A Multiple Model Adaptive Wave Filter for Dynamic Ship Positioning ★

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Abstract: This paper addresses a filtering problem that arises in the design of dynamic positioning systems for ships subjected to the influence of sea waves. Its key contribution is the use of a multiple model adaptive wave filter that relies on measurements of the ship's position and heading only. To this effect, a bank of Kalman filters is designed for a finite number of parameter values, each corresponding to a different peak frequency of the assumed wave spectrum model. Tools from multiple model adaptive estimation (MMAE) theory are exploited to blend the information provided by the different observers, yielding position and velocity estimates of the ship. These estimates are then be used in an appropriately designed feedback control law. Simulations illustrate the efficacy of the MMAE techniques proposed.

Keywords: Multiple-Model Adaptive Wave Filtering; Dynamic Positioning, Marine Vessels.

1. INTRODUCTION

Dynamic positioning systems came to existence in the 1960s for offshore drilling applications, due to the need to drill in deep waters and the realization that Jack-up barges and anchoring systems could not be used economically at such depths. Early dynamic positioning systems were implemented using PID controllers. In order to restrain thruster trembling caused by the wave-induced motion components, notch filters were used with the controllers. However, notch filters restrict the performance of closed-loop systems because they introduce some phase lag around the crossover frequency, which in turn tends to decrease phase margin. An improvement in performance was achieved by exploiting more advanced control techniques based on optimal control and Kalman filter theory, see Balchen et al. [1976]. These techniques were later modified and extended in Balchen et al. [1980a;b], Grimble et al. [1980a;b], Sælid et al. [1983], Fossen et al. [1996], Sørensen et al. [1996], Grøvlen and Fossen [1996], Strand [1999], Fossen [2000] and Torsetnes et al. [2004]. One of the most fruitful concepts introduced in the course of the body of work referred above was that of wave filtering together with the strategy of modeling the total vessel motion as the superposition of low-frequency (LF) vessel motion and wave frequency (WF) motions. It was further recognized that in order to reduce the mechanical wear and tear of the propulsion system components, the estimates entering the DP control feedback loop should be filtered by using a so-called wave filtering technique so as to prevent excessive control activity in response to WF components. In practice, position and heading measurements are corrupted not only with sensor noise but also with colored noise caused by wind, waves, and ocean currents; thus the need for an observer to achieve wave filtering and "separate" the LF and WF position and heading estimates (see Fossen et al. [1996], Fossen [2002] for details).

In Sørensen et al. [1996], WF filtering is done by exploiting the use of Kalman filter theory under the assumption that the kinematic equations of the ship's motion can be linearized about a set of predefined constant yaw angles (36 operating points in steps of 10 degrees, covering the whole heading envelope); this is necessary when applying linear Kalman filter theory and gain scheduling techniques. However, global exponential stability (GES) of the complete system cannot be guaranteed. In Fossen and Strand [1999], a nonlinear observer with wave filtering capabilities and bias estimation was designed using passivity. An extension of this observer with adaptive wave filtering was described in Strand and Fossen [1999]. Gain-scheduled wave filtering was introduced in Torsetnes et al. [2004].

Except for the work in Strand and Fossen [1999] and Torsetnes et al. [2004], the design techniques mentioned so far assume that sea state (and the WF model parameters) do not change during operation and that the WF model parameters are known a priori. In practice, the sea state may undergo large variations and therefore the observer in charge of reconstructing the LF motion should adapt to the sea state itself. The nonlinear passive observer technique introduced in Strand and Fossen [1999] for recursive adaptive filtering is a very important contribution to-

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wards meeting the above goal. However, the task of filter tuning may meet with difficulties. In Torsetnes et al. [2004] the observer gains are parameterized by the wave peak frequencies and spectral analysis techniques are used to estimate the wave spectrum in surge, sway, and yaw from position and heading measurements. This approach is sensitive to measurement noise and may have latency problems because it requires that the samples acquired be buffered to estimate the Power Density Spectrum of the measurement time series.

