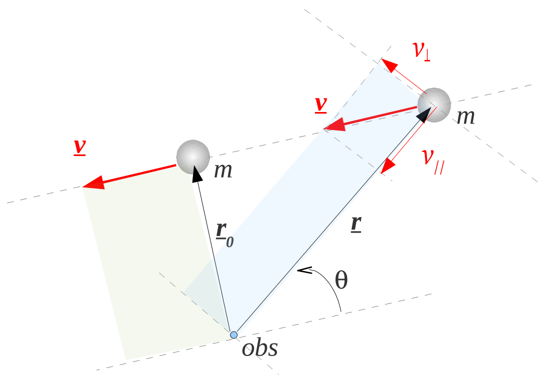



**Fig. 4** Linear uniform motion, and its two components of velocity, *radial* and *angular*. At any point of the trajectory d$\underline{r}$/dt is parallel to $\underline{v}$.

The second question we may ask ourselves is: are there any forces involved? Before answering this question let us manipulate some concepts. From figure 4 we can say that the velocity is the vector $\underline{v} = (v_\perp , v_{//})$, which we may write in terms of angle $\theta$ as:

$$\underline{v} = (v.\ \sin\theta,\ \text{-}\ v.\ \cos\theta\,) = v\ .\ (\sin\theta,\ -\cos\theta\,) \qquad (22)$$

Therefore, the acceleration $d\underline{v}/dt$ is given by:

$$\begin{aligned} d\underline{v}/dt\ &= [d\underline{v}/d\theta\,].[d\theta/dt] \\ &= v\ .\ [d\theta/dt].\ d(\sin\theta,\ -\cos\theta\,)/d\theta \\ &= v\ .\ [d\theta/dt].\ (\cos\theta,\ \sin\theta\,) \end{aligned} \qquad (23)$$

Since $[d\theta/dt]=\omega$ and we know that $\omega = v_\perp /r$, and $v_\perp = v.\ \sin\theta$, we may write:

$$\underline{a} = d\underline{v}/dt\ = (v^2/r).\ \sin\theta\,.\ (\cos\theta,\ \sin\theta\,) \qquad (24)$$

This is a vector always perpendicular to $\underline{v}$ with a magnitude of a *centrifugal acceleration*. It has a maximum along the radial dimension for $\theta = \pi/2$, when its angular component is null, and will be null at infinitum at both sides of the observer. This is obviously the vectorial sum of a centrifugal effect with an angular effect. And it is somehow funny that the same effect can precisely be computed (now in terms of force) by the equation:

$$\underline{F}_0 = (\underline{v} \times \underline{L})/r^2 \qquad (25)$$

So, what is the exact answer to the previous question? We haven't decided yet. But we can already imagine that the observer may probably put the body into orbit if he is be able to act upon the body with a force of the same magnitude as $\underline{F}_0$ projected onto $\underline{r}$.

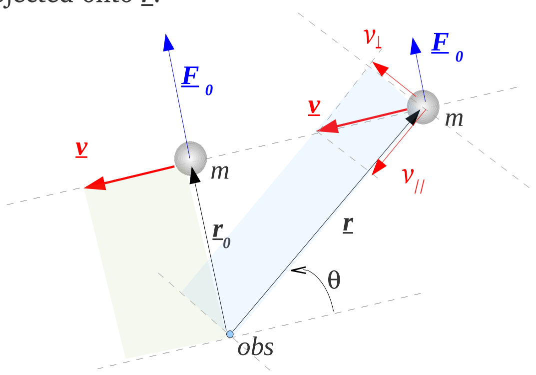



**Fig. 5** Strange force in a linear motion due to $d\underline{v}/dt$.

But now we need to also include the effects of the derivatives of $\underline{r}$ in our equations of motion. For that, let us remind ourselves that the vectorial product of two generic vectors $\underline{a}$ and $\underline{b}$ can also be written as:

$$\begin{aligned} \underline{a} \times \underline{b} &= (\underline{a}_\perp + \underline{a}_{//}) \times (\underline{b}_\perp + \underline{b}_{//}) \\ &= \underline{a}_\perp \times \underline{b}_\perp + \underline{a}_\perp \times \underline{b}_{//} + \underline{a}_{//} \times \underline{b}_\perp + \underline{a}_{//} \times \underline{b}_{//} \\ &= \underline{a}_\perp \times \underline{b}_{//} + \underline{a}_{//} \times \underline{b}_\perp \end{aligned} \qquad (26)$$

And it holds, for the inner product:

$$\underline{a}\ .\ \underline{b} = \underline{a}_\perp\ .\ \underline{b}_\perp + \underline{a}_{//}\ .\ \underline{b}_{//} \qquad (27)$$

Starting again from the general expression of the *Geometric Law of Motion* (5), we can write:

$$\begin{cases} \underline{r} \times \underline{F} = m.\{[d\underline{r}/dt]_\perp \times \underline{v}_{//} + [d\underline{r}/dt]_{//} \times \underline{v}_\perp + \underline{r} \times \underline{a}\} \\ \underline{r}\ .\ \underline{F}\ = m.\{[d\underline{r}/dt]_\perp\ .\ \underline{v}_\perp + [d\underline{r}/dt]_{//}\ .\ \underline{v}_{//} + \underline{r}\ .\ \underline{a}\} \end{cases} \qquad (28)$$

