Dynamic Pedobarography Transitional Objects by Lagrange's Equation with FEM, Modal Matching and Optimization Techniques

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Abstract. This paper presents a physics-based approach to obtain 2D or 3D dynamic pedobarography transitional objects from two given images (2D or 3D). With the used methodology, we match nodes of the input objects by using modal matching, improved with optimization techniques, and solve the Lagrangian dynamic equilibrium equation to obtain the intermediate shapes. The strain energy involved can also be analysed and used to quantify local or global deformations.

1 Introduction

Pedobarography is the measurement of dynamic variations in downward pressure by different areas of the foot sole, using a pedobarograph (apparatus for recording dynamic variations as a person stands upright or walks).

The recording of pedobarographic data during a normal walking step allows the dynamic analysis of the feet's behaviour [1], [2]. For example diabetic patients suffer from irrigation problems which may cause ulcerations [2]. So, it is of interest to determine the conditions that can increase these occurrences, through the analyses of the temporal evolution of the support surfaces, the detection of the plantar hiperpressure zones, and the analysis of the spatial and temporal gradients of zones with higher pressure.

The technical solutions used nowadays to analyse the sequences of pedobarographic images have some deficiencies and are almost subjective [2]. On the other hand, it might be necessary to determine some intermediate pedobarographic images of a given sequence. So to estimate the transitional images we use a physics-based approach: we solve the Lagrange's equation (LE) between the models built by the Finite Element Method (FEM) [3], [4]. With this solution, as more nodes are matched, which can be improved by using modal matching with optimization techniques [5], the obtained intermediate images are coherent with the reality, as can be verified with the experimental results to be presented.

The idea of considering physical restraints, in objects' modelling, has been suggested and used in computational vision by several authors, for example Terzopoulos, to simulate realistic deformation with an elastic model based on the Lagrange equation [6], [7]. It has been seen that when objects are represented according to physical principals, the non-rigid movement can be adequately modelled. In this work, we have attended to what other authors have done, namely: the restrictions that prevent inadequate matches according to the considered criteria [8], the modal matching process proposed by Shapiro [9], and the development of an isoparametric finite element that can be used to adequately model image represented objects [2], [10], [11].

In our previous work we have used the Lagrange's equation to simulate the deformation between 2D represented objects' outlines [4] (some obtained from medical images [3]), which was extended in this paper to the application domain of pedobarography, namely by considering 3D pedobarographic objects and 2D isopressure contours. The criteria that we used to estimate the applied charges also had to be reformulated, by adapting it to these pedobarography objects. In order to obtain more realistic transitional shapes, we applied a previously proposed approach to match the objects nodes based on optimization techniques [5] and then solve the Lagrange's equation with a much higher number of successfully matched nodes. In previous works (see for example [4]) we had analysed the evolution of the global and local strain energy values as the deformation takes place, but in this paper we study the influence of each mode to that evolution.

2 Dynamic Pedobarography

To measure and visualise the distribution of pressure under the foot sole during a step, a system can be used with a glass or acrylic plate trans-illuminated through its polished borders in such a way that the light is internally reflected. This plate is covered on its top by a single or dual thin layer of soft porous plastic material where the pressure is applied (see fig. 1).
In fig. 2 is represented a sample sequence image, and as can be seen the information obtained is very dense and rich [1], [2].

3 From Images to Objects' Models

To obtain 2D/3D objects from the given images (like fig. 2) *standard* image processing and analysis techniques are used [2].

The 2D objects we considered are the contours obtained by the objects' outlines or else isocontours whose pressure values are fixed. On the other hand, the 3D objects we considered are built by modelling each input image as a pressure surface with a membrane that is plane in the zones with no pressure and that deforms itself proportionally to the submitted pressure. So, the virtual 2D contours and 3D surfaces obtained can be analysed as if they where real physical objects [2].