In this paper, inspired by previous pioneering work on dynamic ship positioning, we propose the use of a multiple model adaptive wave filter that relies on measurements of the ship's position and heading only. To this effect, a bank of Kalman filters is designed for a finite number of parameter values, each corresponding to a different peak frequency of the assumed wave spectrum model. The main emphasis of the paper is on the use of MMAE for adaptive wave filtering; however, for the sake of completeness, in the numerical simulations a multivariable PID is used to control the position of the ship.

The structure of the local observers builds upon steady state Kalman filters; see Anderson and Moore [1979]. Tools from multiples model adaptive estimation (MMAE) theory are exploited to blend the information provided by the different observers, yielding position and velocity estimates of the ship. For the necessary background information and the mathematical framework used in the analysis and design of MMAE filters the reader is referred to Aguiar et al. [2008], Hassani et al. [2009a;c;b] and the references therein. In the set-up adopted, the observers run in parallel and at each instant of time their residuals are used to compute, for each observer, the probability that the peak frequency of the assumed wave spectrum model is the true peak frequency of the wave disturbing the ship motion. The state estimate is a probabilistically weighted combination of each observer estimate.

The structure of the paper is as follows. Section 2 is a brief introduction to important issues that arise in dynamic ship positioning. A representative ship model is also described. Section 3 summarizes the main ideas behind MMAE; it also reviews the basic structure of local obsevers. An example is described in Section 4 that illustrates the strategy proposed, via computer simulations. Conclusions and suggestions for future research are summarized in Section 5.

2. DYNAMIC POSITIONING: FILTERING AND SHIP MODELING

In dynamic positioning (DP) systems, the key objective is to maintain the ship's heading and position within desired limits. Central to their implementation is the availability of good heading and position estimates, provided by properly designed filters. In practice, position and heading measurements are corrupted not only by sensor noise but also by colored noise caused by wind, waves and ocean currents. In general, measurements of the vessel velocities are not available. Consequently, estimates of the velocities must be computed from corrupted measurements of position and heading through a state observer. Furthermore, only the slowly-varying disturbances should be counterbalanced by the propulsion system, whereas the oscillatory motion induced by the waves (1st-order wave disturbances) should not enter the feedback control loop. To this effect, the DP control systems should be designed so as to react to the low frequency forces on the vessel only. This is accomplished by using so-called wave filtering techniques, which separates the position and heading measurements into a low-frequency (LF) and a wave frequency (WF) position and heading estimate (Fossen [1994; 2002]). Fig. 1 illustrates this concept graphically. It was this interesting circle of ideas that motivated the work reported in the present paper on multiple model adaptive wave filtering.

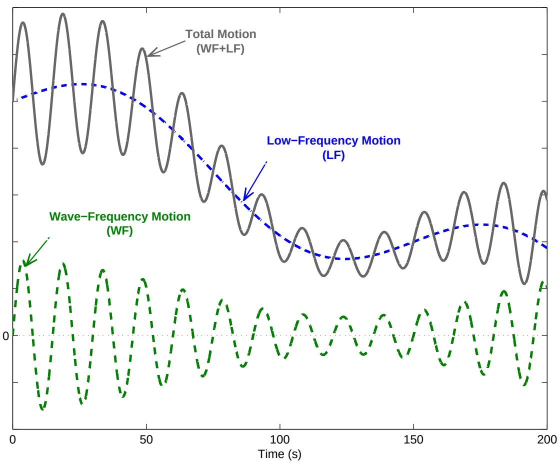

Fig. 1. The total motion of a ship is modeled as a LF response with the WF motion added as an output disturbance (adopted from Fossen and Strand [1999]).