And, in a scalar form[16]:

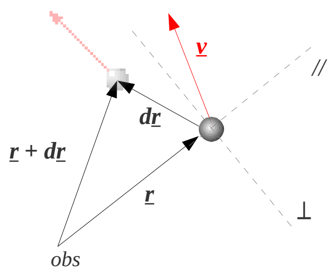
$$\begin{cases} r\ .\ F_\perp = m.\{[dr/dt]_\perp\ .\ v_{//} - [dr/dt]_{//}\ .\ v_\perp\ + r\ .\ a_\perp\} \\ r\ .\ F_{//} = m.\{[dr/dt]_\perp\ .\ v_\perp\ + [dr/dt]_{//}\ .\ v_{//} + r.\ a_{//}\} \end{cases} \qquad (29)$$

In the case of linear motion it simplifies to:

$$\begin{cases} r\ .\ F_\perp = m.\{v_\perp\ .\ v_{//} - v_{//}\ .\ v_\perp\ + r\ .\ a_\perp\} \\ r\ .\ F_{//} = m.\{v_\perp\ .\ v_\perp\ + v_{//}\ .\ v_{//} + r\ .\ a_{//}\} \end{cases} \qquad (30)$$

Using now the results from equation (24) and making a few mathematical manipulations:

[16] Some will argue that $d\underline{r}/dt$ always equals $\underline{v}$, therefore some terms we use in the *Geometric Law of Motion* would vanish. That is not true. Take for instance the *area speed*, given by $\underline{r} \times \underline{v}$, which is a constant of motion for a stationary or equilibrium state. Kepler noticed in the planetary case, and stated his *Law of the Areas*. If we take the derivative, we will have: $[d\underline{r}/dt] \times \underline{v} + \underline{r} \times [d\underline{v}/dt]$. From the figure, we can see that the first term of the derivative considers $\underline{v}=\underline{const}$ and $\underline{r}$ changing; while in the second term $\underline{r}=\underline{const}$ and $\underline{v}$ changes. It is therefore obvious that $[d\underline{r}/dt]$ does not have to be always parallel to $\underline{v}$.



$$\begin{cases} r\,.\,F_{\perp} = m.\{\,0 + v^2.\ \sin\theta\,.\ \cos\theta\,\} \\ r\,.\,F_{//} = m.\{\,v^2 + v^2.\ \sin\theta\,.\ \sin\theta\,\} \end{cases} \quad (31)$$

$$\begin{cases} r\,.\,F_{\perp} = m.\{\,v^2.\ \sin\theta\,.\cos\theta\,\} \\ r\,.\,F_{//} = m.\{\,v^2\ (1 + \sin\theta\,.\ \sin\theta)\,\} \end{cases} \quad (32)$$

$$\begin{cases} F_{\perp} = m.\{\,v^2.\ \sin\theta\,.\cos\theta\,\}/r \\ F_{//} = m.\{\,v^2\ (1 + \sin\theta\,.\ \sin\theta)\,\}/r \end{cases} \quad (33)$$

$$\begin{cases} F_{\perp} = m.\ \sin\theta\,.\{\,v^2.\ \sin\theta\,.\cos\theta\,\}/r_0 \\ F_{//} = m.\ \sin\theta\,.\{\,v^2\ (1 + \sin\theta\,.\ \sin\theta)\,\}/r_0 \end{cases} \quad (34)$$

We get to:

$$\begin{cases} F_{\perp} = m.\ (v^2/r_0\,)\ .\ \sin\theta\,.\{\,\sin\theta\,.\cos\theta\,\} \\ F_{//} = m.\ (v^2/r_0\,)\ .\ \sin\theta\,.\ (1 + \sin\theta\,.\ \sin\theta) \end{cases} \quad (35)$$

And these are the components of the total force $\underline{F}$ acting the body, which can also be expressed as:

$$\underline{F} = m.\ (v^2/r_0\,)\ .\ \sin\theta\,.\ (\,\sin\theta\,.\cos\theta\,,\ 1 + \sin\theta\,.\sin\theta\,) \quad (36)$$

Mysterious... it seems there is something like an intrinsic "force" acting on the body even *before* the body is captured by the observer, dependent only on the speed of the body and its coordinates relatively to the observer. This force will have an intensity $2.mv^2/r$ (double the *normal* centrifugal force) when passing by the observer, which will then be reduced to $mv^2/r$ if captured by the observer. If there is no observer there will be no force. So, the simple presence of the observer interferes with the state of motion of the body? It reminds us of the double slit experiment in the beginning of Quantum Mechanics. Could such a *very strange* behaviour be already impregnated in our macroscopic world? It is difficult to give an answer. Quantum Mechanics accepted such a *very strange thing* as a fact without explaining it, wouldn't it be equally correct to accept this strangeness too? Could Aristotle also be right, in some way? Perhaps this is not a force, but instead a potential? But it is neither a stationary state or an equilibrium state. It is the *free state of motion*. With some little manipulation we may deduce that the intensity of such a "force" is given by:

$$|\underline{F}| = m.\ (v^2/r_0\,).\sin\theta\,.\sqrt{\{1 + 3.\sin^2\theta\,\}} \quad (37)$$

while its components for $0 < \theta < \pi$ behave as:

