Abstracts of the NHOC2014 Workshop

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On quantitative compactness estimates for hyperbolic conservation laws and HJ equations

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Inspired by a question posed by Lax, we wish to study quantitative compactness estimates for the map S_t, $t > 0$ that associates to every given initial data u_0 the corresponding solution $S_t u_0$ of a conservation law or of a first order Hamilton-Jacobi equation. Estimates of this type play a central roles in various areas of information theory and statistics as well as of ergodic and learning theory. In the present setting, this concept could provide a measure of the order of "resolution" of a numerical method for the corresponding equation. In this talk we shall first review the results obtained in collaboration with O. Glass and K.T. Nguyen, concerning the compactness estimates for solutions to conservation laws. Next, we shall turn to the more recent analysis of the Hamilton-Jacobi equation pursued in collaboration with P. Cannarsa and K.T. Nguyen. A control-type analysis of such equations turns out to be fundamental to establish some of these properties.

Normality conditions for state constrained optimal control problems

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A simple example illustrates how, in certain circumstances (when the state constraint is non-smooth), the choice of the left-end point constraint might influence the degeneracy of the necessary conditions for optimality. Is this a common feature for a class of general state constrained optimal control problems? We provide a positive answer to this question, showing that the generalized Euler-Lagrange conditions apply in the normal form for problems in which the state constraint is represented by a convex set, the cost has an integral term, and the left-end point of a minimizer is either away from corners of the state constraint, or it can be freely chosen.

Unbounded controls for stochastic optimal control problems with state constraints

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Recent results related to stochastic optimal control problems with state constraints will be presented involving characterization of the value by the Hamilton-Jacobi approach.

Second order necessary conditions for control-affine problems with state constraints

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We establish new second order necessary conditions for control-affine problems (i.e. with a Hamiltonian that is an affine function of the control) with a scalar control and a scalar state constraint. These optimality conditions extend to the constrained state framework the Goh transform, which is the classical tool for obtaining an extension of the Legendre condition. Then we show how to design a shooting algorithm in order to solve such problems, extending [2]. The presentation is based on the joint work [1] with S. Aronna and B.S. Goh.

References

[1] Aronna S., Bonnans J.F., Goh B.S., Second order necessary conditions for control-affine problems with state constraints. Research report, to appear.

[2] Aronna S., Bonnans J.F., Martinon P., A well-posed shooting algorithm for optimal control problems with singular arcs. *J. Optim. Theory Applications* **158** (2): 419-459, 2013.

Second Order Analysis for
Sparse Optimal Control of Parabolic PDE

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Optimal sparse control problems are considered for parabolic partial differential equations. The non-differentiable objective functional of tracking type includes a quadratic Tikhonov regularization term and the L^1-norm of the control that accounts for the sparsity. Though the objective functional is non-differentiable, a theory of second order sufficient optimality conditions is developed. The second order conditions imply optimality in a L^∞-neighborhood of the state functions. Compared with the calculus of variations, this corresponds to conditions for strong local minima. This is established for Tikhonov regularization parameter $\nu > 0$ and also for the case $\nu = 0$. The second order conditions are used as main assumption for proving the stability of locally optimal solutions with respect to $\nu \to 0$ and with respect to perturbations of the desired state functions.

Maximum principle for optimal control problems with integral equations subject to state and mixed constraints

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On a fixed time interval we consider an optimal control problem with a Volterra type integral equation and a Mayer type cost, subject to endpoint constraints of equality and inequality type, mixed state-control constraints of inequality and equality type, and pure state constraints of inequality type. The main assumption is the uniform linear–positive independence of the gradients of active mixed constraints with respect to the control. For this problem we prove the Maximum Principle as a necessary condition for the strong (or Pontryagin) minimality, which generalize MP for similar problems with ODEs. The proof is based on an extension of the control system by introducing sliding mode controls and using a relaxation theorem that allows one to approximate solutions of the extended system by solutions of the original system. We thus obtain a family of optimal control problems with extended systems, for each of which we apply the stationarity condition (Euler–Lagrange equation, obtained earlier in our joint work with N. Osmolovskii), and then "compress" these conditions into a universal condition that has the form of MP.

High order numerical schemes and value iteration

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Numerical computation of the value function for the infinite horizon problem relies on well understood ideas when monotone schemes are used. In this case, value iteration converges to a unique discrete solution, and in turn the discrete solution converges (via Barles-Souganidis Theorem) to the value function. When more accurate, high-order space approximations are considered, however, things become more complex. We will discuss in this talk uniqueness and convergence issues for such numerical solutions, and single out the property of ε-monotonicity as a key requirement for the numerical scheme.