In what follows, the ship model that we adopt is by now standard. See Fossen and Strand [1999] and Torsetnes et al. [2004]. The model admits the realization

$$\dot{\xi} = A_\omega(\omega_0)\xi + E_\omega w_1 \tag{1}$$
$$\dot{\eta} = R(\psi)\nu \tag{2}$$
$$\dot{b} = 0 \tag{3}$$
$$M\dot{\nu} + D\nu = \tau + R^T(\psi)b \tag{4}$$
$$\eta_\omega = C_\omega \xi \tag{5}$$

where (1) and (5) capture the 2nd-order wave induced motion in surge, sway, and yaw, $w_1 \in \mathbb{R}^3$ is a zero mean Gaussian white noise vector, $\eta_\omega \in \mathbb{R}^3$ is the vessel's WF motion due to first-order wave-induced disturbances, and

$$A_\omega = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ -\Omega_{3\times3} & -\Lambda_{3\times3} \end{bmatrix}, \qquad E_\omega = \begin{bmatrix} 0_{3\times1} \\ I_{3\times1} \end{bmatrix},$$
$$C_\omega = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \end{bmatrix},$$

with

$$\Omega = \mathrm{diag}\{\omega_{01}^2, \omega_{02}^2, \omega_{03}^2\},$$
$$\Lambda = \mathrm{diag}\{2\zeta_1\omega_{01}, 2\zeta_2\omega_{02}, 2\zeta_3\omega_{03}\},$$

where ω_{0i} and ζ_i are the dominating wave frequency and relative damping ratio, respectively. The vector $\eta \in \mathbb{R}^3$ consists of earth-fixed position (x, y) and heading ψ of the vessel relative to an earth-fixed frame, $\nu \in \mathbb{R}^3$ represents the velocities decomposed in a vessel-fixed reference, and $R(\psi)$ is the standard orthogonal yaw angle rotation matrix (see Fossen [2002] for more details); $b \in \mathbb{R}^3$ is a vector of unknown (but constant) bias terms due to waves, wind, and currents in earth-fixed coordinates. Equation (4) describes the ship's LF motion at low speed (see Fossen [1994; 2002]), where $M \in \mathbb{R}^{3\times3}$ is the generalized system inertia matrix, $D \in \mathbb{R}^{3\times3}$ is the linear damping matrix and $\tau \in \mathbb{R}^3$ is a control vector of generalized forces generated by the propulsion system, that is, the main propellers aft of the ship and thrusters which can produce surge and sway forces as well as a yaw moment. Finally, in (5) $\eta_\omega \in \mathbb{R}^3$ is the vessel's WF motion due to 1st-order wave-induced disturbances.

Clearly, the model (1)-(5) is nonlinear because of the presence of the rotation matrix R. However, it can be linearized dynamically by defining a new coordinates of *vessel parallel coordinates* introduced in Fossen [2002] and Fossen and Perez [2009]. Vessel parallel coordinates are defined in a reference frame fixed to the vessel, with axes parallel to the earth-fixed frame. Vector $\eta_p \in \mathbb{R}^3$ consists of position (x_p, y_p) and heading ψ_p of the vessel expressed in body coordinates and is defined as

$$\eta_p = R^T(\psi)\eta. \tag{6}$$

Using the low-speed assumptions ($\dot{\psi} \approx 0$), which is the case in dynamic positioning, it follows that:

$$\begin{aligned} \dot{\eta}_p &= \dot{R}^T(\psi)\eta + R^T(\psi)\dot{\eta} \\ &= \dot{R}^T(\psi)R(\psi)\eta_p + R^T(\psi)R(\psi)\nu \\ &= \dot{\psi}S\eta_p + \nu \\ &\approx \nu, \end{aligned}$$

where

$$S = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Exploiting the vessel parallel coordinates, the vessel model (1)-(5) is given by

