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Self-deflection of steady-state bright spatial solitons in biased photorefractive crystals

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Abstract

The self-bending process of steady-state bright spatial solitons in biased photorefractive media is investigated by taking into account diffusion effects. By integrating numerically the nonlinear propagation equation, it is found that the soliton beam evolution is approximately adiabatic. The self-deflection process is further studied using perturbation analysis, which predicts that the center of the optical beam moves on a parabolic trajectory and, moreover, that the central spatial frequency component shifts linearly with the propagation distance. Relevant examples are provided.

Since their first experimental observation [1], optical spatial solitons in photorefractive (PR) media have been the focus of considerable attention [2–7]. Self-trapped optical beams of this sort are possible when the process of diffraction is exactly balanced by light-induced PR waveguiding. In the particular case where the PR crystal is externally biased, both bright and dark as well as gray solitary wave domains have been predicted under steady-state conditions [8,9]. These optical solitons are associated with a nonuniform screening of the external electric field and, as a result, they are also known as screening solitons [8]. Thus far, the theory of these steady-state soliton domains has proceeded by entirely neglecting diffusion effects [10–12]. More specifically, under strong bias conditions, it has been implicitly assumed [8,9] that the drift process dominates the transport picture and, thus, one can ignore any effects arising from diffusion. As previously pointed out however [8,9], the diffusion process introduces an asymmetric tilt in the light-induced PR waveguide, which in turn is expected to affect the propagation characteristics of PR solitons.

In this Communication we investigate the self-deflection of steady-state bright PR solitons by taking into account diffusion effects. By employing numerical procedures, we find that the shape of these optical beams remains approximately invariant during propagation. The self-bending process is further studied using perturbation methods which involve the conservation laws of the nonlinear wave equation. Our analysis indicates that the beam center shifts quadratically with the propagation distance, whereas the angle between the central wavevector and the propagation axis varies linearly. These results are in good agreement with those obtained using numerical techniques.

To analyze the self-deflection process of a planar bright soliton, let us first consider an optical beam that propagates in a PR crystal along the z-axis and is allowed to diffract only along the x-direction. The PR crystal is taken here to be strontium barium niobate [13] (SBN) with its optical c-axis oriented along the x-coordinate. Moreover, let us assume that the optical beam is linearly polarized along x and that the external bias electric field is applied in the same direction. Under

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these conditions, the perturbed extraordinary refractive index is given by [14] $n_e'^2 = n_e^2 - n_e^4 r_{33} E_{sc}$ where r_{33} is the electro-optic coefficient involved, n_e is the unperturbed extraordinary index of refraction and $\boldsymbol{E}_{sc} = E_{sc}\hat{\boldsymbol{x}}$ is the induced space-charge field. In typical PR media and for relatively broad beam configurations, the value of the space-charge electric field can be obtained from the Kukhtarev–Vinetskii model [15] and it is approximately given by [9]

$$E_{sc} = E_0 \frac{I_d}{I + I_d} - \frac{K_B T}{e} \frac{\partial I / \partial x}{I + I_d}, \tag{1}$$

where $I = I(x, z)$ is the power density of the optical beam and it is related to the slowly-varying envelope ϕ through Poynting's vector, i.e. $I = (n_e / 2\eta_0) |\phi|^2$. In Eq. (1), I_d is the so-called "dark irradiance" which phenomenologically accounts for the thermal generation of electrons in the conduction band and E_0 is the value of the space-charge electric field in the dark regions of the crystal. If the spatial extent of the optical wave is much less than the x-width W of the PR crystal, then under a constant voltage bias V, E_0 is approximately given by $\pm V/W$ [9].

