# A Neural network approach to damage detection in Euler-Bernoulli beams subjected to external forces

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***Abstract*—The aim of this contribution is to present two methods for online damage detection in Euler-Bernoulli beams subjected to external forces. Both methods detect damage by tracking changes in the beam parameters. Here, this change is assumed to occur in time, but not in space; that is, it occurs at a certain time instant, being the same along the beam. The input to the methods consists of the beam vibration data collected at different points. The first method is based on the use of a single Hopfield neural network. At each time instant, this network produces an estimate of the beam parameters and this estimate is the same for all beam points. In turn, the second method combines several Hopfield neural networks. At each time instant, each network produces an initial estimate of the parameters at a certain beam point and the estimates of neighbouring points are then combined to produce a final estimate at each point.**

***Index Terms*—Euler-Bernoulli beam, Damage detection, Hopfield neural network.**

## I. Introduction

Detecting structural damage in an early growth stage is important for economical and life-safety reasons. Damage detection can be regarded as an identification problem, where the monitored process is represented by a parametric model. It is assumed that damage corresponds to a change in the model parameters and that it can be detected by tracking parameter variations through appropriate estimation techniques [1]. More concretely, the basic idea is that model parameters (notably frequencies, mode shapes, and modal damping) are functions of the physical properties of the structure (mass, damping, and stiffness). Therefore, changes in these physical properties will cause changes in the model parameters. Thus, parameter identification and monitoring constitutes a good method for early damage detection. Online damage detection, as determined by changes in the dynamic properties or response of structures, is a subject that has received considerable attention. Traditional techniques include the recursive least-squares and the Kalman filter [2]. A more recent technique, based on the Hopfield neural network originally proposed in [3], has already shown a superior performance in practice [4].

Here two methods for online damage detection in Euler-Bernoulli beams subjected to external forces are presented. Both methods detect damage by tracking changes in the beam parameters. Here, this change is assumed to occur in time, but not in space; that is, it occurs at a certain time instant, being the same along the beam. The input to these methods consists of the beam vibration data collected at different points. The first method is based on the use of a single Hopfield neural network. At each time instant, this network produces an estimate of the beam parameters and this estimate is the same for all beam points. In turn, the second method combines several Hopfield neural networks. At each time instant, each network produces an initial estimate of the parameters at a certain beam point and the estimates of neighbouring points are then combined to produce a final estimate at each point.

The model to describe the relationship between the beam's deflection and the applied load used here is the damped Euler-Bernoulli model. Due to its simplicity this model is an important tool in structural and mechanical engineering [5].

This paper is organized as follows. Section II presents the damped Euler-Bernoulli model using to simulate the beam data. Section III describes the identification methods by a single Hopfield Neural Network as well as by the combination of several Hopfield Neural Networks. The Section IV contains the simulation results, and the conclusions are drawn in Section V.

## II. Euler-Bernoulli model

In this paper the Euler-Bernoulli (E-B) model is considered to simulate the beam vibration data in order to apply the proposed identification method. This section presents a method for obtaining a state-space representation of the model, which is more convenient for simulation purposes.

### A. Model description

The E-B model can be represented by the following fourth order differential equation:

$$\begin{aligned}\frac{\partial^2}{\partial t^2}\,w(t,x) \quad &+ \quad \theta_1 \frac{\partial}{\partial t} w(t,x) \\ &+ \quad \theta_2 \frac{\partial^4}{\partial x^4} w(t,x) \;= \theta_3\, q(t,x), \qquad (1)\end{aligned}$$

where $w(t,x)$ is the transversal displacement, $q(t,x)$ is the external force at location $x$ and at instant $t$, and $\theta_1 = \frac{c}{\mu}$, $\theta_2 = \frac{EI}{\mu}$ and $\theta_3 = \frac{1}{\mu}$ are the beam parameters to be identified, which are related with physical properties of the structure. The

parameters are considered to be constant along the beam and the external force $q(t,x)$ is a harmonic excitation, $F\,cos(\omega\,t)$, with frequency $\omega$ and amplitude $F$. This external excitation is applied at the middle of the beam. The beam with length $L$ is assumed to be clamped-clamped, which is in terms of boundary conditions translated as follows,

$$\begin{aligned} w(t,x)\big|_{x=0,L} &= 0 \\ \frac{\partial}{\partial x} w(t,x)\bigg|_{x=0,L} &= 0\ . \end{aligned} \tag{2}$$