$$\dot{\xi} = A_\omega(\omega_0)\xi + E_\omega w_1 \tag{7}$$
$$\dot{\eta}_p = \nu \tag{8}$$
$$M\dot{\nu} + D\nu = b_p\tau + w_2 \tag{9}$$
$$\dot{b}_p = w_3 \tag{10}$$
$$\eta_\omega = C_\omega\xi \tag{11}$$
$$y = \eta_p + \eta_\omega + v \tag{12}$$

where $w_i \in \mathbb{R}^3$, $i = 1, 2, 3$, are independent zero mean Gaussian white noise processes that capture model uncertainty and (12) represents the position and heading measurement equation. In the above, $\eta_\omega \in \mathbb{R}^3$ is the vessel's WF motion due to 1st-order wave-induced disturbances and $v \in \mathbb{R}^3$ is a zero-mean Gaussian white measurement noise vector. Equations (7)-(12) can be rephrased in a standard form as

$$\dot{x} = A(\omega_0)x + Bu + Gw, \tag{13a}$$
$$y = Cx + v, \tag{13b}$$

where $x = [\xi^T \eta_p^T b_p^T \nu^T]^T \in \mathbb{R}^{15}$ is the state vector, $Bu = \tau$ and $u \in \mathbb{R}^3$ is the control vector, $w = [w_1^T w_2^T w_3^T]^T \in \mathbb{R}^9$ is a zero-mean Gaussian white noise vector that represents the plant disturbance vector, $v \in \mathbb{R}^3$ is a zero-mean Gaussian white measurement noise vector, and

$$A(\omega_0) = \begin{bmatrix} A_\omega & 0_{6\times3} & 0_{6\times3} & 0_{6\times3} \\ 0_{3\times6} & 0_{3\times3} & 0_{3\times3} & I_{3\times3} \\ 0_{3\times6} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times6} & 0_{3\times3} & M^{-1} & M^{-1}D \end{bmatrix}, \quad B = \begin{bmatrix} 0_{6\times3} \\ 0_{3\times3} \\ 0_{3\times3} \\ M^{-1} \end{bmatrix},$$
$$C = [\, C_\omega \;\; I_{3\times3} \;\; 0_{3\times3} \;\; 0_{3\times3} \,], \quad G = \begin{bmatrix} E_\omega & 0_{6\times3} & 0_{6\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times6} & I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & M^{-1} \end{bmatrix}. \tag{14}$$

3. THE CONTINUOUS-TIME MULTIPLE-MODEL ADAPTIVE ESTIMATOR

One of the earliest uses of multiple-models was motivated by the need to accurately estimate the state of a stochastic dynamic system subjected to significant parameter uncertainty. In many applications, the estimation accuracy provided by standard KFs was not adequate. This led to the consideration of Multiple Model Adaptive Estimation (MMAE) techniques. For some early references on Multiple-Model Adaptive Estimator (MMAE) see Magill [1965], Anderson and Moore [1979], Baram and Sandell [1978]. Fig. 2 shows the architecture of a MMAE system.

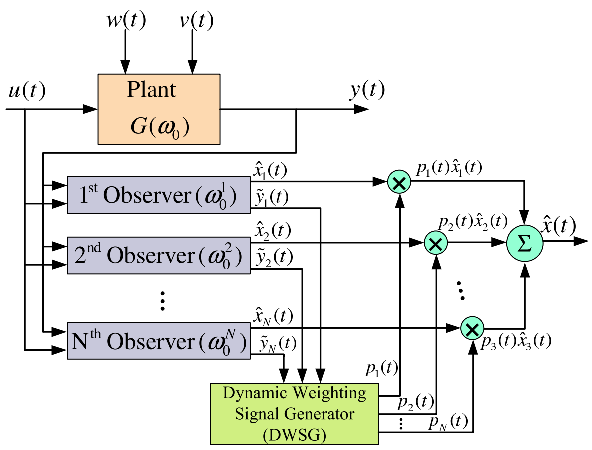

Fig. 2. The MMAE architecture.