In turn, the envelope propagation equation can be obtained by substituting the expression for the perturbed refractive index (induced by the space-charge field) into the paraxial wave equation. After appropriate normalization, the envelope U is then found to obey the following dynamical evolution equation:

$$\mathrm{i} U_\xi + \tfrac{1}{2} U_{ss} - \beta \frac{U}{1 + |U|^2} + \gamma \frac{(|U|^2)_s U}{1 + |U|^2} = 0, \tag{2}$$

where $U = (n_e / 2\eta_0 I_d)^{1/2} \phi$, i.e. the power density is normalized with respect to the dark irradiance, and $U_\xi = \partial U / \partial \xi$, etc. The dimensionless transverse coordinate s is given by $s = x/x_0$, where x_0 is an arbitrary spatial scale, and the normalized coordinate ξ is related to the actual propagation distance z through $\xi = z / k_0 n_e x_0^2$, where $k_0 = 2\pi/\lambda_0$ is the free-space wavevector of the lightwave employed. The dimensionless quantities β and γ are associated with the processes of drift and diffusion respectively, and are given by $\beta = (k_0 / x_0)^2 (n_e^4 r_{33}/2) E_0$ and $\gamma = (K_B T / 2e)(k_0^2 x_0 n_e^4 r_{33})$. For the purpose of simplicity, loss effects have been omitted in Eq. (2).

Under strong bias conditions, E_0 is expected to reach appreciable values and, as a result, the drift component of the current dominates. Thus, by neglecting the process of diffusion, i.e. $\gamma = 0$, Eq. (2) takes the form of a nonlinear Schrödinger equation with a higher-order nonlinearity. In this case, the bright solitary wave solutions of Eq. (2) can be readily obtained by expressing as usual the beam envelope as $U = r^{1/2} y(s) \exp(\mathrm{i}\mu\xi)$. μ represents a nonlinear shift of the propagation constant and $y(s)$ is a normalized real function bounded between $0 \leqslant y(s) \leqslant 1$. The positive quantity r is defined as $r = I_{max}/I_d$ where I_{max} is the maximum power density of the solitary beam. Substitution of this latter form of U in Eq. (2) (with $\gamma = 0$) and posterior integration yields

$$\left(\frac{\mathrm{d}y}{\mathrm{d}s}\right)^2 = \frac{2\beta}{r} \left[\ln(1 - r y^2) - y^2 \ln(1 + r)\right]. \tag{3}$$

In obtaining Eq. (3), we have employed the y-boundary conditions, that is $y(0) = 1$, $y(s \to \pm\infty) = 0$ and $\mathrm{d}y/\mathrm{d}s = 0$ at $s = 0$. Moreover, a close inspection of Eq. (3) reveals that bright planar PR spatial solitons are only possible when β or E_0 are positive quantities [9]. The functional form $y(s)$ of these self-trapped waves can then be determined by numerically integrating Eq. (3). As previously shown [8,9], the intensity full-width half-maximum (fwhm) of these solitary beams depends only on two variables, namely E_0 and r.

In order to investigate the effects of diffusion on the propagation of these PR soliton states, Eq. (2) is solved numerically using a beam propagation method [16,17]. The solitary states of Eq. (2) are used as the input beam profiles. As an example, let us assume that the SBN crystal is used at $\lambda_0 = 0.5$ µm. Its parameters are taken to be $n_e = 2.35$ and $r_{33} = 224 \times 10^{-12}$ m/V and the external field strength E_0 is assumed to be 40×10^3 V/m. The arbitrary scale x_0 is 40 µm. For this set of values, β and γ are found to be $\beta = 34.5$ and $\gamma = 0.56$. Fig. 1 illustrates the self-bending action of the diffusion process on such a bright PR soliton obtained at $r = 10$ and $\beta = 34.5$. On the other hand, Fig. 2 depicts the evolution of the angular power spectrum of this domain under the same conditions. Moreover, Fig. 1 indicates that this optical soliton tends to evolve in an adiabatic fashion, i.e. its beam intensity profile remains approximately invariant during propagation. A close examination of Figs. 1 and 2 also reveals that the center of the beam moves approximately on a parabolic trajectory, whereas the central wavevector of the

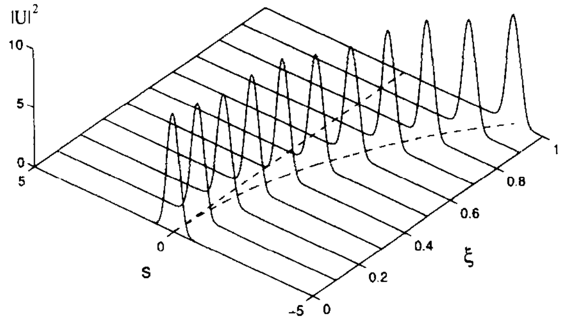

Fig. 1. Intensity profile evolution of an $r=10$ soliton for $\beta=34.5$, when the diffusion parameter is $\gamma=0.56$.

