A Physical Simulation of Objects' Behaviour by Finite Element Method, Modal Matching and Dynamic Equilibrium Equation

R. Pinho*, J. Tavares

LOME - Laboratório de Óptica e Mecânica Experimental
DEMEGI - Departamento de Engenharia Mecânica e Gestão Industrial
FEUP - Faculdade de Engenharia da Universidade do Porto, R. Dr. Roberto Frias, 4200-465, Porto, Portugal
E-mail: {rpinho, tavares}@fe.up.pt

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Abstract

This paper presents a physical approach to simulate image represented objects' behaviour. The Finite Element Method (FEM) is employed to physically model the given objects, then modal analysis is used to match some objects' nodes (by solving the related eigenvalue/vector problem and analysing each node displacement in the respective modal space (Sclaroff, Tavares)), and finally the dynamic equilibrium equation is solved to estimate the object's displacement field.

To solve the Dynamic Equilibrium Equation different integration methods can be used, therefore the obtained results may differ. In this paper we briefly present the used approach and focus on the results obtained by three numerical integration methods: Central Difference, Newmark's and Mode Superposition (Cook). The foremost method has first order precision, as the mass and stiffness matrixes are not diagonal and the damping effect is non-negligible, and we used an algorithm where the velocity is delayed in half time step. On the other hand, with Newmark's method the equation resolution can be unconditionally stable, with no numerical damping but with second order precision. The latter method was solved either with the Central Difference Method (usual algorithm used because the Mode Superposition transformed mass and stiffness matrixes are diagonal) or with Newmark's Method.

For an experimental result, we can consider the initial surface α represented in figure 1, obtained from a real pedobarography image (Tavares), and the target surface β in figure 2, obtained from α by applying a rigid transformation, with all nodes (124) successfully matched. With all mentioned integration methods, four intermediate shapes can be obtained: The Central Difference method's last shape approaches the target surface in less than 700 pixels (which means than in average each node is less than 6 pixels away from its final position), figure 3. The closest approach of the target surface obtained by Newmark's method is at 1600 pixels from β, figure 4. When the Mode Superposition method is used with 75% of the model's modes, the Central Difference's last shape is 1800 pixels from β, figure 5, while with the Newmark's method is 1700 pixels, figure 6.

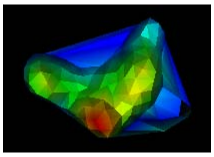
Figure 1: Surface α.

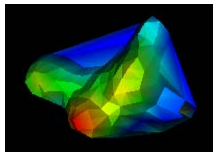
Figure 2: Surface β.

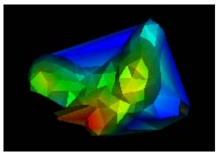
Figure 3: Last shape obtained with Central Difference Method.

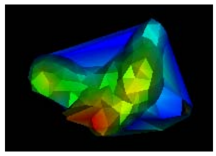
Figure 4: … with Newmark's method.

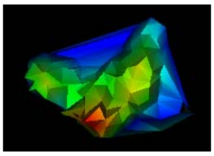
Figure 5: … with Mode Superposition Method and Central Difference Method when 75% of the model's modes are used.

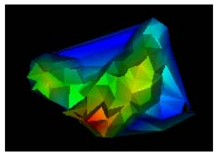
Figure 6: … with Mode Superposition Method and Newmark's method when 75% of the model's modes are used.

Although their might be some exceptions, from several experimental examples considered we have noticed that the closest approaches to the target shape are obtained with the Central Difference method. This might be explained by the existence of numerical damping in the Central Difference Method, while Newmark's method was used as an unconditionally stable method. We have also verified that the results obtained by Newmark's method and by Mode Superposition method (combined either with Central Difference or with Newmark's method) do not differ significantly; but with the latter, the computational cost is lower because the number of used modes can be reduced without a considerable accuracy loss.

Some References

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