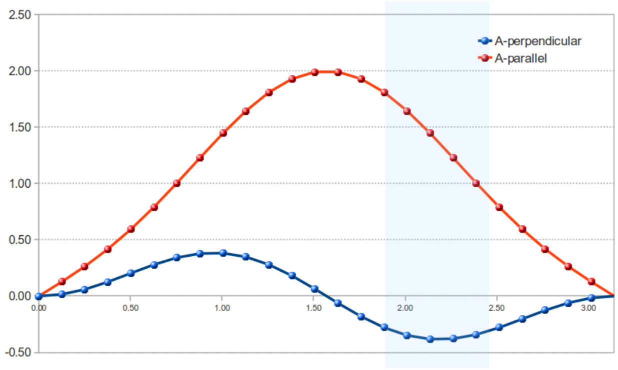



**Fig. 6** Strange *radial* and *angular* forces in a linear motion?

Intuitively, this graph seems to suggest that the best moment to capture the body into orbit would be around $\theta = 2.2$, the shaded zone in the figure.

**Simplistic free fall (under gravity)**:

Free fall makes no sense, in our opinion, without first considering the entire movement of the body, including the way up through the space. The apple of Newton did not only fall. It was first of all slowly pushed up in order to rise in the gravitational field, and only after so doing did it fall. The whole process is one process. But it is not a simple process. In truth, it may be as complicated as elliptical orbiting, for example. So, let us start to simplify it and assume that, contrary to what figure 7 shows, there is only an impulse in the radial direction[17] with the objective of elevating the body against gravity ($g_{//}$). So, $v_{\perp}$ will be null and only $[dr/dt]_{//} \neq 0$, and our equations will reduce to:

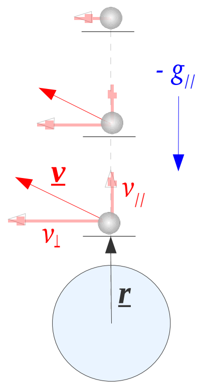



**Fig. 7** Simplistic rise.

$$\begin{cases} r\,.\,F_{\perp} = m.\{0 - 0 + 0\} = 0 \\ r\,.\,F_{//} = m.\{[dr/dt]_{//}\,.\ v_{//} - r\,.\ g_{//}(r)\} \end{cases} \quad (38)$$

[17] In effect, things are more complex since from the beginning. In the initial condition (at rest), the body is acted upon by a centrifugal force due to the spinning of the Earth which is not enough to compensate the gravitational attraction, so the equilibrium is only achieved by blocking its "fall" by means of a reaction ($R_{//}$) from Earth's surface. The equations:

$$\begin{cases} r\,.\,F_{\perp} = m.\{\,0 + 0\,\} = 0 \\ r\,.\,F_{//} = m.\{\,v_{\perp}.\,v_{\perp} - r\,.\,g_{//}\} = -\,r\,.\,R_{//} \end{cases}$$

From where we can say (since in this rectilinear case we have $[dr/dt]_{//} = v_{//}$) that the body is acted upon by a force $(m.v_{//}^2/r)$ surpassing gravity but decreasing with the distance. The body will stop and invert its motion when this force will precisely match the gravitational force of the actual position of the body, that is, when $F_{//} = 0$:

$$\begin{cases} F_{\perp} = m.\{0 - 0 + 0\} = 0 \\ F_{//} = m.\{v_{//}^2/r \ - \ g_{//}(r)\} = 0 \end{cases} \qquad (39)$$

What is to say, when:

$$v_{//}^2/r \ = \ g_{//}(r) \ => \ v_{//} = \sqrt{\{r \, . \, g_{//}(r)\}} \qquad (40)$$

This is the condition for transporting the body from $r = 0$ till a generic $r$ along the radial dimension through the gravitational field. It is interesting to notice that this will not free the body from the source of attraction (gravitational), nevertheless, it is precisely the same value as the condition for orbiting at the same distance, which is:

$$v_{\perp}^2/r \ = \ g_{//}(r) \ => \ v_{\perp} = \sqrt{\{r \, . \, g_{//}(r)\}} \qquad (41)$$

Since in the orbiting situation the body is already "angularly" free from the source of attraction, it is not difficult to imagine that it will get *completely free* from the attractive source when these two conditions hold: free in the *angular* sense and in the *radial* sense (here we consider only two degrees of freedom). That is, when it moves with a velocity ($v$) with the same radial and angular components, that is, when:

$$\underline{v} = (v/\sqrt{2}, v/\sqrt{2}) = (v_{\perp}, v_{//}) = (v_{orb}, v_r) \qquad (42)$$