It is assumed that a linear time-invariant plant G is driven by white noise and a known deterministic input signal and that it generates measurements that are corrupted by white measurement noise. If there is no parameter uncertainty in the plant, then the Kalman filter (KF) is the optimal state-estimation algorithm in a well-defined sense; see, for example, Anderson and Moore [1979]. Moreover, under the usual linear-gaussian assumptions, the KF state-estimate is the true conditional mean of the state, given the past controls and observations. If the plant has an uncertain real-parameter vector, say ω_0, one can imagine that it is "close" to one of the elements of a finite discrete representative parameter set, $\Omega := \{\omega_0^1, \omega_0^2, \ldots, \omega_0^N\}$. One can then design a bank of standard KFs, where each KF uses one of the discrete parameters ω_0^i in its implementation, $i \in \{1, \ldots, N\}$. It turns out that, if indeed the true plant parameter is one of the discrete values, then the conditional probability density of the state is the sum of gaussian densities. In this case, the MMAE of Fig. 2 will generate the true conditional mean of the state and one can compute the true conditional covariance matrix; see, for example, Anderson and Moore [1979]. The structure of MMAE, in Fig. 2, consists of: i) the dynamic weighting signal generator (DWSG) and ii) a bank of N KFs, where each local estimator is designed based on one of the representative parameters. The state estimate is generated by a probabilistically weighted sum of the local state-estimates produced by the bank of KFs.

Multiple Model Adaptive Estimation for continuous time LTI systems was introduced in Lainiotis [1971]. However, no proof of convergence of the state estimate or dynamic weights was given. Later on, in Dunn and Rhodes [1973] and Dunn [1974], a new approach to compute dynamic weights in a stochastic setup was presented. Again, no proof of convergence of the state estimate or the dynamic weights was offered. In Aguiar et al. [2007], using the dynamic weights introduced previously in Dunn and Rhodes [1973] and Dunn [1974], it was shown for the first time that the estimation error is bounded; however, there was no proof of convergence of the dynamic weights. To tackle this problem, in Aguiar et al. [2008] a different method was proposed to generate dynamic weights. Interestingly enough, with the reformulated expression for dynamic weights, convergence of the latter was proven under a certain distinguishability condition. It was further shown that the estimation error is bounded and converges to zero when the true plant parameter is one of the discrete values. The distinguishability condition was later relaxed in Hassani et al. [2009b]. The reader will find in Hassani et al. [2009b] a thorough discussion of convergence analysis when the actual plant parameter is not one of the discrete values adopted during the design phase.

In what follows, we assume the plant model G is subjected to parameter uncertainty $\omega_0 \in \mathbb{R}^l$, that is, $G = G(\omega_0)$. In what follows we consider multiple-input-multiple-output (MIMO) linear plant models of the form

$$\dot{x}(t) = A(\omega_0, t)x(t) + B(t)u(t) + G(t)w(t), \tag{15a}$$
$$y(t) = C(t)x(t) + v(t), \tag{15b}$$

where $x(t) \in \mathbb{R}^n$ denotes the state of the system, $u(t) \in \mathbb{R}^m$ its control input, $y(t) \in \mathbb{R}^q$ its measured noisy output, $w(t) \in \mathbb{R}^r$ an input plant disturbance that cannot be measured, and $v(t) \in \mathbb{R}^q$ is the measurement noise. Vectors $w(t)$ and $v(t)$ are zero-mean white Gaussian signals, mutually independent with intensities $E\{w(t)w^T(\tau)\} = Q\delta t - \tau$ and $E\{v(t)v^T(\tau)\} = R\delta t - \tau$. The initial condition $x(0)$ of (13) is Gaussian random vector with mean and covariance given by $E\{x(0)\} = 0$ and $E\{x(0)x^T(0)\} = \Sigma(0)$. Matrix $A(\omega_0)$ contains *unknown constant parameters* indexed by ω_0.
Consider a finite set of candidate parameter values $\Omega = \{\omega_0^1, \omega_0^2, \dots, \omega_0^N\}$ indexed by $i \in \{1, \dots, N\}$. We propose the following MMAE. The state estimate is given by