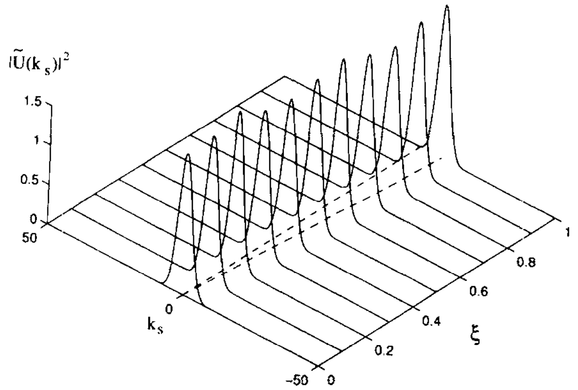

Fig. 2. Evolution of the angular power spectrum of an $r=10$ soliton, for $\beta=34.5$ and $\gamma=0.56$. k_s is the spatial frequency with respect to the normalized coordinate s.

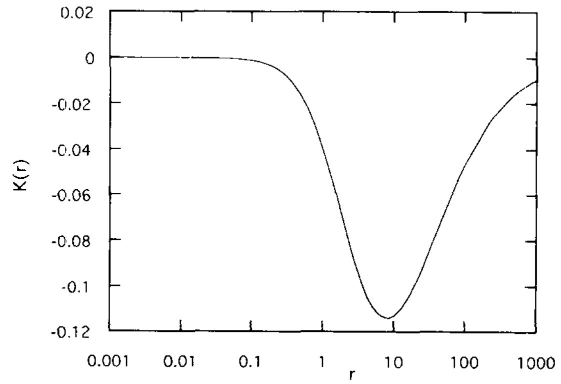

Fig. 3. Dependence of the K function on r.

angular power spectrum shifts linearly with the propagation distance.

This self-bending effect can be further studied using perturbative procedures. It is interesting to note that similar methods have been previously employed within the context of nonlinear fiber optics [18,19]. Keeping in mind that the beam evolution under the action of diffusion is approximately adiabatic, we make the following ansatz for the solution of Eq. (2):

$$U(\xi,s)=r^{1/2}y[s+\nu(\xi)] \times\exp(\mathrm{i}\{\mu\xi+\omega(\xi)[s+\nu(\xi)]+\alpha(\xi)\})\,, \tag{4}$$

where $U(\xi,s)=r^{1/2}y(s)\exp(\mathrm{i}\mu\xi)$ is the steady-state bright PR soliton of Eq. (2) (when $\gamma=0$). In Eq. (4), $\nu(\xi)$ represents a shift in the position of the beam center, $\omega(\xi)$ is associated with the angle between the central wavevector of this beam and the propagation axis, and $\alpha(\xi)$ is a phase factor which is allowed to vary during propagation. The equations of motion of these real variables can then be obtained by substituting Eq. (4) into the two complex conservation laws of Eq. (2) [18,19]. These are established by multiplying Eq. (2) with U^* and $\mathrm{i}U_s^*$ and integrating over the coordinate s. A straightforward calculation yields the following results: $\mathrm{d}\nu/\mathrm{d}\xi=-\omega$, $\mathrm{d}\alpha/\mathrm{d}\xi=\omega^2/2$ and $\mathrm{d}\omega/\mathrm{d}\xi=4\beta\gamma K(r)$, where the dimensionless function $K(r)$ is given by

$$K(r)=\int_{-\infty}^{+\infty}\mathrm{d}s\,\frac{2ry^2(s)}{1+ry^2(s)} \times\{y^2(s)\ln(1+r)-\ln[1+ry^2(s)]\} \times\left(\int_{-\infty}^{+\infty}\mathrm{d}s\,ry^2(s)\right)^{-1}. \tag{5}$$