This velocity is usually called the *escape velocity* ($v_{esc}$) and from here it becomes obvious it must hold that:

$$v_{esc} = \sqrt{2}. \, v_{orb} \quad and \quad v_{esc} = \sqrt{2}. \, v_r \qquad (43)$$

$$v_{esc}^2 = ½ \, v_{esc}^2 + ½ \, v_{esc}^2$$

$$\begin{cases} m. \, v_{orb}^2 = ½ \, m. \, v_{esc}^2 \\ m. \, v_r^2 \ = ½ \, m. \, v_{esc}^2 \end{cases} \qquad (44)$$

$$m.v_{esc}^2 = m. \, v_{orb}^2 + m. \, v_r^2 \qquad (45)$$

Could this equation be more simply stated as: "the total mechanical energy needed to free a body from a source of attraction is the sum of the energies needed to free it from each of the degrees of freedom".

When it reaches the maximum distance on its trajectory, the body has lost all its centrifugal power, and from then on it will be acted upon only by the gravitational force, which will bring it back to the Earth's surface. It is like an elastic with an elastic constant dependent on $r$. Since in the process of *free fall* no tension exists in the body (no contact forces of reaction[18]), the body will have again the impression of floating. But only till it reaches the surface, where suddenly it will feel the force of the impact ($R_{//}$) as being precisely the same as that with which it was launched upwards ($m.v_{//}^2/r$). So, during the free fall, the motion is simply governed by:

$$\begin{cases} F_{\perp} = m.\{0 - 0 + 0\} = 0 \\ F_{//} = m.\{- \ g_{//}(r)\} \ \neq \ 0 \end{cases} \qquad (46)$$

And when it hits the ground, this equation must be converted into an equilibrium equation by means of the reaction of the Earth's surface:

$$\begin{cases} F_{\perp} = m.\{0 - 0 + 0\} = 0 \\ F_{//} = m.\{- \ g_{//}(r)\} \ = -R_{//} \end{cases} \qquad (47)$$

If there is no loss of energy, we will have:

$$R_{//} = m.v_{//}^2/r \qquad (48)$$

If the Earth would be a *ghost mass*[19], however, the body would continue into the centre of the Earth, traverse to the other side, and then return, and it would eternally oscillate between the positions $+r$ and $-r$ almost as an harmonic oscillator. That already would be a *stationary state*. And if we now imagine a certain amount of angular force acting on the body during that period of time, it is not difficult to visualize an elliptical trajectory being described around the centre of attraction. All types of movements seem simply different expressions of the

[18] If the body is inside a case, both the case and the body accelerate from zero speed, thus, no forces will exist between them.
[19] We like to use this term to mean a gravitational effect produced by a mass which does not create any material restrictions to motion.

same thing.

In reality, the free fall is more complex than this, there is also a centrifugal energy due to the angular velocity of the rotating Earth, which also contributes to the elevation of the body in the gravitational field [20], and the Coriolis effect that helps to ensure the constancy of the total angular momentum.

## 6. The Geometric Law in matrix format

There may be several forms for representing these geometric equations of motion. We have already mentioned the Geometric Algebra representation, which compacts them and reduces them to the concept of a *spinor*[21], but also they can be written in the form of an abstract vector $\underline{\mathcal{M}}$ with *angular* and *radial* components, and use the usual algebra. The matrix notation, however, seems also interesting, at least because it shows the system in a clearer perspective. In effect, we may also write these equations in the general form:

$$\underline{\mathcal{M}} = r.\begin{pmatrix} F_\perp \\ F_{//} \end{pmatrix} = m.\begin{pmatrix} v_{//} & -v_\perp \\ v_\perp & v_{//} \end{pmatrix}\begin{pmatrix} [dr/dt]_\perp \\ [dr/dt]_{//} \end{pmatrix} + m.\,r.\begin{pmatrix} a_\perp \\ a_{//} \end{pmatrix} \quad (49)$$

In which $\underline{\mathcal{M}}$ is now a vector related to the sum of the acceleration vector ($\underline{a}$) with the vector resulting from the operation of the actual *velocity matrix* over the displacement vector ($d\underline{r}/dt$). Notice that Newton is only related to the shaded part of this equation. We can now clearly see that the *modifier* of the state of a system can be computed as long as we know the actual velocity ($\underline{v}$), the acceleration ($\underline{a}$), the distance to the observer ($r$) and the displacement tendency ($d\underline{r}/dt$) due to any sort of actions. We may therefore represent the *new* state ($\mathbb{S}$) of our system as:

$$\mathbb{S}(\underline{L}, \mathcal{S}) = \mathbb{S}_0(\underline{L}_0, \mathcal{S}_0) + \int \underline{\mathcal{M}}\,.\,dt \quad (50)$$