$$\hat{x}(t) := \sum_{i=1}^{N} p_i(t)\hat{x}_i(t), \tag{16}$$

where $\hat{x}(t)$ is the estimate of the state $x(t)$ (at time t) and $p_i(t)$ is the conditional probability that $\omega_0 = \omega_o^i$, given the measurements record. In (16), each $\hat{x}_i(t)$; $i = 1, \dots, N$ corresponds to a "local" state estimate generated by the i^{th} steady state Kalman filter (Anderson and Moore [1979]),

$$0 = A(\omega_0^i)\Sigma_{\omega_0^i} + \Sigma_{\omega_0^i}A^T(\omega_0^i) + GQG^T - \Sigma_{\omega_0^i}C^TR^{-1}C\Sigma_{\omega_0^i}, \tag{17a}$$
$$\dot{\hat{x}}_i(t) = A(\omega_0^i)\hat{x}_i(t) + Bu(t) + H_{\omega_0^i}\big(y(t) - C\hat{x}_i(t)\big), \tag{17b}$$
$$\hat{y}_i(t) = C\hat{x}_i(t), \tag{17c}$$

where $[A(\omega_0^i), G]$ and $[A(\omega_0^i), C]$ are assumed to be stabilizable and detectable, respectively for $i = 1, \dots, N$.

In the sequel we introduce dynamic weights that weigh the local estimations (16).

As mentioned before, the (stochastic) continuous-time MMAE (CT-MMAE) was introduced in Lainiotis [1971], Dunn and Rhodes [1973], Dunn [1974]. A priori probabilities for each estimator were derived, but no convergence results were given either for the dynamic weights or the estimation error. In Aguiar et al. [2008], the continuous counterpart of the weight generator that was introduced in discrete-time MMAE by Baram and Sandell [1978], and Anderson and Moore [1979] was used in the CT-MMAE structure for the first time. The new resulting structure shed light into the process of proving not only convergence of dynamic weights in the CT-MMAE but also boundedness of the estimation error. In this case, the dynamic weights are generated by a continuous time differential equation, the structure of which can be found in Aguiar et al. [2008], Hassani et al. [2009b]. This structure will be used in the sequel.

In the proposed MMAE, the dynamic weights $p_i(t) \in \mathbb{R}$, $i = 1, \dots, N$ satisfy

$$\dot{p}_i(t) = -\lambda\Big(1 - \frac{\beta_i(t)e^{-m_i(t)}}{\sum_{j=1}^{N} p_j(t)\beta_j(t)e^{-m_j(t)}}\Big)p_i(t), \tag{18}$$

where λ is a positive constant, $\beta_i(t)$ is a signal assumed to satisfy the condition $c_1 \le \beta_i(t) \le c_2$ for some positive constants c_1, c_2, and $m_i(\cdot)$ is a continuous function called an *error measuring function* that maps the measurable signals of the plant and the states of the i^{th} local estimator to a nonnegative real value. An example of an error measuring function and a $\beta_i(t)$ function, which used throughout this paper, are $m_i(t) := \frac{1}{2}\|y(t) - \hat{y}_i(t)\|^2_{S_i^{-1}}$ and $\beta_i(t) := \frac{1}{\sqrt{\det S_i}}$, respectively, where S_i is a uniformly bounded positive definite weighted matrix and $\|x\|_S = \left(x^TSx\right)^{\frac{1}{2}}$. The matrices S_{θ_i} are important to scale the energy of estimation error signals in order to make them comparable. In what follows, we refer to equation (18) as the dynamic weighting signal generator (DWSG).

