NONLINEAR MOTION CONTROL OF MULTIPLE AUTONOMOUS UNDERWATER VEHICLES

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Abstract: This paper addresses the problem of steering a group of underactuated Autonomous Underwater Vehicles (AUVs) along given spatial paths, while holding a desired inter-vehicle formation pattern (coordinated path-following, abbreviated CPF). Exploiting Lyapunov-based techniques and graph theory, a decentralized control law is derived that takes into account the dynamics of the cooperating vehicles, the constraints imposed by the topology of the inter-vehicle communications network, and the cost of exchanging information. The CPF problem is divided into the motion control task of making each vehicle track a virtual target moving along the desired path, and the dynamic assignment task of adjusting the speeds of the virtual targets so as to achieve vehicle coordination. At the path-following level, the controller derived exhibits an inner-outer loop structure. Convergence and stability of the overall system are proved formally. Simulations results are presented and discussed.

Keywords: Coordination of Autonomous Underwater Vehicles, Nonlinear Control, Path Following

1. INTRODUCTION

The past few decades have witnessed considerable interest in the area of motion control of autonomous underwater vehicles (AUVs) (Fossen, 1994; Leonard, 1995; Encarnação and Pascoal, 2000; Alonge *et al.*, 2001; Jiang, 2002; Lefeber *et al.*, 2003; Aguiar and Pascoal, 2007*b*). For underactuated AUVs, *i.e.*, vehicles with a smaller number of control inputs than the number of independent generalized coordinates, the problem of control system design continues to pose considerable challenges. To tackle this issue and to deal with fact that the dynamics of the AUVs are nonlinear, a popular strategy is to simplify the vehicle's design model using linearization-based techniques. The key assumption is that the range of operation of the vehicle is restricted to a small region for which a linearized model remains valid. However, as a consequence, adequate control is only guaranteed in a neighborhood of the selected operating points. Moreover, performance can suffer significantly when the required operating range is enlarged to encompass manoeuvres that bring out the strong nonlinearities and cross-couplings of the AUV being controlled.

Nonlinear Lyapunov-based designs can overcome some of the limitations mentioned above. Recently, in (Aguiar and Hespanha, 2007) the authors have derived control algorithms for motion control of Underactuated Autonomous Vehicles

[1] Research supported in part by project GREX / CECIST (Contract No. 035223), project MAYA-Sub of the AdI (PT), the FREESUBNET RTN of the CEC, and the FCT-ISR/IST plurianual funding program (through the POS-C Program initiative in cooperation with FEDER).

that include land and marine vehicles, in two and three-dimensional space (see also (Aguiar *et al.*, 2003) for experimental results conducted at Caltech). The important common feature that these designs share is the fact that they explicitly exploit the physical structure of the AUVs instead of “fighting” it.

Current research goes well beyond single vehicle control. Spawned by the advent of small embedded processors and sensors, advanced communication systems, and the miniaturization of electro-mechanical devices, considerable effort is now being placed on the deployment of groups of networked autonomous marine vehicles operating in a number of challenging scenarios. A specially important scenario is automatic ocean exploration and monitoring for scientific and commercial applications, by resorting to multiple autonomous vehicles acting in cooperation. In this scenario, one can immediately identify two main disadvantages if one were to use a single, heavily equipped vehicle: lack of robustness to vehicle failure and inefficiency in sense that the vehicle might need to wander significantly to collect rich enough data. A cooperative network of vehicles has the potential to overcome these limitations and can also adapt its behaviour/configuration in response to the measured environment.