where ($\underline{L}=\underline{r}\times m\underline{v}$) is the angular momentum (or angular action) and ($\mathcal{S}=\underline{r}.m\underline{v}$) is the same concept of action used in the Lagrangian mechanics (radial action). In this way we would represent the state of a system based on a single and generic concept of *action*, instead of based on angular momentum and energy, as Quantum Mechanics does, for example. Such a perspective seems also interesting when we review the elliptical motion: if what defines a state is *action*, then a stationary state implies a constant action in each dimension, or degree of freedom. That is, a state is stationary only if it holds $\underline{L}=\underline{r}\times m\underline{v}=const$ and $\mathcal{S}=\underline{r}.m\underline{v}=const$. When this is not true[22], we may consider the system evolving from a state to another state. In the elliptical motion, $\underline{L}$ is a constant along all the trajectory, but not $\mathcal{S}$. In fact it happens that $\mathcal{S} = 0$ at the *perihelion* and the *aphelion* (see Fig. 8) and it changes between these points. In the case of a pure circular orbit, however, pointed by Galileo as the ideal orbit, both $\underline{L}$ and $\mathcal{S}$ are kept constant along all the trajectory. This would make us look at the elliptical trajectory as a kind of a degenerated circular motion which continuously oscillates along two different *pure* stationary states, $\mathbb{S}_0$ and $\mathbb{S}_1$, which in fact are circular orbits, as shown in the figure:

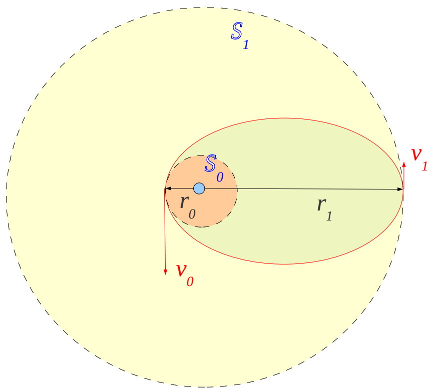



**Fig. 8** An elliptical orbit as a perpetual oscillation between two circular states, $\mathbb{S}_0$ and $\mathbb{S}_1$, of different kinetic energies but the same angular momentum.

If for some reason we are not interested in the parametric trajectory but instead on its geometry, and on the distribution of the probabilities of finding the body in the space around the gravitational centre, we can expect such probabilities $Pb(\mathbb{S}_i)$ to be proportional do the radius of each pure state. So, we would approximately write:

$$\mathbb{S}(\underline{L}, \mathcal{S}) = Pb(0)\,\mathbb{S}_0(\underline{L}_0, \mathcal{S}_0) + Pb(1)\,\mathbb{S}_1(\underline{L}_0, \mathcal{S}_0) \quad (51)$$

which is a superimposition of those two pure

[20] In reality, this velocity is around 1800 Km/h, but the centrifugal effect is very small due to its long distance to the center of the Earth.

[21] A *spinor* in the conjunction of a *scalar* and a *bivector*, which is a n-dimensional *rotator* of vectors, in this case a two dimensional rotator.

[22] It is interesting to notice that if we consider $m$ constant, $\underline{L} = \underline{const}$ and $\mathcal{S} = const$ means $\underline{r}\times\underline{v} = \underline{const}$ and $\underline{r}.\underline{v} = const$, which again shows that the the pair $\underline{r}$ and $\underline{v}$ are a kind of "conjugate" rulers of all motion.

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states. To each pure state obviously corresponds a different kinetic energy of the body. Now, since the angular momentum is constant, we must have:

$$r_0 \,.\, m.\, v_0 = r_1 \,.\, m.\, v_1 \tag{52}$$
$$v_0 \,/\, v_1 = r_1 \,/\, r_0$$
$$m.\, v_0^{\,2} = (r_1/r_0)^2.\; m.\, v_1^{\,2}$$

$$E_1 = (r_0/r_1)^2.\; E_0 \tag{53}$$

Or, using the proper angular speed $\omega$:

$$\omega_1 = (r_0/r_1)^2.\; \omega_0 \tag{54}$$

which leads us to conclude that it makes real sense to use energy ($E$) and the pair ($\underline{\mathbf{L}}$, $E$) to represent the states of a system, instead of the pair ($\underline{\mathbf{L}}$, $\mathbf{s}$) as we previously thought. If now $Pb(1)/Pb(0) = r_1 / r_0$, as we have suggested, the elliptical state may be written in terms of the kinetic energies of its *pure states* as:

$$\mathbb{S}(\underline{\mathbf{L}}, E) = (r_0/r_1).Pb(1).\; \mathbb{S}_0(\underline{\mathbf{L}}_0, E_0) + Pb(1).\; \mathbb{S}_1(\underline{\mathbf{L}}_0, E_1) \tag{55}$$

Or, equivalently, in terms of their angular speeds:

$$\mathbb{S}(\underline{\mathbf{L}}, \omega) = (r_0/r_1).Pb(1).\; \mathbb{S}_0(\underline{\mathbf{L}}_0, \omega_0) + Pb(1).\; \mathbb{S}_1(\underline{\mathbf{L}}_0, \omega_1) \tag{56}$$

