PARAMETRIC STUDY OF FATIGUE CRACK GROWTH

Luís D. C. Ramalho[1,2,*], Paulo M. S. T. de Castro[2]

[1] Faculdade de Engenharia da Universidade do Porto, Rua Dr. Roberto Frias, 4200-465, Porto, Portugal
[2]Instituto de Ciência e Inovação em Engenharia Mecânica e Engenharia Industrial, Rua Dr. Roberto Frias, 4200-465, Porto, Portugal Portugal
[*] Persona de contacto: lramalho@inegi.up.pt

ABSTRACT

Systematic parametric studies of fatigue crack propagation laws are not commonly found in the technical literature. Nevertheless, such studies are interesting to make explicit the dependence of fatigue life with the variation of the constants that characterize the mechanical behaviour of the material, such as the Paris law constants, fracture toughness or the applied force/stress.

The parametric studies should contemplate the influence of several important aspects such as the various possible forms of the relationship $K = f(a, \dots)$. As concerns this aspect, while remote loading generally implies K increasing with a, point loading acting upon the crack faces and opening the crack imply that K increases with the reduction of a, assuming the remaining conditions constant. These circumstances led to the interest in broadening existing parametric analyses by including explicit treatment of aspects such as those mentioned above.

It is shown that C and m, the Paris' law parameters, are the most influential on life, followed by a_0 and last by K_c which has a small influence on life. This parametric study should help designers choose appropriate materials for their desired applications by considering their properties and the effect of those properties on fatigue crack propagation life, systematically shown in the communication.

KEYWORDS: Fracture, Fatigue, Micro-tests, Notches

INTRODUCTION

Engineers generally want to have durable structures, but in some applications, like cars and airplanes, they also need to be as lightweight as possible to reduce fuel consumption. Knowledge about fatigue is essential to achieve both those goals simultaneously, because it allows to determine how many loading cycles a given structure can support before it fails.

Fatigue mechanics can be traced back to the XIX century [1] to authors such as Albert [2] and Wohler [3]. Four different approaches can be taken to fatigue, namely, the stress-life $(S-N)$ model, the strain life $(\varepsilon-N)$ model, the fatigue crack growth model $(da/dN-\Delta K)$, and the two stage model which combines the strain model and the crack growth model [4] The present report focuses on the fatigue crack growth model, which combines the stress intensity factor (K) from Linear Elastic Fracture Mechanics (LEFM), proposed by Irwin [5], with fatigue using Paris' Law [6], [7].

Parametric studies of Paris' law were presented by Mínguez and his colleagues [8], [9]. These studies are interesting to make explicit the dependence of fatigue life with the variation of the constants that characterize the mechanical behaviour of the material, such as the Paris' law constants and the critical stress intensity factor (K_c). But they do not contemplate the influence of several important aspects such as the various possible forms of the relationship $K = f(a, \dots)$. As concerns the latter aspect, recall that while remote loading generally implies K increasing with a, point loading acting upon the crack faces and opening the crack imply 'ceteris paribus' - i.e. assuming the remaining conditions constant - that K increases with the reduction of a. These circumstances led to the interest in broadening the previous work by [8], [9]. The main aim of this work is to determine the influence of various material parameters in the fatigue life of three different cases, namely, an infinite plate with a central crack subjected to a remote stress loading and an infinite plate under point loading at the crack centre.

FATIGUE

2.1. Infinite Plate Under Stress Loading

Even though crack propagation occurring under fatigue shows some yielding at the crack tip, it is small when compared to monotonic loading, even for materials with significant plasticity [4], so crack growth in fatigue can be described similarly to Linear-Elastic Fracture Mechanics (LEFM), except on very high loads and low cycle fatigue cases [10]. The stress intensity factor range (ΔK) is defined as follows:

$$\Delta K = Y\Delta\sigma\sqrt{\pi a} \tag{1}$$

Y being the geometry factor of the problem, which is equal to 1 in the infinite plate, $\Delta\sigma$ is the stress range and a is the half-crack length. The critical half-crack size (a_c) is a crack for which K exceeds its critical value (K_c) for a given maximum stress σ_{max}, in the analysed case of $R = 0$ and $\sigma_{max} = \Delta\sigma$. So a_c is defined as:

$$\mathrm{a_c} = \frac{\mathrm{K_c^2}}{(\mathrm{Y}\sigma_{\max})^2\pi} \tag{2}$$

In the early 1960s Paris and his colleagues proposed what is now known as Paris' law [6], [7], which relates ΔK to the crack growth rate, being described as follows for the infinite plate problem:

$$\frac{da}{dN} = C(\Delta K)^m \tag{3}$$

This equation can be integrated for the infinite plate subjected to a remote stress loading. Its final form is commonly written as:

$$N = \frac{1}{\left(\frac{m}{2}-1\right)C\left(\Delta\sigma\sqrt{\pi}\right)^m}\left(\frac{1}{a_0^{m/2-1}} - \frac{1}{a^{m/2-1}}\right) = \frac{1}{B}\left(A - \frac{1}{a^M}\right) \tag{4}$$

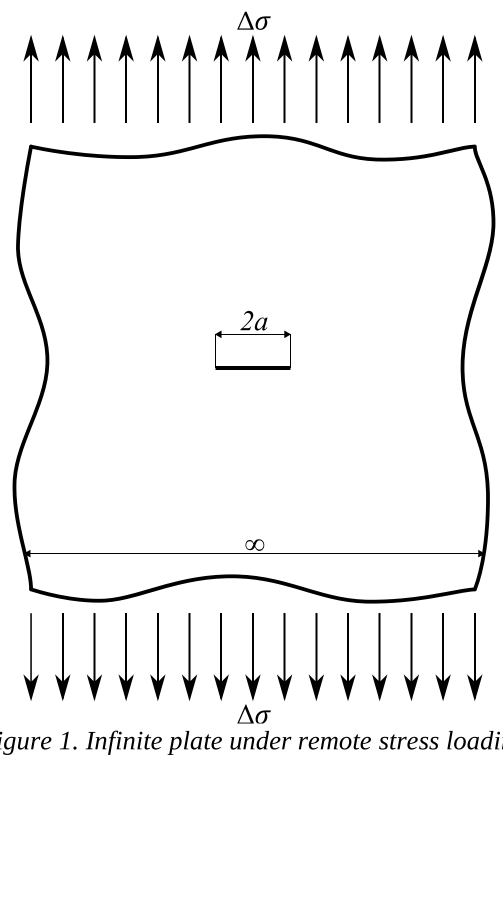

Figure 1. Infinite plate under remote stress loading

Substituting the critical crack size obtained from Eq. (1) into Eq. (4) it is possible to obtain the first critical number of cycles:

$$N_{c1} = \frac{1}{B}\left(A - \frac{1}{a_c^M}\right) \tag{5}$$

This integration can also be written with respect to a after some transformations:

$$a = \frac{1}{(A - BN)^{\frac{1}{M}}} \tag{6}$$

This means that the crack length will tend to infinity when the denominator tends to zero, so the second critical number of cycles is $N_{c2} = A/B$, which can also be considered as the number of cycles needed for the plate to fail, however N_{c1} is always lower than N_{c2}, and in reality a crack cannot propagate until infinity, so N_{c1} is a more correct failure assumption.

2.2. Infinite Plate Under Point Loading

A particular case in fracture mechanics is the infinite plate under point loading whose Westergaard function [11], can be written as [12]:

$$\mathrm{Z} = \frac{\mathrm{P}}{\pi\mathrm{z}}\sqrt{\frac{\mathrm{a}^2}{\mathrm{z}^2 - \mathrm{a}^2}} \tag{7}$$

Considering that the point load is located at the centre of the crack the stress intensity factor in this case can be used to obtain it according to the following equation, [12]:

$$K = \frac{P}{\sqrt{\pi a}} \tag{8}$$

this equation can be used to determine a_c by knowing the material K_c and P_{max}, which is equal to ΔP for $R = 0$ as in the present case:

$$a_c = \frac{{P_{max}}^2}{K_c^2\pi} \tag{9}$$

By substituting (8) into Paris' law:

$$\frac{da}{dN} = C\left(\frac{\Delta P}{\sqrt{\pi a}}\right)^m \tag{10}$$

Paris' law is then integrated similarly to the previous example to obtain the number of cycles needed to reach a particular crack length:

$$N = \frac{1}{\left(1+\frac{m}{2}\right)C\left(\Delta P/\sqrt{\pi}\right)^m}\left(a^{1+\frac{m}{2}} - a_0^{1+\frac{m}{2}}\right) \tag{11}$$