Motivated by the above considerations, the problem of coordinated path-following (CPF) control has recently come to the forum. See for example (Skjetne *et al.*, 2002; Ihle *et al.*, 2006; Ghabcheloo *et al.*, 2007) and the references therein for an introduction to the subject and an overview of important theoretical issues that warrant consideration. Different approaches to the solution of this and similar problems have been reported in the literature. They share a common strategy in that the problem of CFP is partially decoupled into two: *i) path-following*, where the objective is to find local closed loop control laws to steer each vehicle to its path at a reference speed, and *ii) multiple vehicle coordination*, where the goal is to adjust the reference speeds of the vehicles about the desired formation speed, so as to reach formation. Currently, many schemes are available for path-following control. However, the coordination problem lacks a complete solution addressing explicitly practical constraints that arise from the characteristics of the supporting inter-vehicle communications network. This is particularly relevant in the case of underwater applications, where fleets of vehicles must coordinate themselves by exchanging information over low bandwidth, short range communication channels that are plagued with intermittent failures, multi-path effects, and distance-dependent delays.

In this paper we derive new nonlinear motion control strategies for single and multiple under-actuated AUVs. We are motivated by and build on the previous work reported in (Ghabcheloo *et al.*, 2006*a*) The solutions adopted are rooted in Lyapunov-based theory and, in the case of multiple vehicle control, address explicitly the vehicle dynamics as well as the constraints imposed by the topology of the inter-vehicle communications network. The latter are tackled in the framework of graph theory (Godsil and Royle, 2001), which seems especially suited to study the impact of communication topologies on the performance that can be achieved with coordination (Fax and Murray, 2002).

For vehicle coordination we exploit the techniques proposed in (Ghabcheloo *et al.*, 2006*b*), and expand the circle of ideas advanced in (Yook *et al.*, 2002; Xu and Hespanha, 2006) to cope with asynchronous, discrete-time communications. In particular, we avail ourselves of the results presented in the above publications, where decentralized controllers for a distributed system were derived by using, for each system, its local state information together with estimates of the states of the systems that it communicates with. To minimize the requirements of inter-vehicle data exchange we include a logic-based communication strategy that borrows from the results in (Aguiar and Pascoal, 2007*b*). Here, we introduce the key constraint that a vehicle is only allowed to communicate with a set of immediate neighbors.

Due to space limitations, all the proofs are omitted. These can be found in (Vanni, 2007).

2. PROBLEM STATEMENT

The common approach to coordinated path-following is to divide the problem into *i)* a motion control task, to be solved individually for every vehicle, each having access to a set of local measurements, and *ii)* a dynamic assignment task, consisting in synchronizing the parametrization states that capture the along path distances between the vehicles. This strategy results in decoupling path-following (in space) and inter-vehicle coordination (in time) (Ghabcheloo *et al.*, 2007).

2.1 AUV model

Following standard practice, the general kinematic and dynamic equations of motion of an AUV in the horizontal plane can be developed using a global inertial coordinate frame $\{\mathcal{U}\}$ and a body-fixed coordinate frame $\{\mathcal{B}\}$, the origin of which coincides with the vehicle's center of mass. In the horizontal plane, the kinematic equations of motion of the vehicle can be written as

$$\dot{x} = u\cos(\psi) - v\sin(\psi), \qquad (1a)$$

$$\dot{y} = u\sin(\psi) + v\cos(\psi), \qquad (1b)$$

$$\dot{\psi} = r, \qquad (1c)$$

where u (surge speed) and v (sway speed) are the body-fixed frame components of the vehicle's velocity, x and y are the cartesian coordinates of its center of mass, ψ defines its orientation (heading angle), and r is the vehicle's angular speed. In the presence of a constant and irrotational ocean current, expressed in the body-fixed reference frame as $\boldsymbol{v}_c = [u_c, v_c]'$, u and v are given by $u = u_r + u_c$ and $v = v_r + v_c$ respectively, where $[u_r, v_r]'$ is the relative body-current linear velocity vector. Introducing a position vector $\boldsymbol{p} = [x, y]'$, a linear velocity vector $\boldsymbol{v} = [u, v]'$ and an orthonormal transformation matrix $R(\psi)$ from $\{\mathcal{B}\}$ to $\{\mathcal{U}\}$, eqs. (1a,1b) can be written in compact form as

