Adaptive Leader-Follower Formation Control of Autonomous Marine Vehicles

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Abstract— **This paper addresses the problem of adaptive leader-follower formation control (ALFC) of autonomous marine vehicles. The rationale for this study can be found in a number of challenging geotechnical surveying missions aimed at mapping geological structures under the seabed. Mission specifications require that a group of surface vehicles equipped with acoustic receivers (hydrophones) maneuver in formation and acquire acoustic data emitted by one or more vehicles that carry acoustic emitters. To this effect, we adopt a basic set-up for ALFC previously proposed for mobile robots modeled as single integrators and extend it to include explicitly the full dynamic equations of a representative type of marine vehicles. The paper offers a formal proof of convergence of the resulting formation control system. Results of simulations illustrate the performance that can be obtained with the control law proposed.**

I. Introduction

This paper addresses the general problem of making a group of follower vehicles reach a desired (possibly time-varying) geometric formation with respect to the reference frame defined by two leader vehicles that move rigidly with the same linear velocity. The latter is unknown to the follower vehicles. In the set-up adopted, the two leader vehicles play the role of guides and each of the follower vehicles has access to its relative position with respect to either the two leader vehicles or a number of neighboring vehicles. Control laws are derived for each of the follower vehicles that take directly into consideration their dynamic equations of motion. In the sequel, because the overall motion of the group of follower vehicles is effectively guided by that of the leading vehicles and the motion of the latter is not known in advance, the problem will simply be referred to as Adaptive Leader-Follower Formation Control (ALFC); see [1] for a lucid presentation of this circle of ideas in the context of autonomous mobile robots.

The rationale for this problem can be found in a number of extremely challenging geotechnical surveying missions aimed at mapping geological structures under the seabed; see for example [2], [3], and the references therein. In this context, there is considerable interest in affording a group of surface or semisubmerged vehicles equipped with acoustic receivers (hydrophones) the capability to maneuver in formation and acquire acoustic data emitted by one or more vehicles that carry acoustic emitters. Post-processing of the data acquired will reveal the types of sediments and (if there is enough penetration of the acoustic waves) of the geological structures or extent of hydrocarbon reservoirs under the seabed, [3]. In these conditions, allowing for the emitter vehicles and receiving vehicles to change their relative positions (in contrast to what is classically done during offshore seismic surveys, in which for example one emitter vehicle actually carries an array of acoustic hydrophones and there is no acoustic source / receiver separation) paves the way for the exploitation of spatial diversity yielding better accuracy in the mapping of relevant structures under the sediments.

The ALFC problem falls in the scope of cooperative networked motion control, a topic that has been the subject of considerable research effort recently. A network of autonomous robots can perform tasks more efficiently than a single agent or can accomplish more challenging tasks not executable by a single individual. Potential applications for multi-robot systems include collaborative search and rescue, environmental monitoring, exploration and distributed reconfigurable sensor networks, see [4], [5]. Some of the work done has been focused on cooperative path following, where a group of vehicles is required to maneuver along prespecified paths while keeping a desire formation pattern (absolute formation control). See for example [6] for work along these lines with applications in the marine field. In the work reported, each vehicle is required to know its absolute position and those of the neighboring vehicles. This is in contrast with the work in [7], where the objective is not to do absolute formation control but relative formation control instead. In this situation, each vehicle is only required to know the relative position of some of the neighbors in its own reference frame. From a practical point of view, however, this still poses formidable challenges when the vehicles move underwater. In the same vein, the work in [8] addresses the challenging problem of keeping an autonomous underwater vehicle in a moving triangular formation with respect to two autonomous surface leader vehicles by relying on the computation of the ranges between the follower vehicle and the leaders, that is, without computing explicitly the relative position of the follower in the moving frame established by the leaders. Even though the efficacy of the Range-Only Formation (ROF) control strategy described in [8] has been shown in practice, the approach suffers from the drawback that the solution is not easily scalable for a large number of

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follower vehicles.

Motivated by the above considerations, in this paper we address a cooperative motion control problem where all vehicles move at the water surface, thus making the task of computing their relative positions straightforward by relying on GPS data and an inter-vehicle aerial communications network. Previous work on related problems -albeit with applications to vehicles that are not necessarily marine robots- appears in the literature under the heading of leader-follower control [9], [10]. In some of the publications, it was tacitly assumed that there is only one group leader and all the follower robots have access to the leader's velocity or position. A different approach based on a cascade control strategy has also been considered in some of the literature. The key idea is that each follower vehicle tracks its precedent neighbor and the first follower robot tracks the leader [11], [12]. The work in [1] goes one step further and the adaptive formation control is addressed for a network of autonomous mobile robots in which only two leaders know the prescribed reference velocity while the others play the role of followers. Notice however that the models for the follower vehicles are restricted to single integrators, a simplification that is not warranted in the case of autonomous marine robots that exhibit complex dynamics.

