

SLAM for an AUV using vision and an acoustic beacon^{*}

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Abstract: The estimation of the position and attitude of an autonomous underwater vehicle (AUV) is a challenging and important problem in marine robotics. It is well known that the underwater environment poses considerable problems, that include i) the fact that there is no GPS signal, ii) the communication is usually done through acoustic signals, which suffers from faults, delays and low bandwidth, and iii) the use of vision and/or laser is very limited due to poor visibility. In this paper, we combine a multiple set of sensors to address the full state 6DOF pose estimation of an AUV. The problem is formulated assuming that we have partial measurements from an Inertial Measurement Unit (IMU), an acoustic ranging from a single beacon buoy, and a monocular camera attached to the AUV. Using multiple model estimation techniques and the concept of Extended Kalman Filters with Simultaneous Localization and Mapping (EKF-SLAM), we propose an algorithm that integrates the AUV measurements (that arrive at different sampling-times) and compute in real time an estimate of the position and attitude of the AUV. Simulation results are presented and discussed.

Keywords: Underwater Vision, IMU, Extended Kalman Filter, Multiple Model Adaptive Estimator, Simultaneous Localization And Mapping, Pose Estimation

1. INTRODUCTION

Over the last decade, applications with ocean robotics have increased dramatically. The use of remotely operated vehicles (ROV) and, more recently, autonomous underwater vehicles (AUVs) have shown to be extremely important tools to study and explore the oceans. A key enabling element for the use of such robotic vehicles is the availability of advance navigation and positioning systems. Since electromagnetic signals do not propagate well below the sea surface, a solution for communication and positioning is to use acoustic systems. Examples of positioning systems include Ultra Short BaseLine (USBL), Long BaseLine (LBL), and GPS Intelligent Buoy (GIB) [Thomas (1998); Alcocer et al. (2007)], which all use the concept of beacons and transponders. These systems make use of range measurements taken from the time of flight of the acoustic signals. It is worth to stress that acoustic communication suffers from intermittent failures, latency, and multi-path effects.

A complementary approach for AUV navigation is to apply techniques of terrain based navigation, by exploring the correlation between the range measurements from the vehicle to the sea bottom and an apriori known map of the seabed. See for example [Nygren and Jansson (2004); Teixeira and Pascoal (2005)] where terrain aided AUV navigation problem using particle filters have been addressed.

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Another approach is to resort, when possible, to robotic vision. Notice, however, that vision is conditioned to *i*) visibility, which is limited in the underwater environment, and *ii*) richness of the features, which is usually poor, unless the vehicle is close to rocks, cliffs, or sea bottom, that allow to detect features and extracting them from the surrounding environment.

The problem of using vision to estimate the motion of a robotic vehicle has been addressed by several authors in the last decade [Perez et al. (1999); Mallet et al. (2000); Davison and Murray (2002); Burschka and Hager (2004); Garcia and Solanas (2004); Davison et al. (2004, 2007); Aguiar and Hespanha (2009)].

In the underwater field, we point out the work of Ribas et al. (2006), where the authors have tackled the problem for partially structured underwater environments by using a 360° imaging sonar and a feature based SLAM algorithm. Williams et al. (2000) and Folkesson et al. (2007) have also addressed the problem using sonar to detect and localize the features in the environment. In the work by Augenstein and Rock (2009) particle filters were proposed to simultaneously track the pose and reconstruct a 3-D point cloud model of the target, based on SLAM concepts, to enable AUV/ROV station keeping with respect to target. Williams et al. (2009) combines the SLAM trajectory estimates with the stereo image pairs to generate 3D meshes and place them in a common reference frame. At the University of Michigan, Brown et al. (2008) have developed a testbed for multi-AUV SLAM using stereo vision. In [Caccia (2007)] the author addresses the motion control and estimation of an ROV using a laser-triangulation optical-correlation sensor. Karras and Kyriakopoulos (2007) also propose a laser-based vision system for localization of an ROV.