$$\dot{\boldsymbol{p}} = R(\psi)\boldsymbol{v}. \tag{2}$$

We assume that the metacentric height of the AUV is sufficiently large to provide adequate static stability in roll motion, that the AUV is neutrally buoyant, and that the centre of buoyancy coincides with the centre of mass. Then, neglecting the motions in heave, roll, and pitch, the dynamic equations of motion for surge, sway and heading yield (Aguiar, 1996)

$$m_u \dot{u}_r - m_v v_r r + d_{u_r} u_r = \tau_u, \tag{3a}$$

$$m_v \dot{v}_r + m_u u_r r + d_{v_r} v_r = 0, \tag{3b}$$

$$m_r \dot{r} - m_{uv} u_r v_r + d_r r = \tau_r, \tag{3c}$$

where $m_u := m - X_{\dot{u}}$, $m_v := m - Y_{\dot{v}}$, $m_r := I_z - N_{\dot{r}}$ and $m_{uv} := m_u - m_v$ are mass and hydrodynamic added mass terms, and $d_{u_r} := -X_u - X_{|u|u}|u_r|$, $d_{v_r} := -Y_v - Y_{|v|v}|v_r|$ and $d_r := -N_r - N_{|r|r}|r|$ capture hydrodynamic damping effects. The symbols τ_u and τ_r denote the external force in surge and the external torque about the z axis of the vehicle, respectively. The particulars of the AUV used in the simulations at the end of the paper are those of the Sirene underwater shuttle described in (Aguiar, 1996).

2.2 Path-following

To solve the problem of driving a vehicle along a desired path, the key idea exploited is to make the vehicle approach a virtual target that moves along the path with a desired timing law. Let $\boldsymbol{p}_d$ be the position of the target, and v_d its desired rate of progression. We decompose the motion-control problem into an inner-loop dynamic task, which consists of making the vehicle's velocity $\boldsymbol{u} = [u_r, r]'$ track a desired speed reference $\boldsymbol{u}_d = [u_d, r_d]'$, and an outer-loop kinematic task, which assigns the reference speed so as to achieve convergence to the path. The path-following problem can be formulated as follows:

Problem 1. (Path-following). Consider the AUV whose motion is described by (2) and (3), and let $\boldsymbol{p}_d(\gamma) \in \mathbb{R}^2$ be a desired path parameterized by a continuous variable $\gamma(t) \in \mathbb{R}$ and $v_d(\gamma) \in \mathbb{R}$ a desired speed assignment. Suppose also that $\boldsymbol{p}_d(\gamma)$ is sufficiently smooth and its derivatives with respect to γ are bounded. Derive control laws for $\boldsymbol{u}_d$ and $\boldsymbol{\tau} = [\tau_u, \tau_r]$ such that the position error $\|\boldsymbol{p}(t) - \boldsymbol{p}_d(\gamma(t))\|$ and the speed error $|\dot{\gamma}(t) - v_d(\gamma(t))| \to 0$ converge to a neighborhood of the origin that can be made arbitrarily small.

In addition, we introduce the following constraints:

i) The sway velocity (v_r) measurement is not available, as the sensor required is expensive;
ii) The inner-loop controller only receives from the outer loop the speed reference $\boldsymbol{u}_d$. In particular, its time derivative $\dot{\boldsymbol{u}}_d$ is not available. *This is motivated by the fact that it is common for AUVs to be equipped with an inner-loop controller for tracking a speed reference.*

In principle, better results could be achieved, in terms of saturation and smoothness of the control signal, with a single control law for $\boldsymbol{\tau}$ based on both the dynamics and the kinematics of the AUV motion. However, the approach proposed here results in greater portability, since the same kinematic outer-loop controller could be run on a wide range of AUVs, regardless of the parameters that define their dynamics.