It is against this background of ideas that in this paper we extend the methodology exposed in [1] for the ALFC problem to autonomous underactuated surface robots with highly nonlinear dynamics. We do this in two steps: we first derive a leader-follower control law by adopting a unicycle kinematic model for the follower vehicles based on a virtual controller and a tracking control design; after, the control law is modified to take into account the complex dynamics of a marine robot.

II. Problem Formulation

This paper deals with the ALFC problem in which a group of mobile robots aims to keep a rigid formation and track two leader vehicles moving with a given reference velocity. Consider a group of N mobile robots in which the position vector of vehicle $i = 1, \dots, N$ is represented by $\mathbf{p}_i \in \mathbb{R}^2$. The vehicles are divided in two classes, two leaders denoted by $\mathbf{p}_1$ and $\mathbf{p}_2$, and the rest of the vehicles are considered followers.

A. Model of the follower vehicles

At this stage, we consider that the followers are modeled with unicycle kinematics subject to a simple non-holonomic constraint, such that the dynamics of robot $i = 3, \dots, N$ is defined by

$$\begin{aligned} \dot{\mathbf{p}}_i &= R(\psi_i)\,[u_i, 0]^T \\ \dot{\psi}_i &= r_i \end{aligned} \tag{1}$$

where ψ_i is the heading angle of vehicle i, $R(\psi)$ denotes the rotation matrix defined by $R(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}$ and $[u_i, r_i]$ are the control inputs.

Later, in Section IV, we will consider a class of marine vehicles with more complex dynamics.

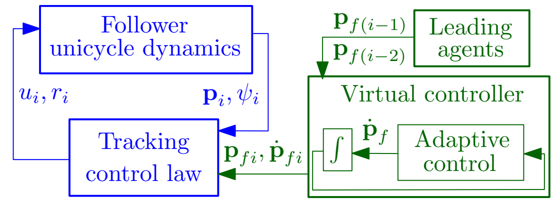

Fig. 1. Structure of the decoupled leader-follower control strategy

B. Leaders' motion

Consider a setup in which the two leaders track a given constant velocity $\mathbf{v}_0 \in \mathbb{R}^2$, and maintain a constant distance between them. In [1], a simple gradient-based control for the leaders, modeled with simple integrator dynamics, is proposed in order to enforce them to maintain a constant distance from each other and track the same constant velocity. However, considering unicycle dynamics for the leaders, other control strategies could be applied, for instance the cooperative path following presented in [6].

C. Control Objectives

In this paper we focus on the control of the follower vehicles. It is assumed that the two leaders keep a constant distance and tack the reference velocity $\mathbf{v}_0$ which is unknown for the followers. The aim for the follower robots is to follow the two leaders' motion keeping a particular rigid formation such that, each vehicle follows its two leading robots and maintains a triangular formation with them. For instance, the aim for vehicle 3 is to reach a triangular formation with the two leaders, i.e., vehicles 1 and 2; the vehicle 4 aims to keep a triangular formation with its leading robots 3 and 2, and so on. The desired formation is achieved when each follower ($i \geq 3$) keeps a triangular formation with respect to its neighbor robots $i-1$ and $i-2$. Let's d_{ij} define the desired distance between vehicles i and j. The objective is thus to design the control inputs $[u_i, r_i]$ such that the distances between each follower and its two neighbors converge to the desired values $d_{i(i-1)}$ and $d_{i(i-2)}$ respectively, i.e. $\|\mathbf{p}_i - \mathbf{p}_{i-1}\| \to d_{i(i-1)}$ and $\|\mathbf{p}_i - \mathbf{p}_{i-2}\| \to d_{i(i-2)}$ when $t \to \infty$.

III. Control Design

The authors of [1] proposed an adaptive strategy to solve the leader-follower formation problem for a group of single integrator robots. Our approach is based on these previous ideas and on a tracking control design. The first contribution of the paper is thus, to extend the previous result presented in [1] considering the nonlinear dynamics (1). In Section IV the new control strategy is applied to the complex nonlinear dynamics of an autonomous marine vehicle.

We propose a decoupled control strategy divided in two steps, as explained in Fig. 1. The main idea is to introduce a virtual variable $\mathbf{p}_{fi}$ with simple integrator dynamics for each follower ($i \geq 3$) in order to apply the adaptive leader-follower control from [1]. The trajectory of the virtual vehicle becomes a reference for the real robot $\mathbf{p}_i$ and designing a tracking controller, the real vehicle will track the trajectory of the virtual agent, see Fig. 1.