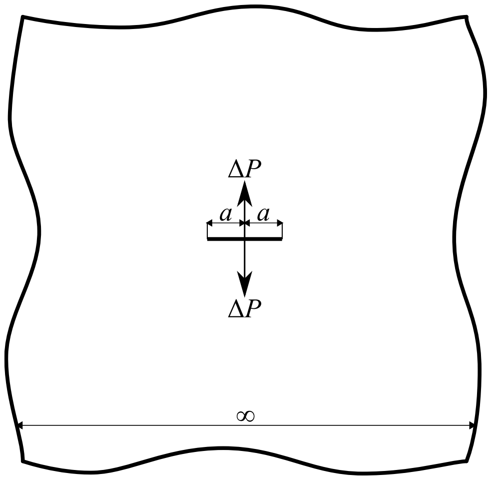

Figure 2. Infinite plate under point loading

PARAMETRIC STUDY

3.1. Infinite Plate Under Stress Loading

In the parametric study conducted for the infinite plate case the default values are: $\Delta\sigma = 100$ MPa; $m = 2.5$ mm/cycle; $C = 1 \times 10^{-11}$; $K_c = 3000$ N/mm$^{3/2}$; $a_0 = 10$ mm, one at a time, these values were changed to determine which ones have a greater influence on the number of cycles. The choice of default values was made based on the range of material properties of various metals presented in [4], being these values within that range.

The analysis of Figure 3 shows that m has the biggest influence in N_f when compared to the other parameters for high loads, low cycle life. However, C also greatly influences fatigue life, and for lower loads it is more influential than m. For both m and C smaller values result in higher N_f, and the inverse happens for higher values of m and C. This happens because m is the slope of the logarithmic plot of Paris' law [8], a lower slope leads to smaller crack growth rates per cycle, and C is the crack growth rate value close to the origin [8], so if it is lower the crack growth rate will remain lower throughout fatigue life.

In Figure 3 a $\Delta\sigma$ plateau is also visible, if $\Delta\sigma$ is higher than that the plateau the plate supports no load cycles, this plateau is increased by increasing K_c or decreasing a_0, and it is decreased by the opposite changes to K_c and a_0. This is due to Eq. (2), by replacing a_c with a_0 and rearranging it is possible to determine the $\Delta\sigma$ at which the plateau occurs ($\Delta\sigma_p$):

$$\Delta\sigma_p = \frac{K_c}{\sqrt{\pi a_0}} \tag{12}$$

which corresponds to a load that implies unstable crack propagation, causing failure after just one load, the monotonic load case. K_c has an influence in the number of cycles before failing only for $\Delta\sigma > 200$ MPa, having minimal effect for loads lower than that.