Error estimates for the Euler discretization of an optimal control problem with first-order state constraints

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We study the error introduced in the solution of an optimal control problem with state constraints, for which the trajectories are approximated with a classical Euler scheme.

Although some results in the subject have been already presented in 2001 by Hager and Dontchev our analysis is proposed here as an another point of view on the issue with some special features, peculiarities, and original aspects.

We derive our results using some techniques coming from the sector of perturbation analysis; such procedure, under general assumptions which will be studied and discussed, provides some tight estimates in the uniform norm. (Joint work with F. Bonnans)

Nondegenerate and Normal forms of the Maximum Principle for Optimal Control Problems with State Constraints

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When applying well-known forms of the maximum principle to optimal control problems with state constraints, we might obtain the so-called abnormal forms – forms in which the scalar multiplier is zero and so the conditions do not involve the objective function –, or degenerate forms – forms in which both the scalar and adjoint multipliers are zero and so the conditions are satisfied for any candidate. For problems satisfying certain constraint qualifications, we can establish necessary optimality conditions in which the scalar and adjoint multipliers do not vanish everywhere, guaranteeing that the necessary conditions are informative to select minimizers. These nondegenerate and normal forms of the necessary conditions are relevant not only to be used in optimization algorithms, but also to establish regularity properties and devise second-order conditions. Several types of constraint qualifications have been proposed in the literature, permitting to deduce necessary conditions of optimality for problems with different degrees of generality. In this talk, we review the main types of constraint qualifications proposed, introduce a recently developed type, and explore relations between them.

First and second order sensitivity relations in optimal control

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Sensitivity analysis in optimal control brings on relations between the co-state of the maximum principle and various generalized differentials of the value function along a given optimal trajectory. For Frechet superdifferentials these relations help to express the optimal synthesis and lead to sufficient optimality conditions. In this talk I will discuss some very recent results on sensitivity relations obtained in collaboration with P. Bettiol and R. Vinter for optimal control problems under state constraints and also new second order sensitivity relations obtained together with P. Cannarsa and T. Scarinci.

Sparse Mean-Field Optimal Control

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Starting with the seminal papers of Reynolds (1987), Vicsek et. al. (1995) Cucker-Smale (2007), there has been a flood of recent works on models of self-alignment and consensus dynamics. Self-organization has been so far the main driving concept. However, the evidence that in practice self-organization does not necessarily occur leads to the natural question of whether it is possible to externally influence the dynamics in order to promote the formation of certain desired patterns. Once this fundamental question is posed, one is also faced with the issue of defining the best way of obtaining the result, seeking for the most "economical" manner to achieve a certain outcome. The first part of this talk precisely addresses the issue of finding the sparsest control strategy for finite dimensional models in order to lead us optimally towards a given outcome. In the second part of the talk we introduce the rigorous limit process connecting finite dimensional sparse optimal control problems with ODE constraints to an infinite dimensional optimal control problem with a constraint given by a system of ODE for the leaders coupled with a PDE of Vlasov-type, governing the dynamics of the probability distribution of the followers. The technical derivation of the sparse mean-field optimal control is realized by the simultaneous development of the mean-field limit of the equations governing the followers dynamics together with the Gamma-limit of the finite dimensional sparse optimal control problems.

Optimal control techniques for trajectory generation in robotics and autonomous vehicles

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The talk addresses the problem of providing suitable reference trajectories in motion planning of autonomous vehicles. Among the various approaches to compute a reference trajectory, our aim is to find those trajectories which optimize a given performance criterion (fuel consumption, comfort, safety, time, etc) and obey constraints (collision avoidance, safety regions, control bounds, etc). This task leads to optimal control problems, which need to be solved efficiently. To this end we use direct discretization schemes and model-predictive control in combination with sensitivity updates to predict optimal solutions in the presence of perturbations. Applications in robotics with collision avoidance, flight path optimization, and autonomous driving will be presented. In particular, a distributed control algorithm for traffic scenarios with many autonomous vehicles that use car-to-car communication will be investigated.

Generalized Nash Equilibrium Problems in Banach Spaces

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A class of noncooperative Nash equilibrium problems is presented, in which the feasible set of each player is perturbed by the decisions of their competitors via a convex constraint. In addition, for every vector of decisions, a common "state" variable is given by the solution of an affine linear equation. Due to the presence of an additional constraint on the state, the problem cannot be reduced to the classical setting as considered in work by Nash. The resulting problem is therefore a generalized Nash equilibrium problem (GNEP).

The existence of an equilibrium for this problem is demonstrated and first order optimality conditions are derived under a constraint qualification. An approximation scheme is proposed, which involves the solution of a parameter-dependent sequence of standard Nash equilibrium problems. This leads to the need for a new type of path-following strategy, which uses a value function based in part on the Nikaido-Isoda function. Function-space-based numerics for parabolic GNEPs and a spot-market model are developed, and numerical results are presented.