Since no closed-form solutions are available for the beam profile $y(s)$, except in the low-amplitude case which will be considered next, the $K(r)$ function is evaluated numerically. The dependence of this function on the parameter r is depicted in Fig. 3. The equations of motion for ω, ν and α can then be integrated, in which case we obtain

$$\omega(\xi)=4\beta\gamma K(r)\xi\,, \tag{6}$$

$$\nu(\xi)=-2\beta\gamma K(r)\xi^2\,, \tag{7}$$

$$\alpha(\xi)=8[\beta\gamma K(r)]^2\xi^3/3\,. \tag{8}$$

As expected, Eqs. (6)–(8) demonstrate that in the absence of diffusion, i.e. when $\gamma=0$, the variables $\omega=\nu=\alpha=0$. On the other hand, be taking diffusion

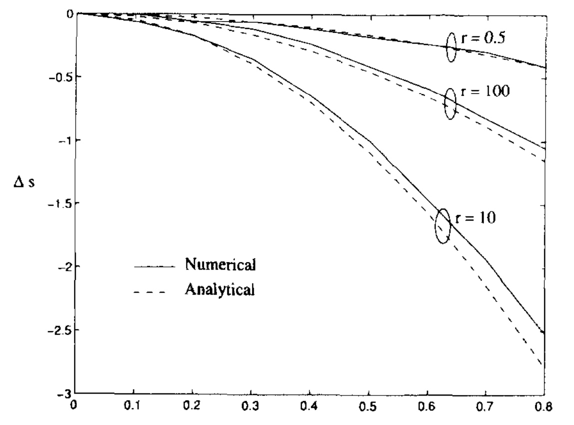

Fig. 4. Evolution of the normalized spatial shift, Δs, for the numerical solution (solid line) and model Eq. (7) (dashed line).

effects into account, Eq. (7) clearly shows that the beam center follows a parabolic trajectory, whereas Eq. (6) implies that the central spatial frequency component shifts linearly with the propagation distance. These results are found to be in good agreement with those previously obtained numerically. From these latter results, one quickly finds that the beam has suffered a lateral displacement given by $x_d = 2\beta\gamma K(r) z^2 / k_0^2 n_e^2 x_0^3$ or

$$x_d = (n_e^3 r_{33} k_0)^2 (K_B T/2e) E_0 K(r) z^2 . \tag{9}$$

where z is the actual propagation distance. Moreover, at this point, the angular deflection, i.e. the angle between the central wavevector of this solitary beam and the z-axis, can be evaluated from Eq. (6) and is given by $\theta_d = 4\beta\gamma K(r) z / k_0^2 n_e^2 x_0^3$ or, more explicitly, by

$$\theta_d = (n_e^3 r_{33} k_0)^2 (K_B T/e) E_0 K(r) z . \tag{10}$$

Both the spatial deflection x_d and the angular deviation θ_d are proportional to the quantities β, γ and $K(r)$. For a given physical system and a fixed soliton parameter r, Eqs. (9) and (10) indicate that the beam self-bending effect, i.e. x_d and θ_d, vary linearly with respect to the external bias field E_0. Since large values of E_0 also imply narrower planar PR solitons, this effect is expected to be more pronounced in this regime. On the other hand, for a constant value of E_0, the self-deflection effect depends on the parameter r through the function $K(r)$. As shown in Fig. 3, this function reaches a maximum close to $r = 10$. This behavior should have been anticipated since the intensity FWHM of these optical PR solitons attains a minimum in this region of r values [9].

Figs. 4 and 5 compare the normalized spatial and angular frequency shifts predicted by our model, i.e. $\Delta s \equiv -\nu$ and $\Delta k_s \equiv \omega$, with those found by numerically solving Eq. (2). These are obtained for three different values of r and for the same system parameters previously considered. As one can see, the results from the two approaches are in good agreement. The small difference between the numerical results and those obtained from the analytical model can be attributed to the fact that the evolution of bright PR solitons under the action of diffusion is not entirely adiabatic, as is clearly seen in Fig. 1.