A. Virtual controller

Firstly, we present the control design based on the adaptive approach from [1] for the virtual agents. The aim for the virtual follower $\mathbf{p}_{fi}$ is to keep a fixed distance $d_{i(i-1)}$ and $d_{i(i-2)}$ with its leading neighbors $i-1$ and $i-2$ respectively. In other words, the aim is to reach a formation satisfying

$$\begin{aligned} \|\mathbf{p}_{f(i-1)} - \mathbf{p}_{f(i-2)}\| &= d_{(i-1)(i-2)} \\ \|\mathbf{p}_{fi} - \mathbf{p}_{f(i-1)}\| &= d_{i(i-1)} \\ \|\mathbf{p}_{fi} - \mathbf{p}_{f(i-2)}\| &= d_{i(i-2)} \end{aligned} \tag{2}$$

for $i = 3, \ldots, N$, where $\mathbf{p}_{f1}$ and $\mathbf{p}_{f2}$ correspond to the leader vehicles $\mathbf{p}_1$ and $\mathbf{p}_2$ respectively. With a view to design a realisable formation the following conditions must be satisfied:

$$\begin{aligned} d_{(i-2)(i-1)} &< d_{(i-2)i} + d_{(i-1)i} \\ d_{(i-2)i} &< d_{(i-2)(i-1)} + d_{(i-1)i} \\ d_{(i-1)i} &< d_{(i-2)(i-1)} + d_{(i-2)i}. \end{aligned}$$

It is assumed that each follower computes the position of its virtual agent and receives the required information to compute the relative position with respect to its two leading neighbors. In the sequel, the following notation is defined

$$\begin{aligned} \mathbf{z}_{i(i-1)} &= \mathbf{p}_{f(i-1)} - \mathbf{p}_{fi} \\ \mathbf{z}_{i(i-2)} &= \mathbf{p}_{f(i-2)} - \mathbf{p}_{fi} \\ \eta_{i(i-1)} &= \|\mathbf{z}_{i(i-1)}\|^2 - d^2_{i(i-1)} \\ \eta_{i(i-2)} &= \|\mathbf{z}_{i(i-2)}\|^2 - d^2_{i(i-2)}. \end{aligned}$$

Using previous notation the following adaptive control law is proposed for the virtual follower ($i \geq 3$):

$$\begin{aligned} \dot{\mathbf{p}}_{fi} &= \theta_i + \mathbf{z}_{i(i-1)}\eta_{i(i-1)} + \mathbf{z}_{i(i-2)}\eta_{i(i-2)} \\ \dot{\theta}_i &= \mathbf{z}_{i(i-1)}\eta_{i(i-1)} + \mathbf{z}_{i(i-2)}\eta_{i(i-2)}. \end{aligned} \tag{3}$$

In order to study the stability of the virtual system, firstly we consider the triangular formation problem of one virtual follower $\mathbf{p}_{f3}$ and the two leaders $\mathbf{p}_1, \mathbf{p}_2$. Consider the control law (3) for $i = 3$, we obtain:

$$\begin{aligned} \dot{\mathbf{z}}_{13} &= \dot{\mathbf{p}}_1 - (\theta_3 + \mathbf{z}_{13}\eta_{13} + \mathbf{z}_{23}\eta_{23}) \\ \dot{\mathbf{z}}_{23} &= \dot{\mathbf{p}}_2 - (\theta_3 + \mathbf{z}_{13}\eta_{13} + \mathbf{z}_{23}\eta_{23}) \\ \dot{\theta}_3 &= \mathbf{z}_{13}\eta_{13} + \mathbf{z}_{23}\eta_{23}. \end{aligned} \tag{4}$$

The stability is studied through the Lyapunov function $V_f = \frac{1}{4}\eta_{13}^2 + \frac{1}{4}\eta_{23}^2 + \frac{1}{2}\|\theta_3\|^2$. Taking the time derivative of V_f along the solutions of system (4), one obtains

$$\begin{aligned} \dot{V}_f &= \frac{1}{2}\eta_{13}\dot{\eta}_{13} + \frac{1}{2}\eta_{23}\dot{\eta}_{23} + \theta_3^T\dot{\theta}_3 \\ &= [\eta_{13}\mathbf{z}_{13}^T \quad \eta_{23}\mathbf{z}_{23}^T] \begin{bmatrix} \dot{\mathbf{p}}_1 \\ \dot{\mathbf{p}}_2 \end{bmatrix} - \|\eta_{13}\mathbf{z}_{13} + \eta_{23}\mathbf{z}_{23}\|^2 \\ &\leq \|\eta_{13}\mathbf{z}_{13}^T \quad \eta_{23}\mathbf{z}_{23}^T\| \left\| \begin{matrix} \dot{\mathbf{p}}_1 \\ \dot{\mathbf{p}}_2 \end{matrix} \right\| - \|\eta_{13}\mathbf{z}_{13} + \eta_{23}\mathbf{z}_{23}\|^2. \end{aligned}$$

It can be proven by contradiction that $\dot{V}_f(t_s) \leq 0$ for a large time t_s and consequently V_f is bounded (see details in [1]). Hence, η_{13}, η_{23} and θ_3 are bounded and $\eta_{13}\mathbf{z}_{13} + \eta_{23}\mathbf{z}_{23}$ is uniformly continuous, thanks to Barbalat's Lemma we conclude that

$$\eta_{13}\mathbf{z}_{13} + \eta_{23}\mathbf{z}_{23} \to 0 \quad \text{as} \quad t \to \infty.$$