### B. State-space realization

In order to get a state-space model, the Galerkin method [6] was used. The idea of this method is to expand the function $w(t,x)$ in an orthonormal basis for the spatial components with time-dependent coefficients, and then use the PDE itself to generate ODEs for those time-dependent coefficients. For that purpose the transversal displacement $w(t,x)$ is separated in the time and the space domains, *i.e.*,

$$w(t,x) = f(t)\,g(x), \tag{3}$$

Substituting (3) in (1) and (2), considering $q(t,x) = F\delta\left(x-\frac{L}{2}\right)cos(\omega\,t)$, the Euler-Bernoulli equation and the boundary conditions become:

$$\begin{aligned} f''(t)\,g(x) \;+\; \theta_1\,f'(t)\,g(x) \;+\; \theta_2\,f(t)\,\frac{d^4\,g(x)}{dx^4} \\ = \;\theta_3\,F\delta\left(x-\frac{L}{2}\right)cos(\omega\,t), \end{aligned} \tag{4}$$

$$\begin{aligned} f(t)\,g(x)\big|_{x=0,L} &= 0 \\ f(t)\frac{d}{d\,x}g(x)\bigg|_{x=0,L} &= 0\ , \end{aligned} \tag{5}$$

where $^{(\prime)}$ represents the time derivative. Multiplying (4) by the spatial function $g(x)$ and integrating with respect to $x$ between 0 and $L$, the PDE (4) is converted in the following ODE,

$$f''(t) \;+\; \theta_1\,f'(t) \;+\; \theta_2\,\zeta^4\,f(t) \;=\; \theta_3\,K\,F\,cos(\omega t), \tag{6}$$

where $K = \frac{g(L/2)}{m}$, $m = \int_0^L (g(x))^2\,dx$, and $g(x)$ is computed from the natural vibration form for a clamped-clamped beam, being given by [7]:

$$g(x) = cosh(\zeta\,x) - cos(\zeta\,x) - \alpha\,(sinh(\zeta\,x) - sin(\zeta\,x))\,, \tag{7}$$

where $\zeta$ is one of the solutions of the equation:

$$cos(\zeta\,L)cosh(\zeta\,L) = 1, \tag{8}$$

and $\alpha$ is

$$\alpha = \frac{cos(\zeta\,L)cosh(\zeta\,L)}{sinh(\zeta\,L)sin(\zeta\,L)}. \tag{9}$$

Taking a discretization interval of $\Delta x$, a spatial discretization vector $\overline{W}$ is constructed containing the values of $g$ at the discretization points, *i.e.*,

$$\overline{W} = \begin{bmatrix} g(0) \\ g(\Delta x) \\ \vdots \\ g(L) \end{bmatrix} \in \Re^m, \tag{10}$$

with $m = \frac{L}{\Delta x} + 1$. Multiplying both sides of the equation (6) by the vector $\overline{W}$ and considering $W(t) = f(t)\overline{W}$ as the corresponding spatial discretization of $w(t,x)$, the time evolution of $W(t)$ is then described by the ODE:

$$W''(t) \;+\; \theta_1\,W'(t) \;+\; \theta_2\,\zeta^4\,W(t) \tag{11}$$

$$= \;\theta_3\,K\,F\,cos(\omega t)\overline{W}. \tag{12}$$

Defining $Q(t) = F\,cos(\omega\,t)$, $X_1(t) = W(t)$ and $X_2(t) = W'(t)$ the following state-space model is obtained:

$$\begin{cases} X'(t) &= \Phi(\theta)\,X(t) + B(\theta)\,Q(t), \\ W(t) &= C\,X(t) \end{cases} \tag{13}$$