The low-amplitude case, that is $r \ll 1$ or $|U|^2 \ll 1$, also deserves special attention. As previously shown [9], in this limit Eq. (2) takes the form

$$iU_\xi = \tfrac{1}{2} U_{ss} - \beta U + \beta |U|^2 U + \gamma (|U|^2)_s U = 0 , \tag{11}$$

which is a modified version of the so-called nonlinear Schrödinger equation. The fundamental bright soliton solution of this equation (in the absence of diffusion, $\gamma = 0$) can be readily obtained and it is given by $U = r^{1/2} \operatorname{sech}[(\beta r)^{1/2} s] \exp[i\beta(r/2 - 1)\xi]$. The action of diffusion on the behavior of these Kerr-like bright PR solitons [9] can then be investigated by substituting the low-amplitude beam profile $y(s) = \operatorname{sech}[(\beta r)^{1/2} s]$ in Eq. (5), in which case one obtains $K(r) = -2r^2/15$. In this same limit, the quantities ω, ν and α can be explicitly evaluated and they are given by

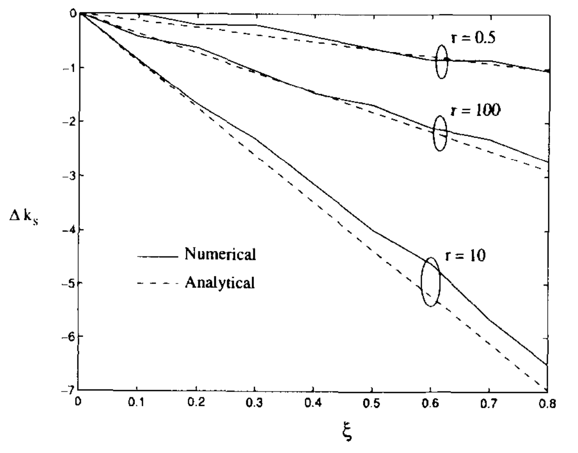

Fig. 5. Evolution of the normalized angular frequency shift, Δk_s, for the numerical solution (solid line) and model Eq. (6) (dashed line).

$$\omega(\xi) = -8\beta\gamma r^2\xi/15, \tag{12}$$

$$\nu(\xi) = 4\beta\gamma r^2\xi^2/15\,, \tag{13}$$

$$\alpha(\xi) = 2(4\beta\gamma r^2/15)^2\xi^3/3\,. \tag{14}$$

Moreover, in this regime, the lateral and angular deflection x_d and θ_d can be obtained similarly from Eqs. (12)–(14). In particular, $x_d = -(n_e^3 r_{33} k_0)^2 (K_B T/15e) E_0 r^2 z^2$ and $\theta_d = -2(n_e^3 r_{33} k_0)^2 (K_B T/15e) E_0 r^2 z$. Furthermore, it is noteworthy pointing out that the low-intensity behavior of $K(r)$, i.e. $K(r) = -2r^2/15$, is in agreement with the results of Fig. 3.

In summary, the self-deflection of steady-state bright PR solitons arising from diffusion effects has been systematically investigated. By employing numerical techniques we have found that the self-bending of these solitary optical beams is approximately adiabatic. This process was further studied using a perturbative model. We have found that the center of the solitary beam moves on a parabolic trajectory, whereas its central spatial frequency component shifts linearly with the propagation distance. Moreover, the dependence of these lateral and angular deflections on the value of the external electric field, E_0, and on the parameter r, where $r = I_{max}/I_d$, was also considered in detail.

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Erratum

Self-deflection of steady-state bright spatial solitons in biased photorefractive crystals (Optics Comm. 120 (1995) 311) ☆

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On p. 312, the quantity β should be

$$\beta = (k_0 x_0)^2 (n_e^4 r_{33}/2) E_0.$$

On p. 312, Eq. (3) should correctly read as follows:

$$\left(\frac{\mathrm{d}y}{\mathrm{d}s}\right)^2 = \frac{2\beta}{r}\left[\ln(1+ry^2) - y^2 \ln(1+r)\right].$$

On p. 313, Eq. (4) should be

$$U(\xi, s) = r^{1/2} y[s+\nu(\xi)] \exp(\mathrm{i}\{\mu\xi + \omega(\xi)[s+\nu(\xi)] + \alpha(\xi)\}).$$

On p. 314, Eq. (11) should correctly read as follows:

$$\mathrm{i}U_\xi + \tfrac{1}{2}U_{ss} - \beta U + \beta |U|^2 U + \gamma(|U|^2)_s U = 0.$$

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