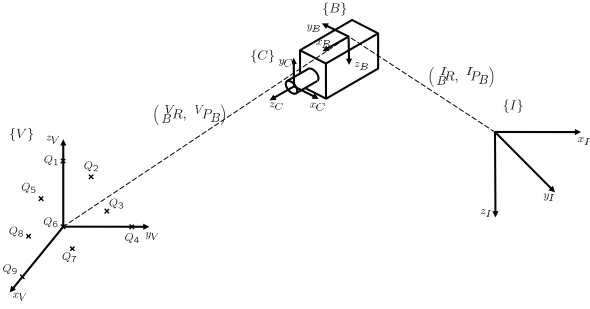


Fig. 1. Schematic diagram of a robotic vehicle equipped with a monocular camera. Body-fixed $\{B\}$, camera-fixed $\{C\}$, earth-fixed (inertial) $\{I\}$, and visual-fixed $\{V\}$ coordinate frames.

In spite of significant progress made in these areas, much work remains to be done to obtain high performance and robust navigation systems for underwater robotic vehicles. In this paper we propose a new and promising algorithm for AUV navigation that make use (when available) of partial measurements from an Inertial Measurement Unit (IMU), an acoustic ranging from a single beacon buoy, and a monocular camera attached to the AUV. The key contribution is the integration of multiple model estimation techniques with the concept of Extended Kalman Filters with Simultaneous Localization and Mapping (EKF-SLAM) to solve the full state 6 Degree Of Freedom (6DOF) pose estimation of an AUV. The algorithm was validated through extensive computer simulations that showed a promising solution to be implemented for real applications.

The paper is organized as follows: Section 2 formulates the pose (position and attitude) estimation problem. Section 3 describes the standard EKF-SLAM. Section 4 describes the proposed multiple model EKF-SLAM algorithm and Section 5 presents simulation results. Concluding remarks are given in Section 6.

2. PROBLEM STATEMENT

This section formulates the position and attitude estimation of an AUV with respect to an inertial coordinate frame. Figure 1 shows a schematic diagram of an AUV equipped with a camera and the coordinates frames needed to formulate the problem. In the figure, $\{B\}$ is the body-fixed coordinate frame whose origin is located e.g., at the center of mass of the vehicle, $\{I\}$ is the earth-fixed inertial coordinate frame, $\{V\}$ denotes another inertial coordinate frame defined by the visual features, and $\{C\}$ is the camera-fixed frame.

In the sequel, the following notation is used. Given two frames $\{A\}$ and $\{B\}$, the symbol ${}^B_R A$ is the rotation matrix from $\{A\}$ to $\{B\}$, ${}^B q$ is the position of the vector q expressed in $\{B\}$, and ${}^B p_A$ is the position of the origin of frame $\{A\}$ expressed in $\{B\}$.

2.1 Kinematic and Dynamic Equations of Motion

Following standard practice, the general 6DOF kinematic and dynamic equations of motion of an AUV can be written in compact form as

Kinematics

$$\begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} = \begin{bmatrix} J_1(\eta_2) & 0_{3 \times 3} \\ 0_{3 \times 3} & J_2(\eta_2) \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \Leftrightarrow \dot{\eta} = J(\eta_2)\nu \quad (1)$$

Dynamics

$$M_{RB}\dot{\nu} + C_{RB}(\nu)\nu = \tau_{RB} \quad (2)$$

where $J_1(\eta_2) = {}^I_B R(\eta_2)$ is the rotation matrix from $\{B\}$ to $\{I\}$ parameterized by vector $\eta_2 = [\phi, \theta, \psi]^T$ of roll, pitch, and yaw angles, and $J_2(\eta_2)$ is the matrix that relates body-fixed angular velocities $\nu_2 = [p, q, r]^T$ with roll, pitch, and yaw rates. The symbols M_{RB} , $C_{RB}(\nu)$ denote the rigid body inertia matrix and the matrix of Coriolis and Centrifugal terms, respectively. The vector $\nu = [\nu_1^T, \nu_2^T]^T = [u, v, w, p, q, r]^T$ consists of the body-fixed linear and angular velocity vectors, and $\eta = [\eta_1^T, \eta_2^T]^T = [x, y, z, \phi, \theta, \psi]^T$ the earth-fixed position and attitude. The vector $\tau_{RB} = [X, Y, Z, K, M, N]^T$ denotes external forces and moments which can be decomposed as

$$\tau_{RB} = \tau + \tau_A + \tau_D + \tau_R + \tau_d \quad (3)$$

where τ_R denotes the term due to buoyancy and gravity, and τ_A is the added mass term. The term τ_D captures the damping and lift effects, and τ represents the forces and moments generated by the thrusters. The symbol τ_d represents input disturbances. The reader is referred to [Fossen (1998)] for detailed explanation of the above equations and further details.