with

$$\begin{aligned} \Phi(\theta) &= \begin{bmatrix} \underline{0}_{(m\times m)} & I \\ -\zeta^4\theta_2\,I & -\theta_1\,I \end{bmatrix} \quad \text{and} \\ B(\theta) &= \begin{bmatrix} \underline{0}_{(m\times 1)}\cdot \\ \\ \theta_3\,K\,\overline{W} \end{bmatrix} \end{aligned} \tag{14}$$

where $I$ is the identity matrix with dimension $m = \frac{L}{\Delta x} + 1$ and $X(t) = \left[(W(t))^T \quad (W'(t))^T\right]^T$. Moreover, $\theta = [\theta_1 \;\; \theta_2 \;\; \theta_3]^T$ with $\theta_1 = \frac{c}{\mu}$, $\theta_2 = \frac{EI}{\mu}$ and $\theta_3 = \frac{1}{\mu}$ is the parameter vector. The initial condition for this model $X(0) = [W(0) \;\; W'(0)]^T = \left[\lambda_0\,\overline{W} \;\; \lambda_1\,\overline{W}\right]$ with $\lambda_0 = f(0)$ and $\lambda_1 = f'(0)$.

## III. Identification method

As mentioned before damage in a beam can be detected by a variation in its parameters. Therefore, in order to detect damage it is necessary to identify these parameters and to monitor their evolution in time and space. In this section two identification methods for online damage detection in Euler-Bernoulli beams are described. The first method is based on the use of a single Hopfield neural network whereas the second method combines several Hopfield neural networks.

### A. Single Hopfield Neural Network

The Hopfield Neural Network (HNN) considered here was proposed in [3]. It is used to produce estimate of the beam parameters at each time instant, which is the same for all beam points. In order to apply this network it is necessary to first write the model (13) in the form $y(t) = A(t)\,\theta$ where $y(t)$ and $A(t)$ are a certain vector and a certain matrix, respectively, computed from the system data, and $\theta$ is the vector of model parameters to be estimated online. This yields,

$$\underbrace{\begin{bmatrix} X_1'(t) - X_2(t) \\ X_2'(t) \end{bmatrix}}_{y(t)} = \underbrace{\begin{bmatrix} \underline{0}_{(m\times 1)} & \underline{0}_{(m\times 1)} & \underline{0}_{(m\times 1)} \\ -X_2(t) & -\zeta^4 X_1(t) & K\overline{W}\,Q(t) \end{bmatrix}}_{A(t)} \underbrace{\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}}_{\theta}. \tag{15}$$

The HNN equations consist in the following discrete time model,

$$\hat{\theta}(t+1) = \hat{\theta}(t) + \frac{\Delta t}{\gamma\beta} D_\gamma\left(\hat{\theta}(t)\right) A^T(t) \times \\ \times \left(y(t) - A(t)\hat{\theta}(t)\right), \tag{16}$$

where $\Delta t$ is the time step, $\gamma$, $\beta$ are parameters that can be tuned to adjust the network performance and $D_\gamma(\hat{\theta}(t,x))$ is the following matrix involving $\gamma$ and $\hat{\theta}(t,x)$

$$D_\gamma(\hat{\theta}(t)) = \begin{bmatrix} \gamma^2 - \hat{\theta}_1(t)^2 & 0 & 0 \\ 0 & \gamma^2 - \hat{\theta}_2(t)^2 & 0 \\ 0 & 0 & \gamma^2 - \hat{\theta}_3(t)^2 \end{bmatrix}.$$