The both leaders track a given velocity $\mathbf{v}_0$ and then, following trivial computations, it can be proven that $\theta_3 \to \mathbf{v}_0$ and consequently $\dot{\mathbf{p}}_{f3} \to \mathbf{v}_0$. Note that $\eta_{13}\mathbf{z}_{13} + \eta_{23}\mathbf{z}_{23} = 0$ means that the agents form a triangular formation ($\eta_{13} = 0$ and $\eta_{23} = 0$) or a collinear formation (η_{13} or $\eta_{23} \neq 0$). The authors of [1] showed that the triangular formation is asymptotically stable, however the collinear one is unstable.

In conclusion, if both leaders 1 and 2 track the given velocity $\mathbf{v}_0$ and the distance between them is constant then, for the virtual follower 3 with the control law (3), the triangular formation satisfying (2) is asymptotically stable. This result can be straightforwardly extended to the case of multiple virtual followers.

B. Tracking control design

We propose a tracking control design to make each mobile robot follow the desired time-varying trajectory generated by its virtual agent. In other words, the aim is to make $\mathbf{p}_i$ converge to the desired position $\mathbf{p}_{fi}(t)$ which is moving with velocity $\dot{\mathbf{p}}_{fi}$. The position error expressed in the body frame, is defined as $\mathbf{e}_i = R(\psi_i)^T(\mathbf{p}_i - \mathbf{p}_{fi})$. Then, the error dynamics satisfies ($i \geq 3$):

$$\begin{aligned} \dot{\mathbf{e}}_i &= R(\psi_i)^T(\dot{\mathbf{p}}_i - \dot{\mathbf{p}}_{fi}) - R(\psi_i)^T S(r_i)\mathbf{e}_i \\ &= [u_i, 0]^T - R(\psi_i)^T\dot{\mathbf{p}}_{fi} - R(\psi_i)^T S(r_i)\mathbf{e}_i \end{aligned}$$

where $S(r_i) = \begin{bmatrix} 0 & -r_i \\ r_i & 0 \end{bmatrix}$. We introduce the vector $\boldsymbol{\delta} = [\delta, 0]^T$, with δ being an arbitrarily small negative constant, to make appear the control input vector $[u_i, r_i]^T$ in the error dynamics

$$\begin{aligned} \dot{\mathbf{e}}_i &= [u_i, 0]^T - R(\psi_i)^T\dot{\mathbf{p}}_{fi} - R(\psi_i)^T S(r_i)(\mathbf{e}_i - \boldsymbol{\delta} + \boldsymbol{\delta}) \\ &= \Delta[u_i, r_i]^T - R(\psi_i)^T\dot{\mathbf{p}}_{fi} - R(\psi_i)^T S(r_i)(\mathbf{e}_i - \boldsymbol{\delta}), \end{aligned}$$

where $\Delta = \begin{bmatrix} 1 & 0 \\ 0 & -\delta \end{bmatrix}$. Based on the tracking control from [13], the following control law is proposed:

$$[u_i, r_i]^T = \Delta^{-1}\left(R(\psi_i)^T\dot{\mathbf{p}}_{fi} - K(\mathbf{e}_i - \boldsymbol{\delta})\right), \tag{5}$$

where K is positive definite. To analyze the convergence of the error system the Lyapunov function $V_{\mathbf{e}_i} = \frac{1}{2}\|\mathbf{e}_i - \boldsymbol{\delta}\|^2$ is proposed. Differenciating along the solutions of (12) yields

$$\dot{V}_{\mathbf{e}_i} = (\mathbf{e}_i - \boldsymbol{\delta})^T\dot{\mathbf{e}}_i = -(\mathbf{e}_i - \boldsymbol{\delta})^T K(\mathbf{e}_i - \boldsymbol{\delta}) \leq 0.$$

Therefore $\mathbf{e}_i = \boldsymbol{\delta}$ is a globally exponentially stable equilibrium point for this system. Assuming that each follower is able to compute its own position $\mathbf{p}_i$ in inertial coordinates then, the trajectory tracking control law (5) can be applied.

C. Stability analysis

The following theorem states the first contribution of the paper dealing with the control law for a unicycle robot to achieve a triangular formation with two leader vehicles.