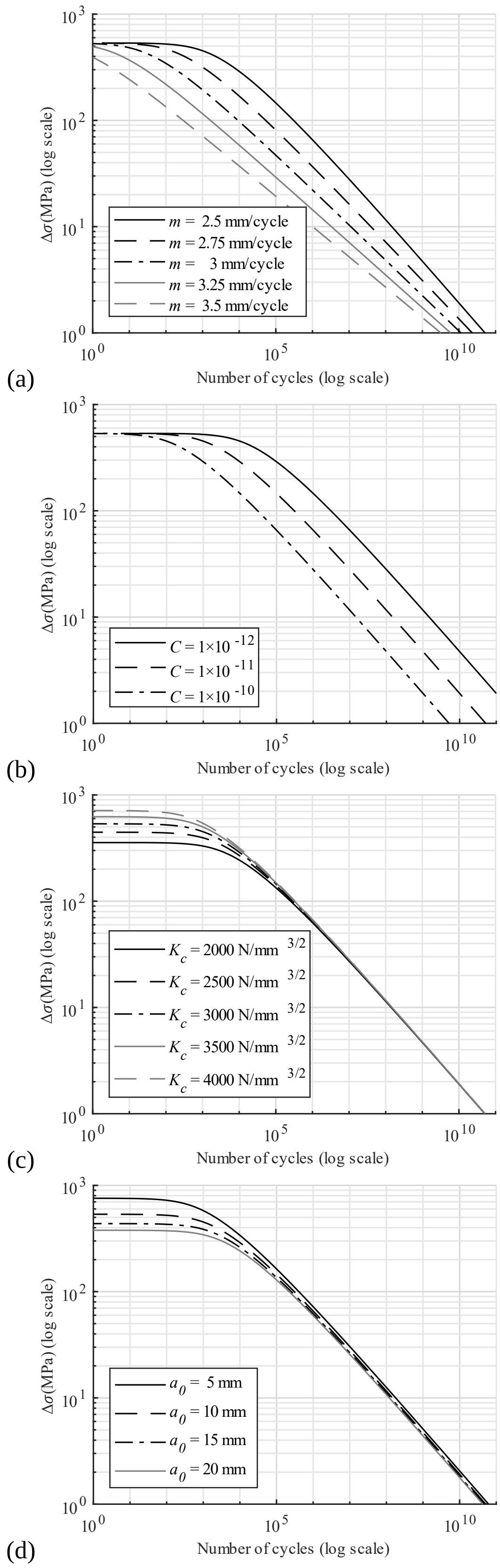

Figure 3. Number of cycles until failure as a function of the stress range, with varying (a) m, (b) C, (c) K_c, (d) a_0

3.2. Infinite Plate Under Point Loading

The second case studied in this work is the infinite plate under a concentrated load at the crack centre. When an infinite plate is subjected to a concentrated load at the crack's centre K tends to decrease with crack size, as shown in Figure 4, made using $P = 2 \times 10^4$ N and $\sigma = 20$ MPa. Therefore, unlike the case of Section 3.1 where after a_c is reached K would continue to increase beyond K_c and the crack propagates in an unstable manner which can be interpreted as complete failure of the plate; in the case of this section, since K decreases with crack length, when everything else remains the same, the interpretation that can be made is that the crack stops propagating unstably when a_c is equal to the K_c of the material, meaning that a_c is in fact the minimum crack length for a given load P and critical stress intensity factor K_c. Beyond a_c the crack grows only due to fatigue according to the Paris law.

In the parametric study conducted for the infinite plate under point loading case the default values are: $\Delta P = 2 \times 10^4$ N; $m = 2.5$ mm/cycle; $C = 1 \times 10^{-11}$; $K_c = 3000$ N/mm$^{3/2}$; $a_0 = 10$ mm, one at a time, these values were changed to assess their influence in the crack evolution, failure was not determined since this type of loading does not have a well-defined failure criterion. Initially, the difference between considering an arbitrary initial crack length or a_c as the initial crack was tested, Figure Figure 5a. It is observed that, as expected, starting from a lower crack length, in the initial cycles there is a difference between both approaches, but as the cycles increase that difference starts disappearing and after 10^5 cycles it is almost null.

In subsequent parametric analysis of this example the $a_0 = a_c$ approach was used because it is more logical in this example. With that in mind, the influence of K_c in crack growth was assessed, Figure 5b, showing that while it influences the initial crack length, it does not change crack length much when the number of cycles is 10^5 or more. The other parametric variations tested,

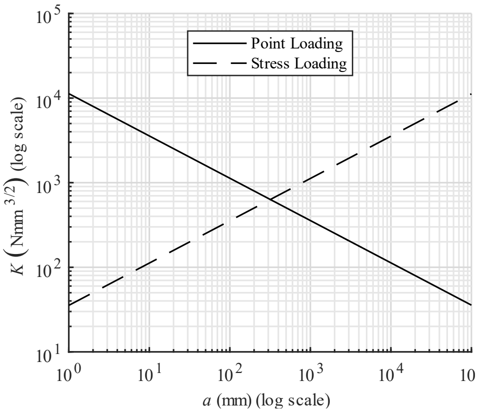

Figure 4. Relationship between crack length and stress intensity factor for the two loading cases