### *B. Combination of Hopfield Neural Networks*

The aim of the combination of Hopfield Neural Networks is to allow sharing information concerning different beam points. To each point a different network is associated. If $n$ points are selected in the beam, there will be $n$ networks. Here it is assumed that the beam is spatially discretized in $n$ equally spaced points $x_1$, $x_2$,...,$x_n$. The corresponding $n$ networks are linearly combined as follows:

$$\begin{bmatrix} \hat{\theta}(t+1,x_1) \\ \hat{\theta}(t+1,x_2) \\ \hat{\theta}(t+1,x_3) \\ \vdots \\ \hat{\theta}(t+1,x_n) \end{bmatrix} = M \begin{bmatrix} H(t,x_1) \\ H(t,x_2) \\ H(t,x_3) \\ \vdots \\ H(t,x_n) \end{bmatrix}, \tag{17}$$

where $H(t,x)$ given by,

$$H(t,x_j) = \hat{\theta}(t,x_j) + \frac{\Delta t}{\gamma\beta} D_\gamma\left(\hat{\theta}(t,x_j)\right) A^T(t,x_j) \times \\ \times \left(y(t,x_j) - A(t,x_j)\hat{\theta}(t,x_j)\right), \tag{18}$$

where $y(t,x_j)$ is a two-component vector formed by the j-th component of $X_1'(t) - X_2(t)$ and the j-th component of $X_2'(t)$, $A(t,x_j)$ is a two-row matrix formed by the corresponding rows of A(t), and the weight matrix, $M$, is

$$M = \begin{bmatrix} \frac{n-1}{n} & \frac{n-(n-1)}{n} & 0 & \cdots & \cdots & \cdots & 0 \\ \frac{n-(n-1)}{2n} & \frac{n-1}{n} & \frac{n-(n-1)}{2n} & 0 & \cdots & \cdots & 0 \\ 0 & \frac{n-(n-1)}{2n} & \frac{n-1}{n} & \frac{n-(n-1)}{2n} & 0 & \cdots & 0 \\ \vdots & & & & & & \\ \vdots & & & & & & \\ \vdots & & & & & & \\ 0 & \cdots & \cdots & \cdots & 0 & \frac{n-(n-1)}{n} & \frac{n-1)}{n} \end{bmatrix}.$$

## IV. Results

This section presents the simulated results obtained by the application of the two proposed methods for online damage detection in Euler-Bernoulli beams subjected to external forces. The model parameters used to simulate the beam vibration data were obtained in [8] as $E = 71.72\times 10^9 N/m^2$, $I = 1.33\times 10^{-11}\,m^4$, $\mu = 0.112\,kg/m$ and $c = 1\,N\,s/m^2$ and this way $\theta_1 = \frac{c}{\mu} = 8.9286\,s^{-1}$, $\theta_2 = \frac{EI}{\mu} = 8.5168\,m^4/s^2$ and $\theta_3 = \frac{1}{\mu} = 8.9286\,m/kg$. The beam length was considered equal to $406\times 10^{-3}\,m$ and the beam was spatially discretized by considering seven equally spaced points $x_1, x_2 \ldots, x_7$.

The damage of the beam is represented by a change in the model parameters. Here the parameter $\theta_2$ is changed and similarly to what happens in [9] this change is a consequence of a variation in the Young's modulus parameter, $E$. A variation of 50% is considered at time $t = 25\,s$.

To analyse the performance of the two methods, the following error measures were considered. For the single Hopfield Neural Network the error was defined as

$$EH(t) = \left\| y(t) - A(t)\,\hat{\theta}(t) \right\|_2 \tag{19}$$

whereas for the combination of the Hopfield Neural Networks, the error was defined as

$$ECH(t) = mean\left\{ \left\| y(t,x_j) - A(t,x_j)\,\hat{\theta}(t,x_j) \right\|_2 \right\} \\ j = 1,\ldots,7 \tag{20}$$

Fig. 1 presents the time evolution of the estimation of the parameter $\theta_2$ obtained by the identification method based on the single Hopfield Neural Network.

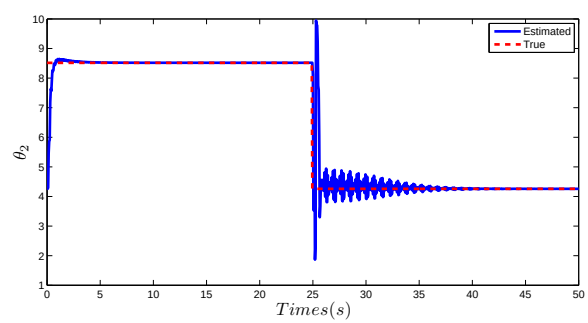


Fig. 1. Time-evolution of the estimation of the parameter $\theta_2$ obtained by a single HNN.