We impose the constraint that the initial conditions $p_i(0)$ be chosen such that $p_i(0) \in (0, 1)$ and $\sum_{i=1}^{N} p_i(0) = 1$. The parameters Q, R and the functions β_i, m_i are tuning parameters/functions of the CT-MMAE chosen by the designer based on the system being modeled.

4. ILLUSTRATIVE EXAMPLE

To illustrate the performance of the DP system with multiple model adaptive wave filtering, the Marine Systems Simulator (MSS) was used (Fossen and Perez [2004]). In the simulation, a model of the CyberShipII was used.

It was assumend that the ship was subjected to wave disturbances with a fixed but unknown spectrum peak frequency in the interval $[0.25\ 1.4]\, rad/s$ covering sea states from calm and glassy to very high and phenomenal. A set of eight candidate values of the peak frequency were selected as $\{0.3, 0.45, 0.6, 0.75, 0.9, 1.05, 1.2, 1.35\}\, rad/s$.[1]

For each candidate value, a KF was developed and a MMAE was derived with the dynamic weights given by (18). A multivariate PID controller was designed that uses $\hat{\eta}_p$ and $\hat{\nu}$ provided by MMAE to control the position of the ship. Because the emphasis of this paper is not on control, but rather on filtering, we eschew the details of controller design. All the parameters used in the simulation are given in Appendix A.

Fig. 3 shows the dynamic weights in the multiple model adaptive wave filtering when the true parameter ω_0 is $0.93\, rad/s$, which is not included in the original design set. Notice how the the fifth observer, which was designed based on $\omega_0 = .9\, rad/s$ is selected in about one minute. Figs. 4 and 5 show the time evolution of the low frequency estimates of the surge and sway motions and the measured motions of the ship. During the simulation, at $t = 40$ (s) the vessel position was commanded to change 10 (m) forward; this simple maneuver was executed with the multivariate PID control law referred to above.

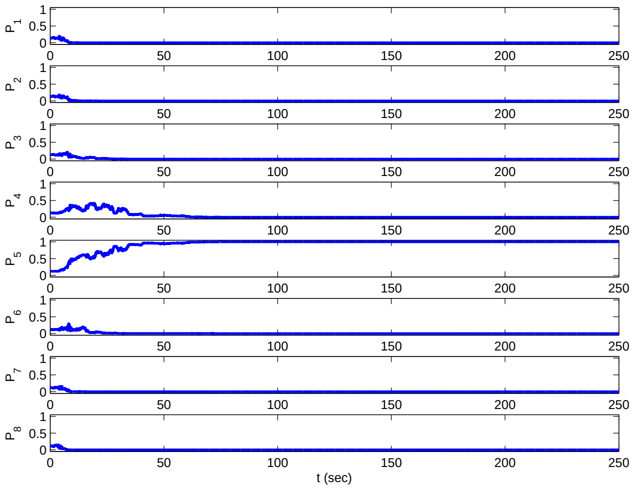

Fig. 3. Evolution of dynamic weights in multiple model adaptive wave filtering ($\omega_0 = 0.85\, rad/s$).

5. CONCLUSIONS AND FUTURE RESEARCH

This paper proposed a new technique for adaptive wave filtering, with application to dynamic ship positioning. Its key contribution was the use of MMAE techniques to yield a filter that adapts to sea state variations. Future work will included the application of the method developed to a more realistic ship model.

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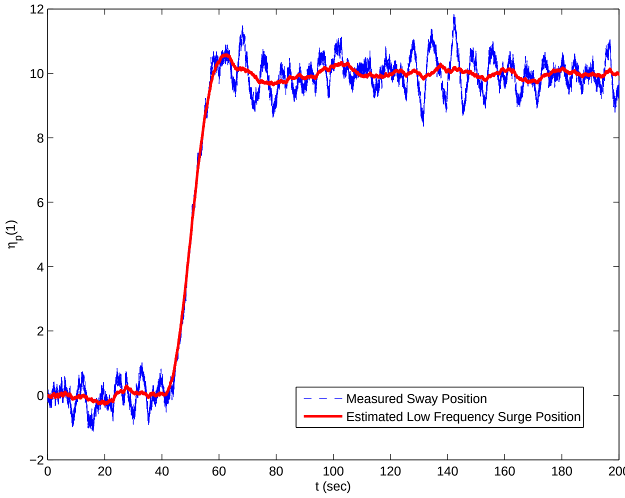

Fig. 4. A comparison of LF surge motion estimation and measured surge motion.