2.3 Coordination

Consider now a group of vehicles $\mathcal{I} := \{1, \ldots, n\}$, each with its own parametrized path $\boldsymbol{p}_{d_i}(\gamma_i), i \in \mathcal{I}$. To achieve coordination between the elements of the group, a common speed profile v_L has to be assigned to all the paths, so that the vehicles move along them while holding a desired inter-vehicle formation pattern. The parameter γ of each vehicle can be seen as a coordination state such that coordination exists between two vehicles i and j iff $\gamma_i(t) = \gamma_j(t)$. The key idea in designing the coordination controller is to introduce a control variable in the form of a correction term $\tilde{v}_d$ that is added to the desired speed of each vehicle, yielding

$$v_{d_i} = v_L + \tilde{v}_{d_i}. \tag{4}$$

Let $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_n]'$ be the vector containing the coordination states of the n vehicles, and $\mathcal{N}_i$ denote the set of vehicles that vehicle i exchanges information with or, in the case of unidirectional communication, receives information from. The coordination problem can be formulated as follows (Aguiar and Pascoal, 2007*a*):

Problem 2. (Coordination). For each vehicle $i \in \mathcal{I}$ derive a control law for the correction speed $\tilde{v}_{d_i}$ as a function of γ_i and γ_j, with $j \in \mathcal{N}_i$, such that for all $i, j \in \mathcal{I}$ the coordination errors $\gamma_i - \gamma_j$ and the formation speed error $|\dot{\boldsymbol{\gamma}} - v_L \boldsymbol{1}|$ converge to a small neighborhood around zero as $t \to \infty$.

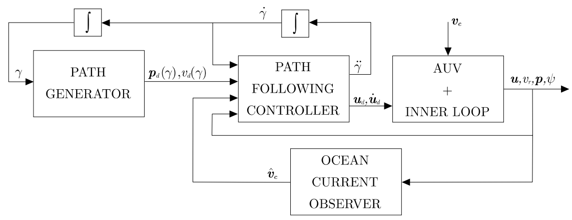

Fig. 1. Structure of the path-following controller.

3. PROBLEM SOLUTIONS

3.1 Path-following controller

Define the position error $\boldsymbol{e}$ expressed in body-frame coordinates as

$$\boldsymbol{e} = R'(\boldsymbol{p}(t) - \boldsymbol{p}_d(\gamma(t))),$$

and a speed error

$$z = \dot{\gamma}(t) - v_d(\gamma(t)),$$

whose time derivative yields

$$\dot{z} = \ddot{\gamma}(t) - \dot{v}_d(\gamma(t)).$$

Borrowing from the techniques of *backstepping*, we introduce the additional control variable $\ddot{\gamma}$ and redefine the path-following problem as that of determining control laws for $\boldsymbol{\tau}$, $\boldsymbol{u}_d$ and $\ddot{\gamma}$ to drive $\boldsymbol{e}$ and z to the origin.

Theorem 3. Consider the system described by (2) and (3) in closed-loop with the control laws

$$\boldsymbol{\tau} = -K_d(\boldsymbol{u} - \boldsymbol{u}_d), \tag{5a}$$

$$\boldsymbol{u}_d = \Delta^{-1}\left(-K_k \tanh(\boldsymbol{e} - \boldsymbol{\delta}) - \boldsymbol{v}_c + R'\frac{\partial \boldsymbol{p}_d(\gamma)}{\partial \gamma} v_d\right), \tag{5b}$$

$$\ddot{\gamma} = -k_z z + \frac{\partial v_d(\gamma)}{\partial \gamma}\dot{\gamma} + (\boldsymbol{e} - \boldsymbol{\delta})' R' \frac{\partial \boldsymbol{p}_d(\gamma)}{\partial \gamma}, \tag{5c}$$

where δ is an arbitrarily small negative constant, $\boldsymbol{\delta} = [0, \delta]'$ and

$$K_d = \begin{bmatrix} k_u & 0 \\ 0 & k_r \end{bmatrix}, \quad K_k = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}, \quad \Delta = \begin{bmatrix} 1 & 0 \\ 0 & -\delta \end{bmatrix}$$

are positive definite matrices. If the gain K_d is sufficiently large then the closed-loop signals are bounded and the control laws (5) solve the path-following problem.