This, of course, reminds us of Quantum Mechanics. Of course there are many ways and models that can be used for representing motion, but this seems a very simple way for representing it geometrically, if needed. Such as when very high velocities or very strong forces are involved that make us lose our usual perception of time, for example. In that case, each *pure state* (a circle) would simply be governed by a force equation of the type[23]:

$$\begin{pmatrix} F_\perp \\ F_{//} \end{pmatrix} = m. \begin{pmatrix} 0 & -\,\omega \\ \omega & 0 \end{pmatrix} \begin{pmatrix} [dr/dt]_\perp \\ [dr/dt]_{//} \end{pmatrix} + m. \begin{pmatrix} a_\perp \\ a_{//} \end{pmatrix} \tag{57}$$

with $\underline{F} = (0, 0)$. That is, we would live in a world of an infinite number of circles, with masses jumping from the present circles to the next circles.

[23] Assuming the pure state as a circle, $v_{//} = 0$ and $w = v_\perp / r$ .

But, what would happen if a gravitational wave of the same period of the orbit (circular) of the body would for some time induce a forced movement into the body? Since that would be a resonant process, we could expect the body to quickly absorb such an interference and transform it into a positive angular acceleration, and kinetic energy, which would make it jump into a more external orbit where a new equilibrium could be established for the system. Since this was a gain of energy coming from the "external world", the total angular momentum of the system would increase accordingly. But we know that there are two ways of increasing angular momentum: by maintaining the orbital radius and increasing the orbital speed; or by maintaining the orbital speed and increasing the orbital radius. The *Geometric Law of Motion* seems to allow both mechanisms to exist. It may in fact be regarded as a kind of a *state-like-machine*, as shown in the next figure (for the simplest cases of circular orbits).

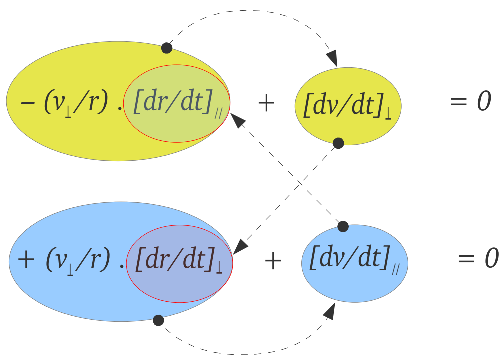


**Fig. 9** *Geometric Law of Motion* as a cinematic *state-like-machine*. Here representing the acceleration exchanges in the case of circular orbits.

Thus, in the case we are describing, the external interference would have entered the system through the $[dv/dt]_\perp$ *state*. This *state*, also called *activity*[24], is responsible for increasing the angular velocity and then passes the control to the *centrifugal-activity*, which forces the body to change its orbital radius against the attractive gravitational force (this, a negative $[dv/dt]_{//}$). While these two forces do not cancel each other, the $[dv/dt]_{//}$ activity is producing a certain change in $[dr/dt]_{//}$, and then the control is passed to the *Coriolis-activity*, which will produce an

[24] The word "state" is used in cybernetics to mean a particular form of *activity* in a *process*; a *state-machine* operates by establishing cycles through those *activities* or *states*. But here we will use the word *activity*.

acceleration in the opposite direction of the external interference, on trying to compensate it. And the process will continue till the moment the *centrifugal-activity* plus the *gravitational-activity* result in an null outcome. At that moment, all the energy received from the "external world" will be stored in the system in the form of potential energy. And the system is now in an *excited state*. Probably the mass is even moving at the same orbital velocity as before, since the strategy used for conserving angular momentum was the changing of $r$, instead of the changing of $v$.

If in reality this works like this or not we do not know yet. But if this mechanism happens to be true, much can be understood on the stability of orbital motion. In effect, in this way the system would naturally fall again into the previous orbit if such a stored potential energy would be lost by some sort of interaction, or even by the emission of energy into the "external world". Another very interesting thing is that by such a theory of motion electrons should not be considered accelerated particles while moving in their orbitals.

## 7. From the world of the astronomic slow into the world of the microscopic fast

This last section we reserve to some discussion and exercises of imagination. We first will try to imagine us flying around our solar system, where distances are enormous and time and motion are apparently slow, and then dive from such an astronomic world into the inside of matter and the domains of atomic and electronic speeds and forces, where Quantum Mechanics rules. Why are there *p-type* orbitals in the atomic world, while they seem not to exist in the planetary world, for example? That is still under study, but who knows if these thoughts will help someone resolve such a mystery.