The sampled-data representation of equation (1)-(3), attained by holding the input constant over $[kT, kT+T)$, where T is the sampling time, can be written in the form

$$x_{k+1} = f_k(x_k, u_k, w_k) \quad (4a)$$

$$y_k = h_k(x_k, u_k, v_k) \quad (4b)$$

where $x_k = \text{col}(\eta(kT), \nu(kT))$ is the state of the system, $u_k = \tau(kT)$ its input, and $w_k = \tau_d(kT)$ the process noise. The output equation (4b) is described in the following section.

2.2 Sensor Measurements

We consider that the AUV is equipped with the following set of sensors:

- An Inertial Measurement Unit (IMU), that only provides measurements of angular velocities $[p, q, r]^T$ and attitude $[\phi, \theta, \psi]^T$ with respect to earth-fixed (inertial) $\{I\}$ coordinate frame.
- A pressure sensor, which gives the depth z .
- An acoustic modem to obtain range measurements from the position of the vehicle to a single fixed buoy with known position.
- A monocular Charged-Coupled-Device (CCD) camera.

Given the heterogeneous sensors described above, it is not realistic to consider that they all provide measurements at the same (and constant) sampling rate. To tackle this issue we first assume that the sampling time T is sufficiently fast so that if there exist measurements provided by the sensors, they occur at some time $t = kT$. The output signal is given by

$$y_k = \text{col}(y_k^1, y_k^2, y_k^3, \dots) \quad (5)$$

where, if at time $t = kT$ the measurement j is not available, we set

$$y_k^j = 0, \text{ and } h_k^j(x_k, u_k, v_k^j) = 0 \quad (6)$$

Otherwise, we have the following:

IMU:

$$y_k^1 = h_k^1(x_k, u_k, v_k^1) = \text{col}(\phi_k, \theta_k, \psi_k, p_k, q_k, r_k) + v_k^1 \quad (7)$$

where v_k^1 denotes measurement noise.

Depth sensor:

$$y_k^2 = h_k^2(x_k, u_k, v_k^2) = z_k + v_k^2 \quad (8)$$

Acoustic beacon: From the measurements given by the IMU and the depth sensor it is clear that the entire position of the AUV cannot be estimated. To tackle this issue, we resort to range measurements provided by an acoustic modem. Let q_b be a priori known fixed position of a buoy with respect to $\{\mathcal{I}\}$. If there is a ranging measurement available at time $t = kT$

$$y_k^3 = h_k^3(x_k, u_k, v_k^3) = \|\eta_1(kT) - q_b\|_2 + v_k^3 \quad (9)$$

CCD camera: The CCD camera attached to the vehicle together with a computer vision algorithm provide measurements of the detected features from the surrounding environment. For each *new* feature i detected, we add an output y^{3+i} that can be written as [Ma et al. (2003)]

$$y_k^{3+i} = \frac{1}{\mu_k^i} F^C q_k^i + v_k^{3+i}, \quad (10)$$

with the constraint

$$[0, 0, 1] y_k^{3+i} = 1 \quad (11)$$

Here, ${}^C q_k^i$ is the unknown position of the detected feature q^i expressed in the camera's frame $\{\mathcal{C}\}$ at time $t = kT$, μ_k^i is the unknown image depth of the point q^i in the camera's frame $\{\mathcal{C}\}$, and F is a matrix transformation that depends on camera's intrinsic parameters such as the focal length, the scaling factors, and the centers offsets.

In what follows, we write (10) in the earth-fixed frame. For simplicity we consider that the vision-fixed $\{\mathcal{V}\}$ and earth-fixed $\{\mathcal{I}\}$ coordinate frames are coincident. Let ${}^{\mathcal{I}}q$, ${}^{\mathcal{B}}q$, and ${}^C q$ be the coordinates of a point q in the frames $\{\mathcal{I}\}$, $\{\mathcal{B}\}$ and $\{\mathcal{C}\}$, respectively. Then, the following holds:

$${}^C q_k^i = {}^C P_B + {}^C R_k {}^{\mathcal{B}} q_k^i, \quad {}^{\mathcal{I}} q^i = {}^{\mathcal{I}} P_{\mathcal{B},k} + {}^{\mathcal{I}} R_k {}^{\mathcal{B}} q_k^i \quad (12)$$