Theorem 1 *Consider two leader vehicles* 1 *and* 2 *tracking the given velocity* $\boldsymbol{v}_0$ *and the distance between them is kept constant. Let* K *be a definite positive matrix and* δ *a small negative parameter which satisfies* $|\delta| << d_{13}$, $|\delta| << d_{23}$. *The follower robot represented by* (1) *with the control law* (5) *and the virtual adaptive controller* (3), *achieves a triangular formation with the two leaders, such that*

$$\begin{aligned} d_{13} - |\delta| \leq \|\boldsymbol{p}_3 - \boldsymbol{p}_1\| \leq d_{13} + |\delta| \\ d_{23} - |\delta| \leq \|\boldsymbol{p}_3 - \boldsymbol{p}_2\| \leq d_{23} + |\delta| \end{aligned} \tag{6}$$

Proof: The control strategy proposed is composed of two independent control loops, the virtual single integrator follower with the adaptive control law (3) and the unicycle robot with the tracking controller (5). The convergence of both controllers has been previously studied using Lyapunov techniques. As it has been proven in [1], the triangular formation ($\eta_{13} = 0$ and $\eta_{23} = 0$) composed by the virtual follower $\mathbf{p}_{f3}$ and the two leaders is asymptotically stable. When this formation is achieved the equalities $\|\mathbf{p}_{f3} - \mathbf{p}_1\| = d_{13}$ and $\|\mathbf{p}_{f3} - \mathbf{p}_2\| = d_{23}$ are satisfied. Thanks to the tracking control law (5) the error position $\mathbf{e}_3$ converges to $\boldsymbol{\delta}$ exponentially and then, vehicle $\mathbf{p}_3$ tracks the virtual follower $\mathbf{p}_{f3}$ such that, $\|\mathbf{p}_3 - \mathbf{p}_{f3}\| = |\delta|$. Consequently, the unicycle robot $\mathbf{p}_3$ forms a triangular formation with leaders 1 and 2. We analyze the properties of the resulting formation in order to find a lower and upper bound for the final distance between the robot and each leader. Rewriting the distance between the follower robot and the leader 1 as $\|\mathbf{p}_3 - \mathbf{p}_1\| = \|(\mathbf{p}_3 - \mathbf{p}_{f3}) - (\mathbf{p}_1 - \mathbf{p}_{f3})\|$ it can be shown that when the virtual follower reaches its equilibrium state then

$$\|\mathbf{p}_3 - \mathbf{p}_1\| \leq \|\mathbf{p}_3 - \mathbf{p}_{f3}\| + \|\mathbf{p}_1 - \mathbf{p}_{f3}\| = |\delta| + d_{13}.$$

The lower bound is a consequence of the reverse triangular inequality:

$$\|\mathbf{p}_3 - \mathbf{p}_1\| \geq \left| \|\mathbf{p}_3 - \mathbf{p}_{f3}\| - \|\mathbf{p}_1 - \mathbf{p}_{f3}\| \right| = \left| |\delta| - d_{13} \right|.$$

Hence, as condition $|\delta| << d_{13}$ is satisfied, we conclude

$$d_{13} - |\delta| \leq \|\mathbf{p}_3 - \mathbf{p}_1\| \leq d_{13} + |\delta|.$$

The same inequalities hold for the leader 2 and thus, the final triangular formation between the unicycle robot 3 and both leaders satisfies (6). ∎

Note that this result can be easily generalized for a fleet of followers defined by (1). Consider that each robot has a virtual agent associated. Thanks to the adaptive control law (3) if the precedent two virtual agents are not coincident, then the virtual follower i converges to form a triangular formation (stable) or a collinear formation (unstable) with the precedent two virtual followers. Therefore the virtual agents form a formation satisfying (2). Due to the tracking controller (5), each robot i converge to a small neighborhood of its virtual agent.

IV. Extension to Marine Vehicles

The aim of this section is to implement the previous ALFC strategy in autonomous marine vehicles. For this purpose, the MEDUSA[1] class of autonomous semisubmerged robotic vehicles is considered.

A. Kinematic and dynamic model of MEDUSA

As the vehicle is bound to the horizontal plane, its kinematic equations take the simpler form

$$\begin{aligned} \dot{x} &= u \cos\psi - v \sin\psi \\ \dot{y} &= u \sin\psi + v \cos\psi \\ \dot{\psi} &= r \end{aligned} \tag{7}$$

where x and y are the Cartesian coordinates of its center of mass, i.e., $\mathbf{p} = [x, y]^T$ denotes its position vector, u (surge speed) and v (sway speed) are the body axis components of the velocity of the vehicle, ψ defines its orientation (heading angle), and r its angular velocity. The motions in heave, roll and pitch can be neglected, as the vehicle has large enough meta-centric height. The resulting simplified dynamic equations of motion for surge, sway and yaw are

$$\begin{aligned} m_u \dot{u} - m_v v r + d_u u &= \tau_u \\ m_v \dot{v} - m_u u r + d_v v &= 0 \\ m_r \dot{r} - m_{uv} u v + d_r r &= \tau_r \end{aligned} \tag{8}$$

where τ_u stands for the external force in surge (thruster common mode), τ_r for the external torque (thruster differential mode), where the terms m_u, m_v, m_r, m_{uv} and d_u, d_v, d_r represent the vehicle masses and hydrodynamic added masses, and linear and quadratic hydrodynamic damping effects respectively. The full set of MEDUSA physical parameters can be found in [14]. While the vehicle is restricted to surface operations, the GPS receiver allows the vehicle to compute its own position $\mathbf{p}$ in the inertial frame.