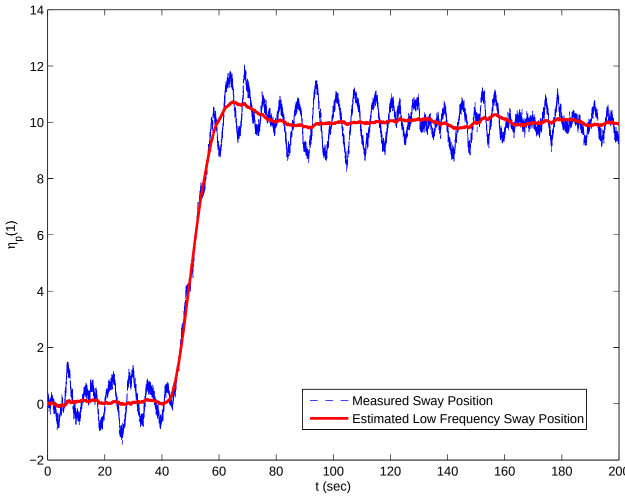

Fig. 5. A Comparison of LF sway motion estimation and measured sway motion.

[1] See Hassani et al. [2009c] for a systematic way of chosing nominal parameters in MMAE.

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Appendix A. SIMULATION PARAMETERS

To illustrate the performance of the DP system with multiple model adaptive wave filtering, the model of the CyberShipII was used (Fossen and Perez [2004]). The dynamics for CybershipII can be described by (9) where

$$M = \begin{bmatrix} 25.8 & 0 & 0 \\ 0 & 33.8 & 1.0115 \\ 0 & 1.0115 & 2.76 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 7 & 0.1 \\ 0 & 0.1 & 0.5 \end{bmatrix}. \tag{A.1}$$

In process of designing bank KFs the intensity matrices $Q = diag\{Q_1, Q_2, Q_3\}$ and R were considered as

$$Q_1 = \begin{bmatrix} 0.09 & 0 & 0 \\ 0 & 0.09 & 0 \\ 0 & 0 & 10^{-4} \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 10^{-3} & 0 & 0 \\ 0 & 10^{-3} & 0 \\ 0 & 0 & 10^{-4} \end{bmatrix},$$
$$Q_3 = \begin{bmatrix} 10^{-3} & 0 & 0 \\ 0 & 10^{-3} & 0 \\ 0 & 0 & 10^{-4} \end{bmatrix}, \quad R = \begin{bmatrix} 10^{-4} & 0 & 0 \\ 0 & 10^{-4} & 0 \\ 0 & 0 & 10^{-8} \end{bmatrix}. \tag{A.2}$$

During the Simulation a multivariable PID controller with the following gains were used to control the position of the ship.

$$K_P = \begin{bmatrix} 2.7685 & 0 & 0 \\ 0 & 3.212 & 0.0445 \\ 0 & 0.0952 & 7.652 \end{bmatrix},$$
$$K_I = \begin{bmatrix} .3162 & 0 & 0 \\ 0 & .3162 & -.0046 \\ 0 & 0.0054 & 4.4721 \end{bmatrix},$$
$$K_D = \begin{bmatrix} 10.1192 & 0 & 0 \\ 0 & 9.3138 & 0.2638 \\ 0 & 0.3949 & 6.0549 \end{bmatrix}. \tag{A.3}$$