Computational Optimal Control Using Galerkin Methods

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In the first part of the talk, I will give a brief review on related computational methods, such as pseudospectral methods and their convergence theorems. Then a new numerical technique will be presented for solving optimal control problems, which is a direct method that calculates optimal trajectories by discretizing the system dynamics using Galerkin numerical techniques and approximating the cost function with quadrature. Then I will introduce some results on the feasibility and convergence of the computational method. Both strong and weak forms of boundary conditions will be addressed. Algorithms will be introduced and illustrated using several numerical examples.

The Theory of Trajectory Tubes - A Mathematical Tool for control under realistic information

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The theory of control under realistic information, especially in closed-loop form and in deterministic setting, inevitably requires to describe the achieved solutions in the form of trajectory tubes instead of isolated trajectories. This presentation indicates the motivations, problem settings and related solution theory for such problems based on appropriate versions of the Hamiltonian formalism. Further discussed are related computational methods applicable to large dimensions that occur in these problems

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“More is Not Necessarily Better”: Optimality of Bang-Singular Controls for Models of Anti-Cancer Therapies

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Mounting medical evidence that in cancer treatments "more is not necessarily better" has generated a search for what could be called the biologically optimal dose (BOD). In this talk, some mathematical models for anti-cancer therapies will be analyzed as optimal control problems with tools and methods that also include geometric optimal control. The models under consideration will take into account tumor heterogeneity as well as elements of the tumor microenvironment like the tumor vasculature or tumor-immune system interactions. Challenges related to finding optimal solutions, like establishing optimality of singular controls, bang-singular junctions, synthesis of controlled trajectories, chattering controls will be addressed. The connections between the resulting structures of optimal controls, particularly singular and bang-singular controls, and new therapy protocols like metronomic chemotherapy, chemo-switch protocols, adaptive therapy, that have arisen in the search for a biologically optimal dose will be discussed.

Joint research with Heinz Schättler (Washington University, St. Louis, USA).

Necessary Conditions of Optimality for Infinite Horizon Constrained Dynamic Control Problems

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In this talk necessary conditions of optimality in the form of a maximum principle for infinite horizon optimal control problems whose solutions converge asymptotically to equilibria at infinity are discussed. The considered class of problems feature constraints at both trajectory endpoints, and a cost functional that depends on the state at the final time. The main novelty of these optimality conditions concerns a new transversality relation at infinity which improves previous comparable results in the sense that a better trade-off between the range of applicability and the information provided by the optimality conditions is achieved.

Spectral Methods for the solution of infinite horizon optimal control problems

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We consider a class of infinite horizon optimal control problems in Lagrange form involving the Lebesgue integral with a weight function in the objective. This special class of problems arises in the theory of economic growth, in processes where the time T is an exponentially distributed random variable and in problem of asymptotic feedback stabilization of a linear control system. The problem is formulated as optimization problem in Hilbert Spaces. The remarkable on this statement is the choice of Weighted Sobolev and Weighted Lebesgue spaces as state and control spaces respectively. These considerations give us the possibility to extend the admissible set and simultaneously to be sure that the adjoint variable belongs to a Hilbert space. For the class of problems proposed, we can proof an existence result as well as Pontryagin's Maximum Principle including transversality conditions. A spectral method is developed to construct a numerical scheme for the solution of the problem.

Joint work with A. Burtchen and M. Bähr.

Sensitivity Analysis and Real-Time Control Techniques for Bang-Bang Control Problems

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We consider parametric optimal control problems, where the control variable enters the system dynamics and objective linearly. This type of optimal control problems arises in various applications in biology, biomedicine, chemical and electrical engineering, economics, mechanics and physics. The evaluation of necessary optimality conditions (Pontryagin's Minimum Principle) shows that the optimal control is a concatenation of bang-bang and singular arcs. This talk focusses on bang-bang controls. First, we review second-order sufficient conditions (SSC) which are based on SSC for the Induced Optimization Problem (IOP). The IOP is a finite-dimensional optimization problem, where switching times of the bang-bang control are taken as optimization variables. When SSC for the IOP hold, we can apply a classical sensitivity result in finite-dimensional optimization which states the the optimization vector and the Lagrange multiplier are locally differentiable functions of the system parameters. The parametric sensitivity derivatives are computed as a byproduct of the direct computation of switching times. Then an efficient real-time control approximation of perturbed bang-bang controls can be based on Taylor expansions of the parametric switching times, where all sensitivity derivatives have been computed off-line. We illustrate sensitivity analysis and real-time control on several numerical examples. The computations are performed using the optimal control package NUDOCCCS developed by C. Büskens. The talk is based on results obtained jointly with C. Büskens (Bremen) and N.P. Osmolovskii (Moscow and Radom).