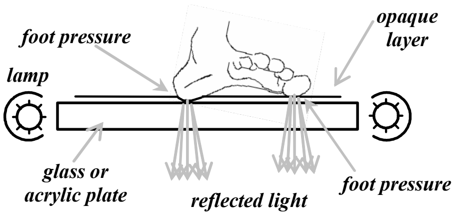


Fig. 1. Basic pedobarography principle [1], [2]

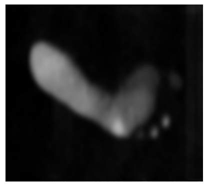


Fig. 2. Image of a pedobarography sample sequence [1], [2]

To physically model each of the given objects we employed the Finite Element Method (FEM), namely by using the Sclaroff's isoparametric element [11]. Using this finite element, the models built for 2D contours delimit a virtual object with elastic properties, and the obtained model is like an elastic membrane. When a 3D surface object is modelled by the same element, it is as if each feature point is covered by a blob of rubbery material [2], [11].

To build the Sclaroff's isoparametric finite element model we used as nodes the objects' data points $X = \left[X_1 \quad \cdots \quad X_m \right]$, and assemble the interpolation matrix H (which relates the distances between objects' nodes) using Gaussian functions:

$$g_i(X) = e^{-\|X - X_i\|^2 / (2\sigma^2)}, \qquad \textbf{(1)}$$

where X_i is the function's n-dimensional center, and σ controls data interaction. The interpolation functions, h_i, are given by:

$$h_i(X) = \sum_{k=1}^{m} a_{ik} g_k(X), \qquad \textbf{(2)}$$

where a_{ik} are the interpolation coefficients, with value 1 (one) at node i, and 0 (zero) at all other nodes, and m is the number of nodes. The interpolation coefficients a_{ik} can be determined by inverting matrix G defined as:

$$G = \begin{bmatrix} g_1(x_1) & \cdots & g_1(x_m) \\ \vdots & \ddots & \vdots \\ g_m(x_1) & \cdots & g_m(x_m) \end{bmatrix}. \qquad \textbf{(3)}$$

This way, the interpolation matrix of Sclaroff's isoparametric element, for a 2D shape will be:

$$H(X) = \begin{bmatrix} h_1 & \cdots & h_m & 0 & \cdots & 0 \\ 0 & \cdots & 0 & h_1 & \cdots & h_m \end{bmatrix}. \qquad \textbf{(4)}$$

The mass and stiffness matrices, M and K respectively, are then built as usually [2], [10], [11], and the used damping matrix, C, was considered as a linear combination of those two matrices.

The process of physically modelling 3D objects is entirely analogous.

4 Matching Objects

To match the initial and target 2D models' nodes, each generalized eigenvalue/vector problem is solved:

$$K\Phi = M\Phi\Omega\,, \tag{5}$$

where Φ is the matrix of the shape vectors ϕ_i (which describes the displacement (u, v) of each node due to vibration mode i) and Ω is the diagonal matrix whose entries are the squared eigenvalues and with the frequency of vibrations' squares increasingly ordered.

After building each modal matrix, by comparing the displacement of each node in the modal eigenspace, some nodes can be matched and so the affinity matrix, Z, is built:

$$Z_{ij} = \left\| u_{1,i} - u_{2,j} \right\|^2 + \left\| v_{1,i} - v_{2,j} \right\|^2 . \tag{6}$$

In this matrix, the affinity between nodes i and j will be 0 (zero) if the match is perfect, and will increase as the match worsens.

In this work, to find the best match two search methods are considered: (1) a local method or (2) a global one. The local method was proposed in [9], [10], [11], and previously used with pedobarography data in [1], [2], [12], and basically it consists in searching each row of the affinity matrix for its lowest value, adding the associated correspondence to the matching solution if that value is also the lowest of the associated column. This method has the principle disadvantage of disregarding the object structure as it searches for the best match for each node. On the other hand, the global method consists in describing the matching problem as an assignment problem, and solving this problem using an appropriate optimization algorithm [5].

With the global search matching approach, cases in which the number of objects' points to match is different can also be considered: initially the global search algorithm add fictitious points to the model with fewer elements, then the points that are matched with the fictitious elements are adequately matched by using a neighbourhood and an affinity criterion. This way matches of type "one to many" or vice versa are allowed for the model's excess points [5].