B. Motion control

In order to apply the previous control laws designed for the unicycle kinematics (1) to the MEDUSA vehicle, a new control strategy is proposed. The main idea is to decompose the motion control problem in two tasks: the inner-loop dynamics task and the outer-loop kinematic task. The inner-loop controller enforces the vehicle to track a desired speed. The outer-loop task consists in assigning the reference speed in order to make the vehicle converge to a desired position.

Based on a full state feedback, a single control law for the vehicle thrusters could be designed and better results could be achieved in terms of saturation and smoothness of the control signal. However, dividing the problem results in simpler control laws and the previous tracking control design, presented in Section III-B, can be easily extended to the complex dynamics of the MEDUSA vehicle.

1) Dynamic control: Consider the MEDUSA dynamics defined by (8). The equations of the actuated dynamics can be written in a compact form as follows:

$$M\dot{\mathbf{u}} + C(v)\mathbf{u} + D\mathbf{u} = \boldsymbol{\tau}, \tag{9}$$

where $\mathbf{u} = [u, r]^T$ is the state vector (velocities) and

[1]MEDUSAs are developed at the Laboratory of Robotics and Systems in Engineering and Science (LARSyS), Instituto Superior Técnico of Lisbon.

$$M = \begin{bmatrix} m_u & 0 \\ 0 & m_r \end{bmatrix}, \quad C(v) = \begin{bmatrix} 0 & -m_v v \\ -m_{uv} v & 0 \end{bmatrix},$$
$$D = \begin{bmatrix} d_u & 0 \\ 0 & d_r \end{bmatrix}.$$

The aim is thus to design the control inputs $\boldsymbol{\tau} = [\tau_u, \tau_r]^T$ such that $\mathbf{u} = [u, r]^T$ converges to the desired speed $\mathbf{u}_d = [u_d, r_d]^T$. The error between the vehicle velocity and desired velocity is denoted by $\tilde{\mathbf{u}} = \mathbf{u} - \mathbf{u}_d$. Hence, the dynamics of the error satisfies

$$M\dot{\tilde{\mathbf{u}}} = -C(v)\mathbf{u} - D\tilde{\mathbf{u}} - D\mathbf{u}_d - M\dot{\mathbf{u}}_d + \boldsymbol{\tau}. \tag{10}$$

Thanks to the following proposed control law

$$\boldsymbol{\tau} = -K_d(\mathbf{u} - \mathbf{u}_d) + C(v)\mathbf{u} + D\mathbf{u}_d + M\dot{\mathbf{u}}_d, \tag{11}$$

where K_d is positive definite, the error dynamics becomes

$$\dot{\tilde{\mathbf{u}}} = -M^{-1}[K_d + D]\tilde{\mathbf{u}}$$

and the origin $\tilde{\mathbf{u}} = 0$ is a globally exponentially stable equilibrium point for this system. To prove the convergence of the error system, consider the following Lyapunov function $V_d = \frac{1}{2}\|\tilde{\mathbf{u}}\|^2$. Defining the positive definite matrix $A = M^{-1}[K_d + D]$, the time derivative of the Lyapunov function satisfies $\dot{V}_d = -\tilde{\mathbf{u}}^T A\tilde{\mathbf{u}} \leq 0$.

2) Kinematic control: Considering the simple unicycle kinematics (1), the tracking control (5) enforce convergence of the vehicle $\mathbf{p}$ to the desired time-varying position $\mathbf{p}_f$ with a small error $\boldsymbol{\delta}$ expressed in the body frame. Consider now the kinematic equations of MEDUSA (7). In this situation, the dynamics of the error $\mathbf{e} = R(\psi)^T(\mathbf{p} - \mathbf{p}_f)$ becomes

$$\dot{\mathbf{e}} = \Delta\mathbf{u} + [0, v]^T - R(\psi)^T\dot{\mathbf{p}}_f - S(r)(\mathbf{e} - \boldsymbol{\delta}) \tag{12}$$

and then, the new tracking control law can be designed as

$$\mathbf{u} = \Delta^{-1}\left(R(\psi)^T\dot{\mathbf{p}}_f - [0, v]^T - K(\mathbf{e} - \boldsymbol{\delta})\right). \tag{13}$$

Following the same analysis as previously, using the Lyapunov function $V_{\mathbf{e}} = \frac{1}{2}\|\mathbf{e} - \boldsymbol{\delta}\|^2$, we conclude that the origin $\mathbf{e} = \boldsymbol{\delta}$ is a globally exponentially equilibrium point for system (12). This outer-loop controller provides the reference speed $\mathbf{u}_d$ for the MEDUSA in terms of $[u, r]$, with a view to tack the time-varying trajectory of the virtual follower $\mathbf{p}_f$.