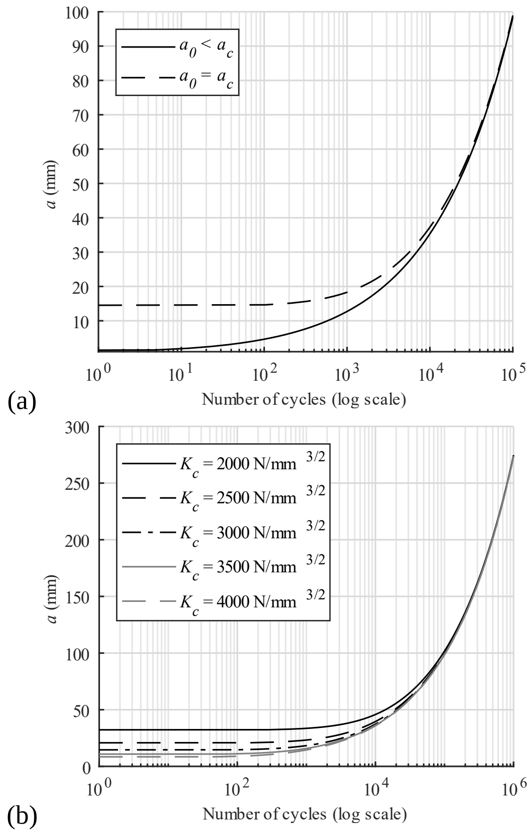

Figure 5. Influence in crack growth considering (a) that the initial crack length is a_c or and initially set crack, (b) different K_c which change a_c

Figure 6, show that m has the biggest influence on crack growth, with high values significantly increasing crack propagation speed. Also as expected, higher ΔP and C values result in accelerated crack propagation.

CONCLUSION

This work presents a parametric study of fatigue crack propagation and resulting life for two different loading cases of an idealized cracked infinite plate, a remote stress loading and a concentrated load at the crack centre.

These remote stress loading example shows a strong influence of the Paris' law parameters m and C on component life while K_c and a_0 have a comparatively smaller effect. However, the K_c and a_0 influence the $\Delta\sigma_p$ values, with it increasing with higher K_c values and lower a_0 values, while the opposite variations result in the opposite effect. The point loading at the crack centre present a very different behaviour when compared to the remote stress loading. In this case K tends to decrease with crack length instead of increasing, this is because as a increases the loading starts being further apart from the crack tip, so it will have a lesser effect there, leading to the lowering K. As K lowers, crack growth rate also lowers as the crack propagates further with each cycle.

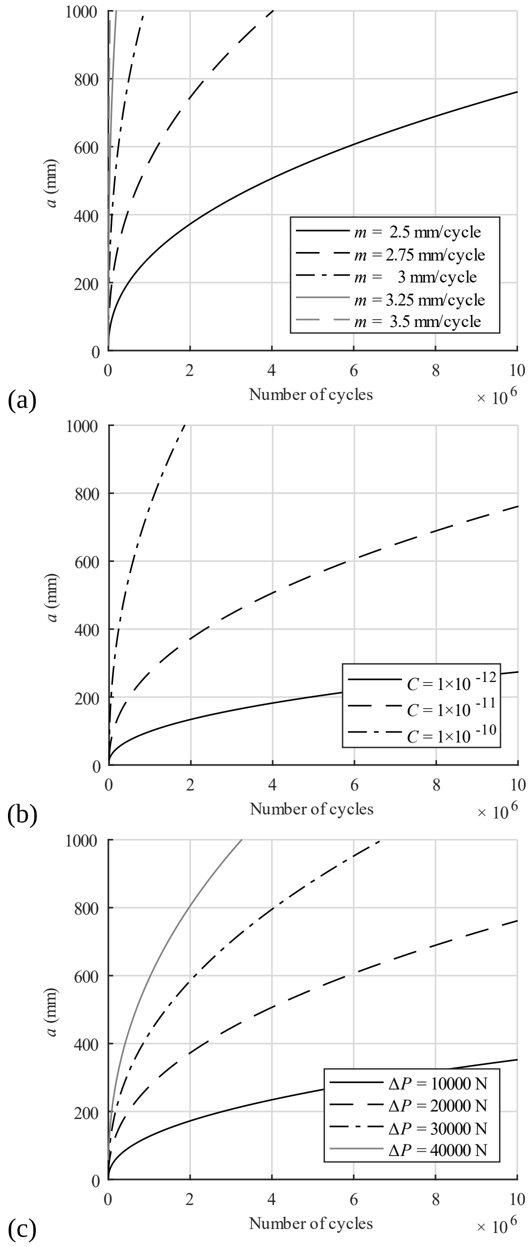

Figure 6. Crack growth with cycles changing (a) m, (b) C, (c) ΔP

In summation this work shows the effect of various parameters in fatigue life considering a fatigue crack growth model. It was shown that m and C, the Paris' law parameters, are the most influential on life. This parametric study should help designers choose appropriate materials for their desired applications by considering their properties and the effect of those properties on life shown here.

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