Combining equations (10)-(12), we finally obtain

$$y_k^{3+i} = \frac{1}{\mu_k^i} [F^C P_B + F^C R_k {}^{\mathcal{I}} R_k' ({}^{\mathcal{I}} q^i - {}^{\mathcal{I}} P_{\mathcal{B},k})] + v_k^{3+i} \quad (13)$$

$$[0, 0, 1] y_k^{3+i} = 1 \quad (14)$$

The next sections address the problem of computing an estimate of state x for dynamical system described by (4).

3. EKF-SLAM

The past few years have witnessed an increased research effort on the problem of Simultaneous Localization And Mapping (SLAM). In the literature we can find two main forms of SLAM: online SLAM and full SLAM. Online SLAM involves computing the posterior estimation using the current pose combined with the map. On the other hand, full SLAM estimates a posterior over the entire path combined with map. [Thrun et al. (2005)].

In EKF formulation, it is considered that the size of the state x is fixed. This is not the case in the EKF-SLAM where the state estimate \hat{x} (and covariance matrix P) are continuously augmented as new features are discovered. Roughly speaking the EKF-SLAM algorithm can be decomposed in three steps:

- (1) Predict the state estimate using the process model and the input signal u .
- (2) update the current state estimate using the measurements including the re-observed features.
- (3) add new features to the current state.

It is important to stress that to implement the EKF-SLAM the following two tasks must be carefully addressed: *i*) the feature detection task and *ii*) the data association task, that is responsible to classify if the detected feature is a new one or an old one, and which one. These two tasks are out of the scope of this paper.

The methodology adopted to augment the state \hat{x} for the specific problem addressed in the paper is now briefly described. At the initial time, the state \hat{x} is given by $\hat{x}_0 = \text{col}(\hat{\eta}_0, \hat{\nu}_0)$, where $\hat{\eta}_0$, and $\hat{\nu}_0$ are the initial guesses for the attitude, and linear and angular velocities respectively. As soon as a new feature $q_i \in \mathcal{R}^3$ at time $t = kT$ is detected, the state \hat{x} is augmented with an estimate of the location of the feature ${}^{\mathcal{I}} \hat{q}_i$. To compute the initial estimate of ${}^{\mathcal{I}} \hat{q}_i$ we resort to the measurement provided by the vision system y_k^{3+i} and equation (13). Notice however that in this step we have four unknown variables (location of feature ${}^{\mathcal{I}} q_i$ and depth of image μ_i) and only three independent equations. To solve this issue we set $\hat{\mu}_i = \hat{\mu}_{i0}$, where $\hat{\mu}_{i0}$ is an initial estimate of the depth (distance to the plane) of the feature q_i in the image. As it will be shown in Section 5, this initial value plays a key role on the convergence of the EKF-SLAM algorithm.

The covariance matrix P has also to be augmented to

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{21} \end{bmatrix} \quad (15)$$

where $P_{11} = P_k$, $P_{12} = P_{21}^T = \Pi_1 J_v^T$, and $P_{22} = J_v \Pi_2 J_v^T + R_f$. In the above expressions Π_1 is a mapping which selects the columns of P_k corresponding to η , Π_2 is a mapping which selects columns and rows of P_k corresponding to ν . J_v represents the Jacobian of vision measurement (13) with respect to AUV states (η, ν) , and R_f is the covariance of measurement noise on the vision measurements.

The algorithm used for EKF-SLAM can be described as follows:

- Initialize AUV states x and covariance matrix P .
- Predict the AUV states using previous estimated states, given inputs and the AUV model, presented in (4).
- Propagate states covariance matrix according to EKF covariance matrix prediction.
- If measurements, or re-observed features are available.
 - Update the EKF using the measurements available and re-observed features.
- If feature i has been observed for the first time.
 - Extract an estimate of ${}^{\mathcal{I}} q_i$ from y_{3+i} and an initial estimate of μ_i using (13), (14).
 - Augment the state vector x with the estimate of ${}^{\mathcal{I}} q_i$ into the states vector together with AUV states.
 - Initialize the elements of covariance matrix (15).
- Go to prediction step.