Once again the process of matching 3D objects is entirely analogous [2], [5], [11].

5 Resolution of Lagrange's Equation

In this work, to obtain the transitional objects' shapes attending to physical properties we solve, using the Mode Superposition Method, [4], [13], [14], the Lagrange's Equilibrium Equation:

$$M\ddot{U}^t + C\dot{U}^t + KU^t = R^t \quad (7)$$

where U are the nodal displacements and R are the implicit applied charges.

The Mode Superposition Method requires the initial displacement and velocity vectors to solve the Lagrange's Equation. The solution found to estimate the first one is to consider it as a part of the expected modal displacement. On the other hand, the initial modal velocity was estimated as a part of the initial modal displacement. For the implicit applied charges on each node, we considered them as proportional to the expected displacement.

6 Experimental Results

The used approach was included in a software platform previously built, to develop and test image processing and computer graphics algorithms (for a detailed presentation see [15]). In this section, we will present some of the obtained results when the approach previously described is applied on real dynamic pedobarography images.

For the first example, consider contours 1 and 2 (represented in fig. 3) obtained from two distinct images [2]. If the modal matching is done on a local search basis, we obtained 35 matched nodes (fig. 4), while when optimization techniques are used, all of the 75 nodes of the initial contour are successfully matched (fig. 5). In fig. 6 are represented the seven intermediate contours obtained with the presented approach to estimate the intermediates shapes.

For the second example, consider surfaces 1 and 2 in fig. 7, obtained from two pedobarography images [2], with 103 and 117 nodes respectively. When modal matching is done without optimization techniques, only 43 nodes are matched (fig. 7), however if these techniques are considered then all nodes are successfully matched (fig. 8). In fig. 9 are represented the fifth and the last (eighth) intermediate shapes obtained by our physically-based approach. When the strain energy involved is analysed [2], we notice that its values are proportional to the displacement in each step, that is as the simulation starts to evolve, the displacements tend to increase but as the target shape is approached, the displacements/strain energy decrease [4]. Moreover, as can be seen in fig. 10, where the strain energy involved in the all (8) estimated steps of the deformation between surfaces 1 and 2 is distributed by twelve groups of modes (The number of groups of modes considered is just for representation and analysis purposes.). We noticed that: when the deformation between objects is almost global, then the strain energy is more concentrated in the first groups of modes (which happens in this example); if rather local deformation is significant, then higher modes are more influential in the strain energy values [2].