C. Closed-loop controllers

In this subsection, we derive the final controllers that will be implemented in the MEDUSA vehicle and the stability of the closed-loop dynamics is analyzed.

Previous inner and outer-loop controllers have been designed independently. In addition, to compute the control laws (11) and (13), full information about the reference velocity and its derivative is required and the full state is needed for the feedback control. With a view to implement both the inner and outer-loop controllers in a more realistic way, the sway velocity v and the derivative of the reference velocity $\dot{\mathbf{u}}_d$ are removed from the control laws, and they are viewed as input perturbations that limit the performance of the system.

Theorem 2 *Consider the MEDUSA class vehicle described by equations (8) and (7). Let K and K_d be two positive definite matrix and $\dot{\boldsymbol{p}}_f$, $\ddot{\boldsymbol{p}}_f$ and v be bounded inputs. Consider the control law:*

$$\begin{aligned} \boldsymbol{\tau} &= -K_d(\boldsymbol{u} - \boldsymbol{u}_d) + D\boldsymbol{u}_d \\ \boldsymbol{u}_d &= \Delta^{-1}\left(R(\psi)^T\dot{\boldsymbol{p}}_f - K(\boldsymbol{e} - \boldsymbol{\delta})\right) \end{aligned} \tag{14}$$

If the gains K and K_d are sufficiently large, then the system is input-state stable (see [15]) with restrictions on the initial states $\boldsymbol{e}(0)$ and $v(0)$.

Proof: The stability of the closed-loop system can be studied by the Lyapunov function $V = \frac{1}{2}(\|\tilde{\mathbf{u}}\|^2 + \|\mathbf{e} - \boldsymbol{\delta}\|^2)$. Substituting the control law (14) in (10) and (12) leads to

$$\begin{aligned} \dot{\tilde{\mathbf{u}}} &= -M^{-1}[D + K_d]\tilde{\mathbf{u}} - M^{-1}C(v)\mathbf{u} - \dot{\mathbf{u}}_d \\ \dot{\mathbf{e}} &= -K(\mathbf{e} - \boldsymbol{\delta}) + \Delta\tilde{\mathbf{u}} + [0, v]^T - S(r)(\mathbf{e} - \boldsymbol{\delta}) \end{aligned}$$

Differentiating the Lyapunov function V yields

$$\begin{aligned} \dot{V} = &-\tilde{\mathbf{u}}^T M^{-1}[D + K_d + C(v)]\tilde{\mathbf{u}} - \tilde{\mathbf{u}}^T M^{-1}C(v)\mathbf{u}_d \\ &- \tilde{\mathbf{u}}^T\dot{\mathbf{u}}_d + (\mathbf{e} - \boldsymbol{\delta})^T\left(-K(\mathbf{e} - \boldsymbol{\delta}) + \Delta\tilde{\mathbf{u}} + [0, v]^T\right). \end{aligned}$$

The time derivative of the reference velocity $\mathbf{u}_d$ is

$$\begin{aligned} \dot{\mathbf{u}}_d &= \Delta^{-1}\left(-S(r)R(\psi)^T\dot{\mathbf{p}}_f + R(\psi)^T\ddot{\mathbf{p}}_f - K\dot{\mathbf{e}}\right) \\ &= -K\tilde{\mathbf{u}} + \mathbf{a} \end{aligned}$$

where

$$\begin{aligned} \mathbf{a} = &\Delta^{-1}\left(-K\left([0, v]^T - K(\mathbf{e} - \boldsymbol{\delta}) - S(r)(\mathbf{e} - \boldsymbol{\delta})\right)\right) \\ &+ \Delta^{-1}\left(-S(r)R(\psi)^T\dot{\mathbf{p}}_f + R(\psi)^T\ddot{\mathbf{p}}_f\right) \end{aligned}$$

Applying Young's inequality with $\mu > 0$ it holds

$$(\mathbf{e} - \boldsymbol{\delta})^T\Delta\tilde{\mathbf{u}} \leq \frac{1}{\mu}(\mathbf{e} - \boldsymbol{\delta})^T(\mathbf{e} - \boldsymbol{\delta}) + \mu\, \tilde{\mathbf{u}}^T\Delta^2\tilde{\mathbf{u}}$$

and thus the time derivative $\dot{V}$ can be rewritten as

$$\begin{aligned} \dot{V} \leq &-\tilde{\mathbf{u}}^T\left(M^{-1}[D + K_d + C(v)] - K + \mu\Delta^2\right)\tilde{\mathbf{u}} \\ &- \tilde{\mathbf{u}}^T\left(M^{-1}C(v)\mathbf{u}_d + \mathbf{a}\right)\tilde{\mathbf{u}} \\ &- (\mathbf{e} - \boldsymbol{\delta})^T\left(\left(K - \frac{1}{\mu}\right)(\mathbf{e} - \boldsymbol{\delta}) - [0, v]^T\right). \end{aligned}$$