Note that in SLAM, re-observing a feature does not just improve the estimate of its location, but also improves the estimate of other features locations and the AUV state.

This fact may not hold when the feature detection and data association do not work correctly. Further readings about the EKF-SLAM can be found in [Thrun et al. (2005)].

4. MULTIPLE MODEL EKF-SLAM

In this section we propose a combined approach of multiple model and EKF-SLAM. The motivation for this algorithm arises from the fact that the EKF-SLAM convergence is very sensitive to the initial guess of μ_i . To minimize this problem we propose the following: From a range measurement sensor (or a priori estimate of the image depth) generate as many multiple models as needed (depending on the uncertainty of image depth) to cover the range of uncertainty of μ_i . Then, apply a multiple model adaptive estimation scheme. The models and their corresponding weights are updated by arrival of new measurements according to (16)-(18) [Bar-Shalom et al. (2002)].

$$\beta_k^j = \left((2\pi)^{m/2} \sqrt{\det S_k^j} \right)^{-1} \quad (16)$$

$$w_k^j = r_k^{jT} S_k^{j-1} r_k^j \quad (17)$$

$$p_k^j = p_{k-1}^j \beta_k^j e^{-\frac{1}{2} w_k^j} \left(\sum_{l=1}^N p_{k-1}^l \beta_k^l e^{-\frac{1}{2} w_k^l} \right)^{-1} \quad (18)$$

where r_k^j , S_k^j , and p_k^j represent residual, innovation matrix, and weight of the j^{th} model at time $t = kT$ respectively.

In this approach we have one main model which would estimate AUV states together with all features locations. For each new observed feature we would create a set of N models to extract a good estimative of \mathcal{I}_{q_i} . Whenever these N models corresponding to feature i have converged, the best estimate of \mathcal{I}_{q_i} will be augmented into the main model. All the multiple models corresponding to initialized feature is then deleted. The derived estimate will be used as an initial feature location in the EKF-SLAM of the main model. Whenever all the features have been initialized, we would have only the main model containing all the features estimated locations with AUV states and the update procedure would follow the same as the standard EKF-SLAM.

The pseudo code corresponding to the proposed multiple model EKF-SLAM algorithm is as follows:

- Initialize the main model with initial AUV states and uncertainty covariance matrix.
- Predict AUV states, given inputs and AUV model.
- Propagate states covariance matrix according to EKF covariance matrix prediction.
- If measurements, or re-observed initialized features measurements are available:
 - Update the EKF using the measurements available and re-observed initialized features measurements.
- If new feature i has been observed:
 - Generate a set of probable μ_i with N elements as, $\mathcal{S}_i = \{\mu_{i,1}, \dots, \mu_{i,k}, \dots, \mu_{i,N}\}$, where each $\mu_{i,k}$ represents an initial guess for k^{th} model.
 - Extract an estimative of \mathcal{I}_{q_i} using y_{2+i} and $\mu_{i,k}$ for k^{th} model as $\mathcal{I}_{q_{i,k}}$, using (13), (14).
 - Replicate a copy of main model into N created models.

Table 1. Simulation parameters

T : sample time	25ms
Vision sample time	50ms
Depth sample time	50ms
IMU sample time	25ms
Ranging beacon sample time	1s
AUV initial pose	$[0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$
estimator initial pose	$[1 \ 1 \ 1 \ 0 \ 0 \ 0]^T$
AUV initial velocity	$[0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$
estimator initial velocity	$[0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$
Process noise	1% of input τ
IMU noise covariance	$diag([0.0524[1, 1, 1], 0.01[1, 1, 1]].^2)$
Depth noise covariance	0.1^2
Vision noise covariance	$diag([0.1, 0.1, 0.1].^2)$
Ranging noise covariance	0.1^2
q_b : Beacon position	$[10 \ 10 \ 0]^T$
N : Number of models	11

- Initialize models weights p_k^j with $\frac{1}{N}$.
- Augment estimative of $\mathcal{I}_{q_{i,k}}$ into corresponding k^{th} model.
- Initialize new elements of k^{th} model covariance matrix in i^{th} set of models, using (15).
- If feature i corresponding to \mathcal{S}_i has been re-observed.
 - Update each model of the corresponding set using the measurements available and re-observed feature i measurements.
 - Update the weights corresponding to the models using (16), (17), (18).
 - Check if the states of the set have converged to states of an specific model of the set. If so, use the converged estimative of \mathcal{I}_{q_i} to augment it into the main model and delete all the converged models.
- Go to prediction step.

