Hybrid systems
PDEEC 2009-10

HW #3

Introduction

The project consists of a set of exercises and design problems covering most of the material presented in class. The use of the tools presented in class is strongly recommended, especially in the design problems.

There is an Annex with the models that we will use in the project.

Reach sets

1. What is the reach set at time t = 10 of the system described by the *Simple vehicle model* when the vehicle departs from the origin at time t = 5?

2. Consider the system from the previous question. What is the reach set at time t = 10 when the vehicle departs from the origin at time t = 5 under state constraint $x_1 \leq 0$?

3. Consider the system *"Arrow model"*. What is the reach set at time t = 10 when the vehicle departs from the origin at time t = 0.

4. Consider the system *Simple vehicle model under adversarial behaviour.* What is reach set at time t =1 under adversarial behaviour when the vehicle departs from the origin at time t = 0? [Keep in mind that we are interested in the set of all positions that can be reached independently of what the adversary does].

5. Consider the system *Unicycle model*. Is this system locally controllable? Is $0 \in int(F(x))$ (int(S) denotes the interior of a set S)? [Keep in mind that $F(x):= \{v: \exists u, v = f(x, u)\}$, where in this case $u=(u_s, r)$ and $x=(x_1, x_2, \Theta)$].

6. Consider the system *Linear model* with A=[1, 0; 0, 1] and B=[1, 0]. Is this system controllable? What are the implications in terms of reachability?

Design problem 1 (Food harvesting)

Consider a vehicle modelled by the *"Arrow model"* system. Consider that the vehicle:

1. has the shape of a square with one bumper on each side (to signal collisions with the world);

2. has a smell sensor to detect food (when it bumps into food);

3. has the shape of a square (20 cm x 20 cm)

4. has a sensor which detects when another vehicle is less than one meter away.

5. departs from the origin;

6. evolves in a 10m x 10 m square box (where there are 4 pellets of food placed randomly)

7. collects a food pellet upon hitting it;

Design a hybrid controller for the vehicle to collect the 4 pellets of food and to return to the origin when this is done.

Design problem 2 (Uncooperative food harvesting)

Consider two vehicles of the type described in previous problem and evolving in the same 10m x 10m square box. Assume that there is no communication between the two vehicles. Design one hybrid controller for each vehicle so that the two vehicles collect the 4 pellets of food, while avoiding collisions, and return to the origin when the 4 pellets of food have been collected.

Design problem 3 (Safe controller)

Consider a vehicle modelled by the *"Unicycle model"* system. Design a hybrid controller for the vehicle to move inside a 10m x 10m square box M without colliding with the walls. What can you say about invariance of the new system (obtained from composition of the controller with the vehicle model) with respect to the set M?

Design problem 4 (GoToTarget (GTT)controller)

This example follows closely one example in P. Varaiya and T. Simsek, "A theory of hierarchical, distributed systems".

Consider a vehicle modelled by the *"Mode controlled unicycle model"* system with linear speed u and angular speed Θ bounded by say $U = 5$ and $\Theta = 1$. The vehicle has three sensors:

1. Sensor VREP that gives the global position of vehicle (x_1, x_2, Θ).

2. Sensor LCone which indicates the existence of a target (obstacle) in a large cone in front of the vehicle.

3. Sensor SCone which indicates the existence of a target (obstacle) in a small cone in front of the vehicle.

The hybrid controller GTT receives target position (x_{1T},x_{2T}). Design GTT so that the vehicle reaches target according to following rules:

1. If vehicle has reached target then stop.

2. If target is Left of LargeCone: switch to mode *left* and then when target is in SmallCone switch to mode *straight.*

3. If target is Right of LargeCone: switch to mode *right* and then when target is in SmallCone switch to mode *straight*

Design problem 5 (Dispatcher)

This example follows closely one example in P. Varaiya and T. Simsek, "A theory of hierarchical, distributed systems".

Consider a vehicle modelled by the *"Mode controlled unicycle model"* system with linear speed u and angular speed Θ bounded by say U = 5 and Θ = 1. The vehicle has three sensors:

4. Sensor VREP that gives the global position of vehicle (x_1, x_2, Θ).
5. Sensor LCone which indicates the existence of a target (obstacle) in a large cone in front of the vehicle.
6. Sensor SCone which indicates the existence of a target (obstacle) in a small cone in front of the vehicle.

In Design Problem 4 you designed a a hybrid controller GTT. GTT receives the target position (x_{1T},x_{2T}) and takes care of execution control so that the vehicle reaches target according to following rules:

4. If vehicle has reached target then stop.
5. If target is Left of LargeCone: switch to mode *left* and then when target is in SmallCone switch to mode *straight.*
6. If target is Right of LargeCone: switch to mode *right* and then when target is in SmallCone switch to mode *straight*
7. If target is not reached, target is outside LargeCone, return to Rule 2.

Now consider a dispatcher supervising several GTT controllers. Given a task, i.e. a sequence of targets for each vehicle, the dispatcher assigns next target for GTT of each vehicle. Design the dispatcher.

Note: GTT and dispatcher are dynamic hybrid controllers and cannot be described in language of continuous or Discrete Event Systems (DES) control.

Design problem 6 (Cooperative food harvesting)

Consider Design problem 2. Use the hierarchical approach from Design problems 3 and 5 to come up with a cooperative control structure for a set of n vehicles. If target is not reached, target is outside LargeCone, return to Rule 2.

Annex - Vehicle models

We consider several models of increasing complexity for the motions of our vehicles in R^2.

The state is $x=(x_1, x_2) \in R^2$

The general model is

$dx/dt = f(x, u)$

Simple vehicle model

$dx/dt \in B$ (unit closed ball in R^2)

Simple vehicle model under adversarial behaviour

$dx/dt = u + v$ where $u \in B$ (unit closed ball in R^2) and $v \in 0.5B$ (closed ball with radius 0.5 in R^2)

"Arrow model"

$dx_1/dt \in \{-1, 0, 1\}$

$dx_2/dt \in \{-1, 0, 1\}$

Unicycle model

This model has a third state Θ to model the orientation of the vehicle. The inputs are the speed $u_s \in [0, 1]$ and the angular velocity $r \in [-.1, 1]$.

$dx_1/dt = u_s \cos \Theta$

$dx_2/dt = u_s \sin \Theta$

$d\Theta/dt = r$

Planar motions of autonomous vehicles are often described by this simplified model.

Mode controlled unicycle model

The potential behavior of a vehicle modelled as a unicycle is the set of all smooth curves. This is too unstructured. We restrict its behavior to 4 discrete modes of motion:

stop ($u=0$, $\Theta =0$),

straight ($u=5$, $\Theta =0$),

left ($u=0$, $\Theta =1$),

right ($u=0$, $\Theta =-1$).

The vehicle is controlled by selecting a mode of motion.

Linear model

$dx/dt = Ax + Bu$, $u \in U=[-1,1]$