Denote $A = M^{-1}[D + K_d + C(v)] - K + \mu\Delta^2$ and assume that v is small enough for A to be positive definite. Then, $\dot{V} \leq 0$ when

$$\begin{aligned} \|\tilde{\mathbf{u}}\| &\geq \|A^{-1}\|\|M^{-1}C(v)\mathbf{u}_d + \mathbf{a}\| \\ \|\mathbf{e} - \boldsymbol{\delta}\| &\geq \|K^{-1}\|\left(\frac{1}{\mu}\|\mathbf{e} - \boldsymbol{\delta}\| + |v|\right) \end{aligned}$$

and therefore, the system is input-state stable with respect to the inputs $\mathbf{u}_d$ and $\mathbf{a}$. ■

V. Simulation Results

In this section, several simulation results are presented to show the performance of the control strategy proposed in this paper. First, a group of six robots, two leaders and four followers are considered. The control strategy defined in Fig. 1 is applied, such that the followers modeled by (1) are governed by control law (5) and the virtual agents compute the adaptive controller (3) with the desired distances $d_{ij} = 2$, $\forall i, j$. The reference velocity for the leaders is $\mathbf{v}_0 = [1, 0]^T$ and it changes its value at $t = 10$s to $\mathbf{v}_0 = [1, 1]^T$.

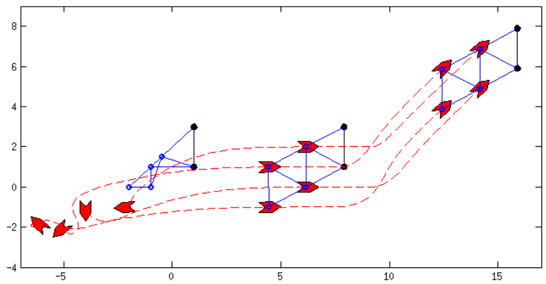

Fig. 2. Simulation of six robots achieving a leader-follower formation. The figure displays three snapshoots. The black dots represent the two leaders following a straigh line. The four red agents follow the trajectories of the virtual followers (blue circles) and the formation is manteined during the motion.

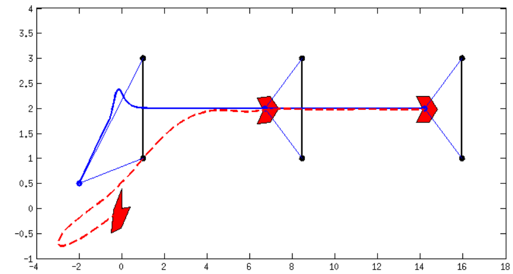

Fig. 3. Leader-follower formation triangular formation with two leaders (blak dots) and one the MEDUSA vehicle as follower (red agent). The figure displays three snapshoots, the initial conditions and two time instants at $t = 15$s and $t = 30$s. The red dashed line represents the trajectory of the MEDUSA traking the virtual follower (blue circle).

Fig. 2 displays the trajectories (red dashed lines) of the four followers tracking the virtual agents (blue circles). The initial positions of robots and virtual agents are arbitrary generated and the control parameters are $\delta = -0.1$ and K equal to the identity matrix. The virtual agents achieve a formation composed of equilateral triangles and move in the plane following the leaders motion. The unicycle robots track their corresponding virtual agents and consequently they keep the formation. After the abrupt change of the reference velocity at $t = 10$s, the robots can be recovered to the formation and move as a whole again with the new velocity.

Second, a simulation with two leaders tracking the velocity $\mathbf{v}_0 = [0.5, 0]^T$ and a MEDUSA class vehicle which motion is described by (8) and (7) is shown in Fig. 3. Again, we apply a decoupled control approach in which a virtual agent governed by the adaptive control law (3) achieves a triangular formation with the two leaders. The tracking controller (14) from Theorem 2 enforces the MEDUSA vehicle to track the trajectory of the virtual agent. The control parameters are $\delta = -0.1$, $K = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ and $K_d = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$.

VI. Conclusions and Future Works

The paper deals with the ALFC problem in which two robots play the roll of leaders and the followers maintain a predetermined rigid formation while moving with a desired reference velocity that is only known for the leaders. Based on [1], we propose a new control strategy composed of two decoupled controllers to solve the ALFC problem considering unicycle dynamics for the followers. Each follower robot tracks a virtual agent which is enforced to keep a triangular formation with its two leading agents. In the second part of the paper the challenging extension to marine vehicles is presented. The full dynamics of an autonomous semisubmerged robotic vehicle of the MEDUSA class is considered and adding an inner-loop controller to the previous ALFC strategy the vehicle is able to keep a triangular formation with its two leaders and adaptively track the reference leaders' velocity.

Currently, we are studying the convergence properties of the proposed control approach using only relative information between the robots. In future research, we aim to extend this work with a view to using only range measurements to compute the multiple vehicle formation control laws. The application of network localization methods based on Euclidean distances matrices will be exploited for this purpose.

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