5. SIMULATION RESULTS

To illustrate the performance of the SLAM algorithm described in the paper, we consider the problem of computing the position and attitude of an AUV that is required to move according to the following mission: The AUV is required to move up and down along the Z axis and also to rotate along the same Z axis so that it can get features from different directions. From time to time it will also move forward and backward along the X axis, which will make it closer and farther from the features that are located stationary on the wall in front of the AUV.

The numerical values used for the physical parameters of the AUV match the ones of the Sirene underwater shuttle described in Aguiar and Pascoal (1997). The features are randomly distributed in front of AUV (see Fig 2). Table 1 shows the simulation parameters used in SLAM algorithm.

Figures 2-4 illustrates the performance of the multiple model EKF-SLAM. Detected features are located on a wall in front of AUV shown by (*). As it can be seen, the estimated features locations (o) are close to the true ones. Figures 3,4 show that the AUV position and attitude estimation have converged to a finite small bound with zero mean in all 3 axes, even though we have started from an initial position farther from the actual initial position of the AUV. In order to compare the results with standard EKF-SLAM, we use the same map of features and the same initial condition as before. Fig. 5 illustrates the results. Since the initial estimate for μ_i is not close to the true ones, the estimated features locations have drifted and

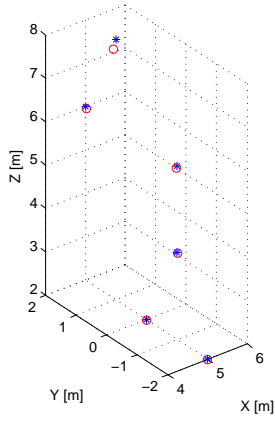


Fig. 2. True (*) and estimated (o) features locations using the multiple model EKF-SLAM.

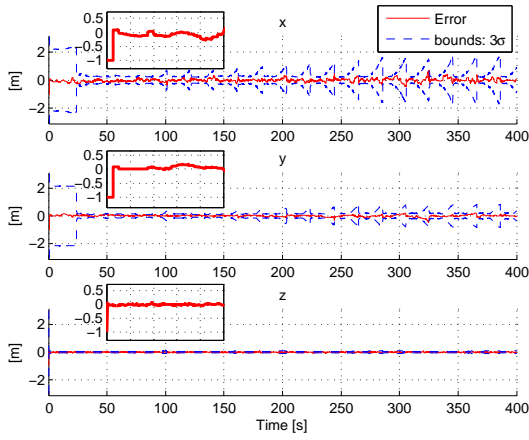


Fig. 3. Time evolution of the position estimation error and the 3σ uncertainty bound for the scenario corresponding to Fig. 2. The first 20 seconds are zoomed in.

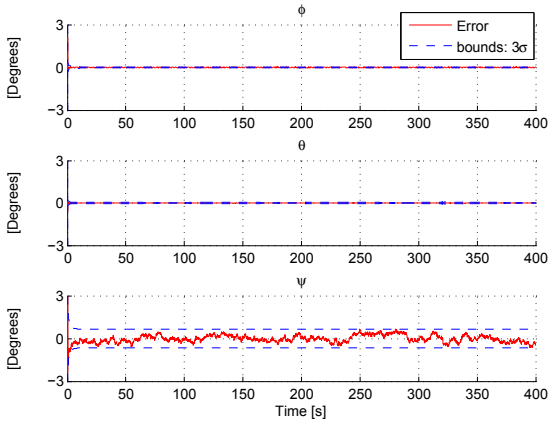


Fig. 4. Time evolution of the attitude estimation error and the 3σ uncertainty bound for the scenario corresponding to Fig. 2.

converged to values with significant error. Also we could notice that, the estimated position of x and y have been drifted along time.