**Earth and the solar system**:

It does not seem so strange that many people sometimes compare our solar system to a kind of giant atom of astronomical dimensions, with the Sun at the center of its narrative. In the core of this atom is this star with a mass of more than 300 000 times the mass of the Earth. In fact, practically 99,9% of the mass of this atom belongs to the Sun. Mean distances from the centre of rotation to the principal planets vary from 0,4 *Astronomic Units* (AU)[25] in the case of Mercury, to 34UA in the case of Pluto. The Sun rotates with an equator velocity at its surface of near 2 km/s, while the other principal planets follow the tendency presented in figure 10. The orbital speeds, however, vary from near 48Km/s in the case of the faster Mercury (0,02% of the speed of light), to 30Km/s for the Earth, and 4,5Km/s for Pluto. So, this already can give us an idea of the distances and velocities that will be involved in our equations of motion. Even if Earth, for example, moves through the space at the incredible velocity of 30Km in a second!, in that second, however, there is practically no increase in the vector position of the planet, due to its long orbital radius. This means that at all times the term $d\underline{\mathbf{r}}/dt$ is parallel to $\underline{\mathbf{v}}$ and $v_{//}$ is basically null. So, orbits around the Sun tend to be mainly circular. A solar system is an "atom" principally made of *s-type* "orbits".

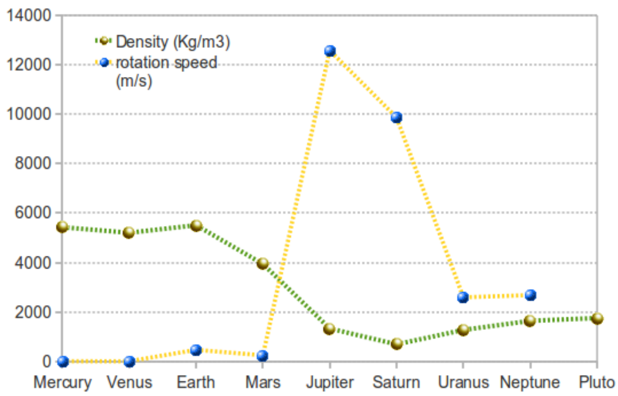


**Fig. 10** Density and rotation speed (at the equator) of the principal planets of our solar system. Denser planets orbit more near the Sun, and have smaller spins than outer planets. All the orbital angular momenta are approximately perpendicular to the ecliptic, except that of Pluto. We wonder if these celestial bodies could have been expelled by our Sun, and our solar system would be a kind of mother with their children moving around, contrary to what *Capture Theory* defends, for example. Some have a view of the universe as a kind of world of debris that slowly reorganize after a sequence of explosions, in order to return to order again; but we have a vague intuition that the universe is nothing like that. It is much calmer and subtle. It is even an intelligent process, something evolving by means of a superior logic based on mechanisms of subtleties, instead by the continuous disorder induced by explosions. But that is already Cosmology.

And what happens when gravitational forces get extremely strong and the radius of the orbits are only some thousand kilometres, as in the centre of a

[25] 1 AU is the distance from Earth to the Sun, near 150 000 000 Km.

galaxy, for example? Is it wise to expect orbits to still be either circular or elliptical? The simple fact that the gravitational forces become extremely intense makes such a region approach a highly bounded structure (more in the direction of a liquid, for example), therefore more the motion will resemble that of the interior of a rotating sphere under an intense internal gravity, like in the case of a star, for example. In such an ambience, more and more the centrifugal, Coriolis and Euler forces will be strong, and will interfere in the motion of the bodies and drive them into trajectories that would never been expected in the case of planetary motion. The laws, though, are the same. But in such an extraordinary situation, motion becomes much more diversified and strange, precisely as it happens in the interior of a “rotating world”. Thus, probably even *p-type* orbits will exist near the center of a galaxy. In effect, we even believe that the *orbit regulatory scheme* is the same scheme responsible for these complex types of orbits.

**A question of perception**:

But we may also question ourselves about how much do we perceive from such an astronomical world. In the period of a second, practically nothing happens for a human in the state of motion of a solar system. Things seem to happen so slow that it would be needed an incredible amount of memory to record all such a motion, and most of the time with very similar data, therefore uninteresting from the point of view of a human being. We know from day-to-day experience that we have some difficulties for perceiving very slow motions.

But, how much do we really perceive in what concerns oscillation, timed events, frequency? Events under 0.01Hz, for example, became so slow that our brain seems to get bored and gives up the attention. Extremely slow movements are almost undetectable by most people. Our brains only start to capture pressure oscillations (sound) above, say, 0,1Hz, and when it reaches 20KHz it is already to much stimulus to process. Dolphins and some other animals are more “intelligent”, in that sense. So, if humans are not able to process a sound above 20KHz, it is also difficult to believe it would be able to process some other kind of stimulus above this same frequency. After this frequency, perhaps our cells still be able to feel something, but not our main processor. This, seems instead to start relying on indirect effects, like intensity, destruction, etc. Humans are systems that perceive frequency from say 0.1Hz till 20KHz.