The structure of the path-following controller is illustrated in Fig. 1. The strategy to deal with the problem that the vehicle is underactuated is to make the position error converge to the point $(0, \delta)$, arbitrarily close to the origin. To measure the water current $\boldsymbol{v}_c$ we use the observer described in Aguiar and Pascoal (2007*b*). Theorem 3 still holds with v_c substituted by its estimate. This follows from the fact that the closed-loop system can be viewed as the cascade of a globally asymptotically stable system (the observer) with output error $\tilde{\boldsymbol{v}}_c$, and an ISS system with input $\tilde{\boldsymbol{v}}_c = v_c - \hat{\boldsymbol{v}}_c$.

3.2 Coordination controller

To take into consideration the communication constraints we resort to graph theory. The vehicles in a formation are the vertices of a graph, the existing communication links are the edges, directed if the communication is unidirectional, undirected if the communication is bidirectional. Letting A and D denote respectively the adjacency matrix and the degree matrix associated with the graph that describes the communication network, we can define an error vector

$$\boldsymbol{\xi} = L_D \boldsymbol{\gamma},$$

where $L_D = D^{-1}(D - A)$ is the normalized Laplacian. The assumption is made that the communication topology does not change in time, *i.e.*, the Laplacian is constant. The i-th element of vector $\boldsymbol{\xi}$ is

$$\xi_i = \gamma_i - \frac{1}{|\mathcal{N}_i|}\sum_{j \in \mathcal{N}_i} \gamma_j,$$

that is, the sum of the coordination errors between vehicle i and the vehicles that communicate with it. The single variable ξ_i captures the communication constraints of the network and can be used for control purposes (Ghabcheloo *et al.*, 2007). From the properties of the Laplacian (Godsil and Royle, 2001), the error $\boldsymbol{\xi}$ is null *only* when the vehicles are in coordination. Furthermore, we will assume that the communication is bidirectional. This implies that the Laplacian is symmetric and that the left eigenvector associated to the null eigenvalue is $\mathbf{1}'$.

Theorem 4. Consider a formation of n vehicles, each guided by the motion control laws (5) along a path parametrized by γ_i, and let L_D be the normalized Laplacian of a graph that describes the inter-vehicle communications network. Assume that any two neighboring vehicles communicate in a continuous manner. Then, the decentralized control law

$$\tilde{\boldsymbol{v}}_d = -k_\xi \tanh\left(L_D \boldsymbol{\gamma}\right), \quad k_\xi > 0 \tag{6}$$

solves Problem 2.

3.3 Discrete communication

The coordination controller described by (6) relies on the continuous exchange of information among the vehicles in the formation. This assumption is unrealistic because underwater communication systems have very low bandwidths and only allow the exchange of data to take place at discrete instants of time. In (Aguiar and Pascoal, 2007*a*), a logic-based communications strategy is proposed, that takes into account both the fact that communications do not occur in a continuous manner and the cost of exchanging information. In between communications, that are regulated by a supervisory logic, each vehicle runs estimations of the coordination states of the rest of the formation. This is done through synchronized estimation blocks, identical for every vehicle, that admit the following dynamics, based on (4) and (6):

$$\dot{\hat{\boldsymbol{\gamma}}} = \boldsymbol{v}_L(\hat{\boldsymbol{\gamma}}) - k_\xi \tanh\left(L_D \hat{\boldsymbol{\gamma}}\ \right). \qquad (7)$$