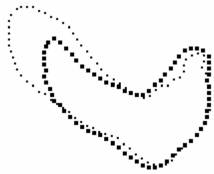


Fig. 3. Contours 1 and 2

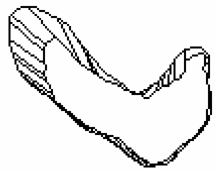


Fig. 4. Matching between contours 1 and 2 with local search

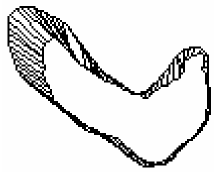


Fig. 5. Matching between contours 1 and 2 with global search

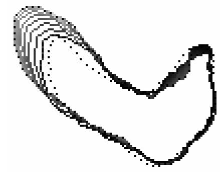


Fig. 6. Estimated intermediate contours

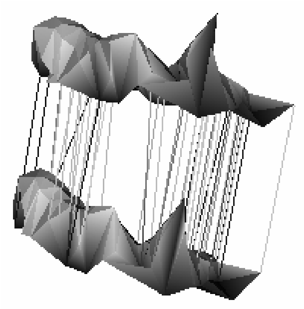


Fig. 7. Matching obtained between surfaces 1 and 2 with local search

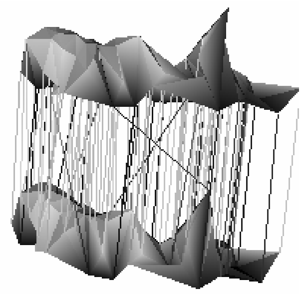


Fig. 8. Matching obtained between surfaces 1 and 2 with global search

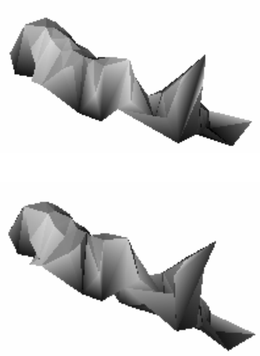


Fig. 9. 5^{th} and 8^{th} intermediate shapes obtained from surfaces 1 and 2

For the third and last example, consider two isocontours extracted from surfaces 1 and 2 of the previous example (fig. 11). If modal matching is used with the usually local search, only 25 of the 68 nodes are matched, but if optimization techniques are considered, then 54 nodes can be matched (fig. 12). In fig. 13 are represented two intermediate shapes of the thirty that can estimate the evolution of the foot's pressure surface at a determined value.

7 Conclusions

In this paper, we have presented an approach that can be used to estimate transitional objects of a given sequence of pedobarographic images: We started by extracting 2D/3D objects from the input images with standard image processing techniques. Then, the obtained objects were physically modelled by using the Sclaroff's isoparametric finite element. After that, correspondences between the models' nodes were established by modal analysis improved with optimization techniques. Finally, we solve the Lagrange's equation to obtain the intermediate shapes according to physical principles.

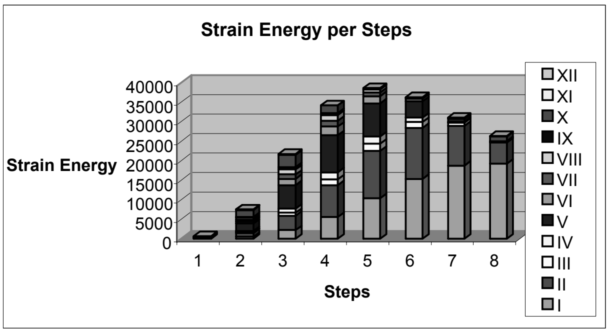
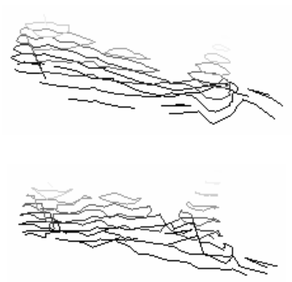
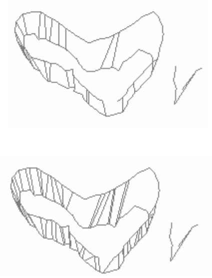


Fig. 10. Strain Energy involved in the deformation between surfaces 1 and 2 during the estimated steps distributed by 12 groups of modes ordered increasingly (I-XII)





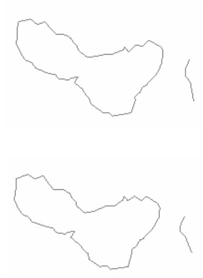


Fig. 11. Isocontours obtained from surface 1 and 2, respectively

Fig. 12. Matching, between isocontours of Fig. 11, with local and global search, respectively

Fig. 13. 10th and 20th intermediate shapes obtained from the isocontours of Fig. 11

The experimental results obtained, some presented in this paper, confirm that satisfactory matching results can be obtained for dynamic pedobarographic image data. It is also verified that the used global search matching strategies improves considerably these results. Another advantage of the improved search strategy based on optimization techniques we used, is that the global matching methodology becomes easier to use and more adaptable to experimental applications [5]. This improved algorithm also matches satisfactorily all excess models' nodes, which can prevent the loss of information along the image sequence.

During experimentations, we have also noticed that: as the correspondence between nodes gets better, the estimated intermediate shapes are more trustworthy and realistic and so this physics–based approach estimates the objects behaviour as it would be physically expected.

When analysing the strain energy values along the estimated deformation, we are able to confirm that global deformation's stain energy is concentrated in low order modes, which are related to rigid deformations, and that local deformation' strain energy is due to the higher modes' influence, responsible for local deformations (such as noise).

Acknowledgments

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