General limit values for long-run optimal control problems

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The asymptotic behavior of the values is a classical problem in ergodic control. Usually it consists in studying the limit of the value functions defined through Cesaro or Abel means when the averaging parameter converges. We will show that - when a limit value exists - it is somehow independent to the particular choice of averaging method.

Decomposition methods for Variational Inequalities

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We consider decomposition methods for variational inequalities, related to the classical Dantzig–Wolfe and Benders decompositions of linear programs. Our approach is rather general, in that it can be used with certain types of set-valued or nonmonotone operators, as well as with various kinds of approximations in the subproblems of the functions and derivatives in the single-valued case. Also, subproblems may be solved approximately. Convergence is established under reasonable assumptions. We also report numerical experiments for computing variational equilibria of the game-theoretic models of energy markets. Our numerical results illustrate that the decomposition approach allows to solve large-scale problem instances otherwise intractable if the widely used PATH solver is applied directly, without decomposition.

This is joint work with P. Luna (Postdoctoral fellow at Federal University of Rio de Janeiro) and M. Solodov (IMPA- Rio de Janeiro).

Weak KAM techniques for singularly perturbed optimal control problems

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We tackle singularly perturbed optimal control problems with slow and fast variables via approximation of the corresponding Hamiltonians by coercive ones. We study the latter using tools issued from weak KAM theory and transfer asymptotically information to the original problems. In this way we recover already known facts assuming the fast variable to vary in a compact ambient, and establish new results replacing compactness by suitable coercivity conditions.

Globalizing stabilized SQP by smooth primal-dual exact penalty function

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An iteration of the stabilized sequential quadratic programming method (sSQP) consists in solving a certain quadratic program in the primal-dual space, regularized in the dual variables. The advantage with respect to the classical sequential quadratic programming (SQP) is that no constraint qualifications are required for fast local convergence (i.e., the problem can be degenerate). However, unlike for SQP, designing natural globally convergent algorithms based on the sSQP idea proved quite a challenge and, currently, there are very few proposals in this direction. For equality-constrained problems, we suggest to use for the task the smooth two-parameter exact penalty function, which is the sum of the Lagrangian with squared penalizations of the violation of the constraints and of the violation of the Lagrangian stationarity with respect to primal variables. Global convergence is established under natural assumptions. Moreover, we show that the globalized algorithm preserves the superlinear rate of sSQP under the same weak condition.

Dynamic optimization challenges in networked vehicle systems: Are we missing something?

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The dynamic optimization challenges arising in networked vehicle systems are discussed in the framework of coupled physical and computational dynamics. The challenges are discussed with reference to classical control problems of optimization, invariance and attainability for systems governed by the laws of physics and computation. Directions for future research are discussed with special emphasis on the aspects of coupled dynamics and dynamic structure that seem to be missing in the literature.

Computational Methods in the Fractional Calculus of Variations

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The fractional calculus of variations and fractional optimal control are generalizations of the corresponding classical theories, allowing problem modeling and formulations with arbitrary order derivatives and integrals. Because of the lack of analytic methods to solve such fractional problems, numerical techniques are developed. Here, we mainly investigate the approximation of fractional operators by means of series of integer-order derivatives and generalized finite differences. Direct and indirect methods for solving fractional variational problems are presented.

Optimal control on infinite time-horizon: the maximum principle revisited

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The talk will begin with a short review of necessary optimality conditions for ODE optimal control problems. Then a recent result by S. Aseev and the speaker will be presented.

The following features of the result are new:

- weak meaning of optimality (allowing for divergent objective integrals);
- normal form of the maximum principle;
- no continuity requirement for the dependence of the data on the control;
- weak regularity assumption (locally unbounded controls are allowed).

Some particular cases will be discussed and applications to several "classical" and new problems will be presented. Finally, extensions for classes of distributed control problems will be discussed.

Averaged Control

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This lecture is devoted to address the problem of controlling uncertain systems submitted to parametrized perturbations.

We introduce the notion of averaged control according to which the average of the states with respect to the uncertainty parameter is the quantity of interest. We observe that this property is equivalent to a suitable averaged observability one according to which the initial datum of the uncertain dynamics is to be determined by means of averages of the observations done.

We will first discuss this property in the context of finite-dimensional systems to later consider Partial Differential Equations, mainly, of wave and parabolic nature.

As we shall see, surprisingly, the averaging process with respect to the unknown parameter may lead a change of type ion the PDE under consideration from hyperbolic to parabolic, for instance, significantly affecting the expected control theoretical properties.

We will also present some open problems and perspectives of future developments.

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ITN SADCO Initial Training Network Sensitivity Analysis for Deterministic Controller Design

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