In particular, every vehicle runs an estimate of its own state. It is by comparing the actual value of its state with this estimate that a vehicle decides when to communicate with the vehicles in its neighborhood. If, at a certain instant t_k, $|\gamma_i - \hat{\gamma}_i| \geq \epsilon^2$, then vehicle i broadcasts the value of γ_i. Assuming that no delays affect the communication links, each vehicle updates its estimate instantly, so that

$$\hat{\boldsymbol{\gamma}}_i(t_k) = \boldsymbol{\gamma}_i(t_k).$$

Remembering the expression of the normalized Laplacian, the control law (6) becomes

$$\tilde{\boldsymbol{v}}_d = -k_\xi \tanh\left(\boldsymbol{\gamma} - D^{-1} A \hat{\boldsymbol{\gamma}}\right), \qquad (8)$$

where it has been explicited that the correction term for every vehicle is the sum of a term that depends on the coordination state of the vehicle itself, which is available at every instant, and a term built on the estimates of the states of the other vehicles. In the instants between communications, a perturbation input $\tilde{\boldsymbol{\gamma}} = \boldsymbol{\gamma} - \hat{\boldsymbol{\gamma}}$ appears. The overall closed-loop system is ISS with respect to $\tilde{\boldsymbol{\gamma}}$, which is bounded by ϵ^2. Selecting a lower tolerance ϵ^2 reduces the neighborhood of the origin to which ξ converges but increases the number of messages exchanged among vehicles. Notice that although the control signal for each vehicle is based on the states of its neighbors, every agent runs an estimation of the states of the whole formation. In the absence of delays, the updates of the coordination states can be retransmitted across the formation and reach instantly every vehicle (the graph is connected), so every vehicle has access to the same estimates, as would happen if one single centralized estimator were shared by all the vehicles.

We now state the main result of the paper.

Theorem 5. Consider the system composed by *i)* a formation of n vehicles of the form (3), guided by the motion control laws (5) along a path parametrized by γ_i, *ii)* the coordination law (8) with the estimator (7), and *iii)* the logic-based communication system described above. Then, the control system proposed solves the coordinated path-following problem, that is, Problem 1 and Problem 2.

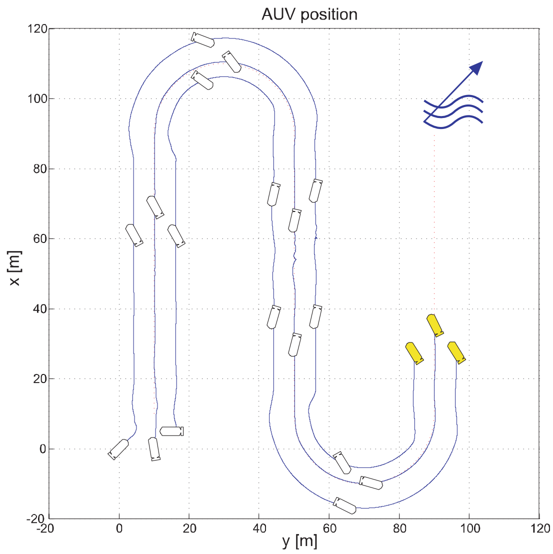

Fig. 2. Trajectory of the AUVs on the x-y plane

3.4 Time delays

Underwater communication channels suffer from intermittent failures, latency and multi-path effects. Notably important are delays introduced both by the processing of data and by the distance among the vehicles. Assume that at time t vehicle i broadcasts its coordination state. Vehicles j and k will receive the message at $t + t_j$ and $t + t_k$ respectively. If the three vehicles were to update their estimate of $\boldsymbol{\gamma}_i$ as soon as they receive a message (or send, in the case of i), then the estimation blocks would cease to be synchronized. The communication strategy must then be modified. One approach is to keep the estimators always synchronized by updating them only at $t_k + \tau_{max}$, where τ_{max} is the maximal delay. See (Vanni, 2007) for more details and other approaches.