The error $EH(t)$ obtained with this method in the identification of the beam parameters is illustrated in Fig. 2.

To analyse the combination of the Hopfield Neural Networks the equation (20) was used and the obtained error profile is presented in Fig. 3. The estimation of the parameter $\theta_2$ for each time instant and for each beam point performed by this last method is illustrated in Fig. 4.

As can be seen by the results illustrated in Fig. 2 and Fig. 3 both the single HNN and the combination of HNNs produced estimation errors that converge to zero and present a

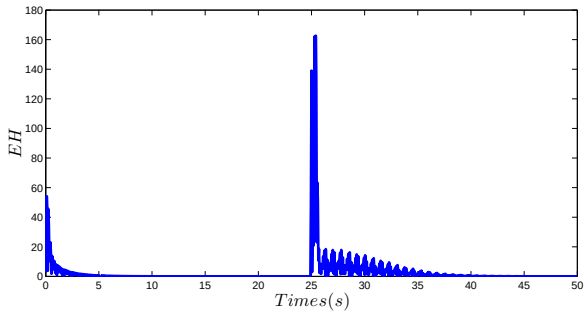


Fig. 2. Time-evolution of the estimation error of the beam parameters obtained by a single HNN.

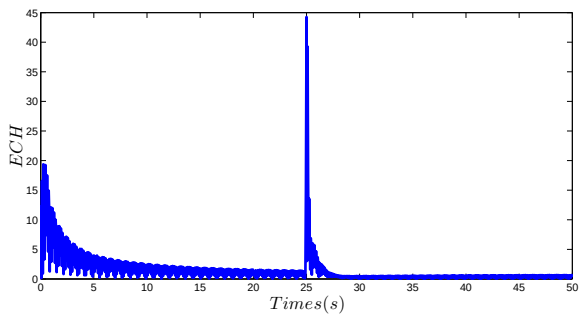


Fig. 3. Time-evolution of the estimation error of the beam parameters obtained by the combination of HNNs.

peak when damage (parameter variation) occurs. Globally the combination of HNNs reacted better to the parameter change. This may constitute an advantage for its use as an estimated even in different contexts. Moreover although this feature has not be exploited in the present work, the combination of several networks seems to be appropriate to treat the case of non-homogeneous beams, where the parameters vary along the beam length.

## V. Conclusion

In this paper preliminary results on the use of Hopfield Neural Networks for online damage detection in Euler-Bernoulli beams actuated by external forces were presented. The first method consists in the use of a single Hopfield Neural Network whose effectiveness was already been proved in [4]. The second method is based on a combination of several Hopfield Neural Networks using at each point information from the neighbouring beam points. This method also turns out to be effective. This encourages its use in the case of non-homogeneous beams, with spatial dependent parameters where a single Hopfield Neural Network cannot be (easily) applied. This is the subject of current investigation.

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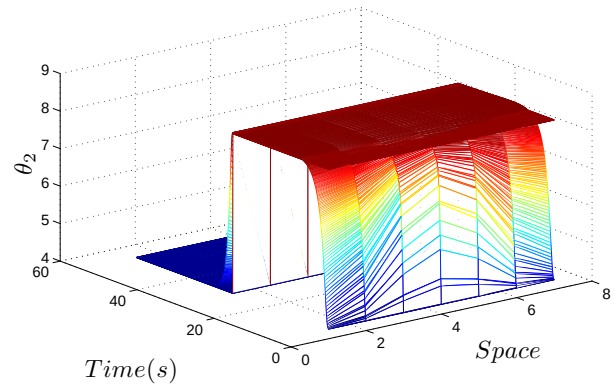


Fig. 4. Time-evolution of the beam parameter estimation obtained by the combination HNNs.

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