But, for the sake of this exercise, let us compute the ratio of a *typical velocity* over a *typical radius* and call it *typical-frequency*. For the Earth moving in its orbit we have:

$$typical\text{-}frequency = (1/2\pi).(30\ Km/s)/(150\ 000\ 000\ Km)$$
$$typical\text{-}frequency = 3 \times 10^{-8}\ Hz = 0{,}03\ \mu Hz$$

This frequency is so small that we could never “dedicate our time” to it. It would be a waste of time. Even the *typical-frequency* related to the spinning of Earth is only around 10 μHz. And of course it is even more difficult for us to capture the details of the motion of the years, and even worse of the decades, and centuries, and millennia. We have absolutely no mental structure to even guess what millions of years of existence means, we have no time for it. Neither to absorb the dimensions of even a single galaxy. We cannot even undoubtedly perceive that our ancestors were in fact real. We live in an illusion, an illusion which is real, in our time. Time, as humans sense it, seems not to be an important parameter for the universe of the infinitely large, neither for the universe of the infinitely small, which, in fact, are the same... as ours. This magnificent circularity is what always confuse human beings.

**Bohr's atom and *p-type* orbitals**:

As long as we dive into the matter and approach the atomic world, velocities substantially increase while masses are millions of times reduced[26]. But distances are reduced millions of times more than masses[27]. The big change is in fact in space; perhaps it is really curved. The *typical-frequency* for Bohr's atom is around:

$$f_{typical} = (1/2\pi).(2 \times 10^{6}\ m/s) / (10^{-10}\ m) = 3 \times 10^{15}\ Hz$$

Something which is not understandable from the human perspective. If in a simulation process each

[26] Surprisingly, the velocity of Bohr's electron ($2 \times 10^{6}$ m/s) is only around 67 times the orbiting velocity of the Earth. And that represents 0,7% of the speed of light. But the mass of the electron ($9{,}1 \times 10^{-31}$ Kg) is already around $1{,}5 \times 10^{7}$ times smaller than the mass of the Earth.

[27] The Bohr's radius ($10^{-10}$ m) is around $58 \times 10^{16}$ times smaller than Mercury's orbital radius.

rotation of the Borh's electron would last 1 second, we would need 100 000 millennia to simulate only 1 second of the real time of the atom! When we enter the subatomic world our concept of time also loses meaning. Therefore it seems an excellent idea to forget time and try understand things in terms of geometry and the possible evolutions of state of such a geometry. Quantum Mechanics is, without doubt, fascinating because of that.

So, in the case of the atomic and subatomic world, orbits are so small and being processed so fast that in all situations our equations of motion will have to be considered in their general form, even in the case of circular orbits. This is because $d\underline{r}/dt$ will have components both parallel and radial, as well as in many cases $\underline{v}$ will have both components too. The description of the atomic world naturally allows more possibilities of motion than the astronomic world. And, in these possibilities it is also included all those strange behaviours detected in the interior of spinning platforms, where particles can in fact be found describing orbits not even crossing the centre of rotation, precisely as it happens in the *p-type* and *d-type* orbitals, in the quantum description. This is due to the high intensity of the centrifugal, Coriolis and Euler effects, acting in those circumstances.

**The singularity of $r = 0$:**

In the present description of motion, $R = 0$ is a very special point. It is a *singularity*. This singularity is expected, not from the force equations, which are only valid for $r \neq 0$, but from the original torque and work equations, represented in the matrix format as:

$$\underline{\mathcal{M}} = r.\begin{pmatrix} F_{\perp} \\ F_{//} \end{pmatrix} = m.\begin{pmatrix} v_{//} & -v_{\perp} \\ v_{\perp} & v_{//} \end{pmatrix}\begin{pmatrix} [dr/dt]_{\perp} \\ [dr/dt]_{//} \end{pmatrix} + m.\, r\, .\begin{pmatrix} a_{\perp} \\ a_{//} \end{pmatrix} \quad (58)$$

From these, we can see that $\underline{\mathcal{M}}$ vanishes at $r = 0$, and the equation leads to the obvious true $0 = 0$, a *null singularity*. We would expect, therefore, that a sort of exchange in dimensions exist in the centre of a system with an intense gravitational field, as if both the *radial* and the *angular* dimensions would collapse and transform into a single *linear dimension* along the angular momentum vector, the $z$ direction. Any particles entering this zone (which must be a zone of very low density, almost a vacuum, since less dense matter also tends to move into the centre of rotation) will experience a constant oscillation back-and-forward perpendicular to the plan of the matter, as in a kind of gravitational *maser*. Due to the need of conserving energy and matter, we may expect this to be a kind of an oscillator for matter and energy where from eventually a beam can emerge. The centre of a galaxy would, in this way, be a powerful gravitational *oscillator* operating along $z$ direction, which in certain conditions will be able to expel *matter* and *energy* into both sides perpendicularly to the galaxy:


**Fig. 11** A suggestive image of a galaxy with a "black-hole" at its centre expelling matter and energy along both sides of $z$ direction, the direction of total angular momentum. Screenshot taken from the *YouTube* video: http://youtu.be/cW7BvabYnn8

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J. Manuel Feliz-Teixeira graduated in Physics in the Faculty of Sciences of University of Porto, Portugal, and received an MSc in Mechanical Engineering and a PhD from the Faculty of Engineering of the same university. More recently he dedicated to researching new approaches for renewable energy, and presently is focused on lecturing Physics and study *gravity* phenomena by means of classical Mechanics.

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