4. ILLUSTRATIVE EXAMPLE

Computer simulations were done to illustrate the performance of the CPF controller proposed, when applied to a group of three AUVs. The numerical values used for the physical parameters match those of the Sirene AUV, described in (Aguiar, 1996). The AUVs are required to follow a lawn mower path, typical in ocean exploration scenarios, while keeping a triangular formation pattern. The parameters of the simulation are: $\boldsymbol{p}_1(0) = (0\text{ m}10\text{ m})$, $\boldsymbol{p}_2(0) = (0\text{ m}0\text{ m})$, $\boldsymbol{p}_3(0) = (5\text{ m}15\text{ m})$, $\psi_1(0) = -10°$, $\psi_2(0) = 45°$, $\psi_3(0) = -90°$, $\|v_c\| = .3\text{m/s}$, $\psi_c = 45°$, $k_u = 4000$, $k_r = 4000$, $\delta = -.5$, $k_x = 1$, $k_y = .6$, $k_z = 1$, $k_\xi = .5$, $\epsilon^2 = 5$, $\bar{\tau} = 5$. To test the robustness of the control system, noise was added to both position and velocity sensor measurements. Vehicle 1 is allowed to communicate with AUVs 2 and 3, but the latter two do not communicate

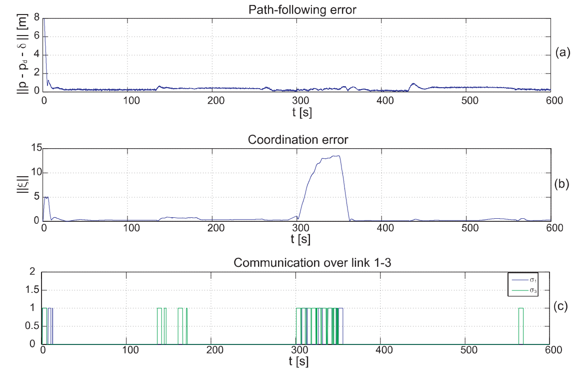

Fig. 3. Norm of the total path-following error ($[\boldsymbol{e}_1 - \delta, \boldsymbol{e}_2 - \delta, \boldsymbol{e}_3 - \delta]$) (a), norm of the coordination error $\boldsymbol{\xi}$ (b) and communication over link 1-3 (c)

between themselves directly. To further illustrate the behavior of the proposed CPF control architecture, we also force the following scenario: from $t = 300$ s to $t = 350$ s, the coordination state of AUV 3 cannot increase, *i.e.*, $\dot{\gamma}_3 = 0$. Fig. 2 shows the trajectories of the AUVs. The orientation of the vehicles is always such that it compensates the effect of the water current. Fig. 3 shows the convergence of the path-following error (a) and of the coordination error (b), that increases when γ_3 is forced to stop. The variable σ in Fig. 3 (c) assumes value 1 when vehicle 1 or vehicle 3 send a message over link 1-3. Note the reduced frequency of data exchanges in the overall period. The vehicles only need to communicate briefly at the beginning of the simulation, to synchronize their parametrization states, and during the curved parts of the paths. The communication rate increases when AUV 3 is forced to slow down.

5. CONCLUSIONS

The paper addressed the problem of coordinated path-following for a group of underactuated autonomous underwater vehicles in the presence of ocean currents. The solution proposed builds on Lyapunov based techniques and is valid for a large class of underwater underactuated vehicles. Furthermore, it takes into account the constraints imposed by the topology of the inter-vehicle communications network, and it led to a decentralized control law with reduced exchange of data among the vehicles. Simulations illustrated the efficacy of the solution proposed.

REFERENCES

Aguiar, A. P. (1996). Modeling, control, and guidance of an autonomous underwater shuttle for the transport of benthic laboratories. Master's thesis. Instituto Superior Técnico - Dept. Electrical Eng. Lisbon, Portugal.

Aguiar, A. P. and A. M. Pascoal (2007*a*). Coordinated path-following control for nonlinear systems with logic-based communication. Submitted for review.

Aguiar, A. P. and A. M. Pascoal (2007*b*). Dynamic positioning and way-point tracking of underactuated AUVs in the presence of ocean currents. *International Journal of Control.* In press.

Aguiar, A. P. and J. P. Hespanha (2007). Trajectory-tracking and path-following of underactuated autonomous vehicles with parametric modeling uncertainty. *Trans. on Automatic Control.* **18**(8). In Press.

Aguiar, A. P., L. Cremean and J. P. Hespanha (2003). Position tracking for a nonlinear underactuated hovercraft: Controller design and experimental results. In: *Proc. of the* 42^{nd} *Conf. on Decision and Control (CDC'03).* Vol. 4. Maui, Hawaii, USA. pp. 3858–3863.

Alonge, F., F. D'Ippolito and F. M. Raimondi (2001). Trajectory tracking of underactuated underwater vehicles. In: *Proc. of the* 40^{th} *Conf. on Decision and Control (CDC'01).* Vol. 5. Orlando, FL, USA. pp. 4421–4426.

Encarnação, P. and A. M. Pascoal (2000). 3-D path following for autonomous underwater vehicle. In: *Proc. of the* 39^{th} *Conf. on Decision and Control (CDC'00).* Vol. 3. Sydney, NSW, Australia. pp. 2977–2982.

Fax, J.A. and R.M. Murray (2002). Information flow and cooperative control of vehicle formations. In: *Proc. of the* 15^{th} *IFAC World Congress.* Barcelona, Spain.

Fossen, T. I. (1994). *Guidance and control of ocean vehicles.* 2nd ed.. John Wiley & Sons Ltd.. Chichester, UK.

Ghabcheloo, R., A. M. Pascoal, C. Silvestre and I. Kaminer (2007). Nonlinear coordinated path following control of multiple wheeled robots with bidirectional communication constraints. *International Journal of Adaptive Control and Signal Processing* **21**(2-3), 133–157.

Ghabcheloo, R., A. P. Aguiar, A. M. Pascoal, C. Silvestre, I. Kaminer and J. P. Hespanha (2006*a*). Coordinated path-following control of multiple underactuated autonomous vehicles in the presence of communication failures. In: *Proc. of the* 45^{th} *Conf. on Decision and Control (CDC'06).* San Diego, CA, USA. pp. 4345–4350.

Ghabcheloo, R., A. P. Aguiar, A. M. Pascoal, C. Silvestre, I. Kaminer and J. P. Hespanha (2006*b*). Coordinated path-following in the presence of communication losses and time delays. Submitted for publication.

Godsil, C. and G. Royle (2001). *Algebraic graph theory.* Springer-Verlag. New York.

Ihle, I.-A. F., M. Arcak and T. I. Fossen (2006). Passivation designs for synchronization of path following systems. In: *Proc. of the* 45^{th} *Conf. on Decision and Control (CDC'06).* San Diego, CA, USA.

Jiang, Z.-P (2002). Global tracking control of underactuated ships by Lyapunov's direct method. *IFAC Automatica* **38**(2), 301–309.

Lefeber, E., K. Y. Pettersen and H. Nijmeijer (2003). Tracking control of an underactuated ship. *Trans. on Control Systems Technology* **11**(1), 52–61.

Leonard, N. E. (1995). Global tracking control of underactuated ships by Lyapunov's direct method. *Journal of Oceanic Engineering* **20**(3), 211–220.

Skjetne, R., S. Moi and T. I. Fossen (2002). Nonlinear formation control of marine craft. In: *Proc. of the* 41^{st} *Conf. on Decision and Control (CDC'02).* Vol. 2. Las Vegas, NV, USA. pp. 1699–1704.

Vanni, Francesco (2007). Coordinated motion control of multiple autonomous underwater vehicles. Master's thesis. Instituto Superior Técnico - Dept. Mechanical Engineering. Lisbon, Portugal.

Xu, Y. and J.P. Hespanha (2006). Communication logic design and analysis for networked control systems. In: *Current Trends in Nonlinear Systems and Control* (L. Menini, L. Zaccarian and C. T. Abdallah, Eds.). pp. 495–514. Birkhäuser. Boston, MA, USA.

Yook, J. K., D. M. Tilbury and N. R. Soparkar (2002). Trading computation for bandwidth: Reducing communication in distributed control systems using state estimators. *Trans. on Ctr. Syst. Tech.* **10**(4), 503–518.