Figures 6-8 show the simulation results where there is no range measurements. As it is expected, in this case the results are not so good as the ones in Fig. 2, but still

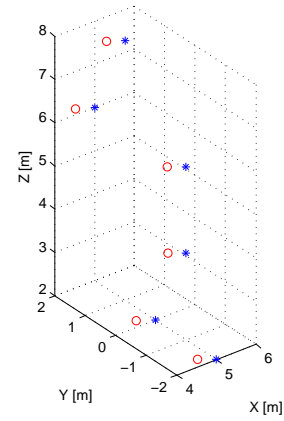


Fig. 5. True (*) and estimated (o) features locations using standard EKF-SLAM.

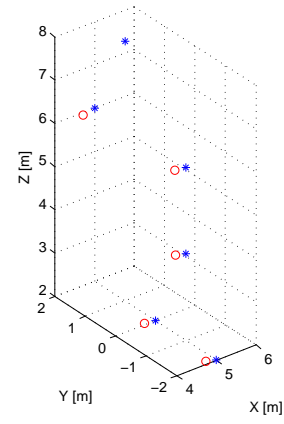


Fig. 6. True (*) and estimated (o) features locations using multiple model EKF-SLAM and without benefiting from ranging measurement.

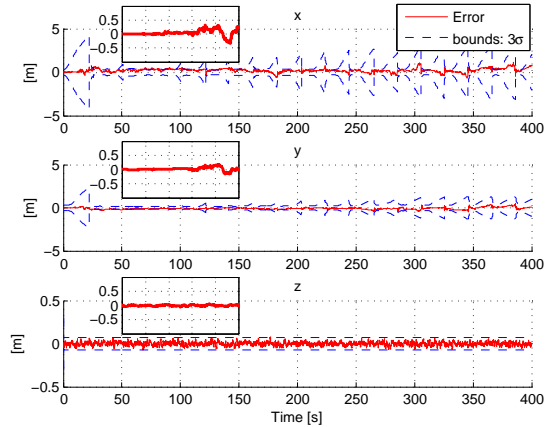


Fig. 7. Time evolution of position estimation error and 3σ uncertainty bound corresponding to Fig. 6. The first 20 seconds are zoomed in.

acceptable. Notice also that, they are better than the EKF-SLAM with range measurements.

6. CONCLUSIONS

In this paper we have proposed a new SLAM algorithm for an AUV that integrates the measurements provided by an IMU, a depth sensor, a buoy range, and a monocular camera attached to the AUV. The key contribution was

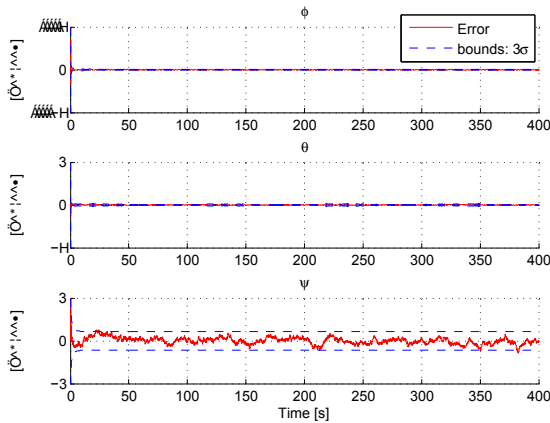


Fig. 8. Time evolution of attitude estimation error and 3σ uncertainty bound corresponding to Fig. 6.

the use of multiple model adaptive estimation tools to extend the standard EKF-SLAM. The simulation results illustrated the efficiency of this new approach. We could also notice that the computation power needed to use the multiple model EKF-SLAM is in the same order of the standard EKF-SLAM after the convergence of the multiple models. During the transient, the computation effort increases and has a direct relation with the number of models multiplied by the landmarks that have not converged yet. Future work will include experimental tests.

REFERENCES

- Aguiar, A. and Pascoal, A. (1997). Modeling and control of an autonomous underwater shuttle for the transport of benthic laboratories. In *Proc. of MTS/IEEE Conference, OCEANS '97*, volume 2, 888–895.
- Aguiar, A.P. and Hespanha, J.P. (2009). Robust filtering for deterministic systems with implicit outputs. *Systems & Control Letters*, 58(4), 263 – 270.
- Alcocer, A., Oliveira, P., and Pascoal, A. (2007). Study and implementation of an EKF GIB-based underwater positioning system. *Control Engineering Practice*, 15(6), 689 – 701.
- Augenstein, S. and Rock, S. (2009). AUV/ROV Pose and Shape Estimation of Tethered Targets without Fiducials. In *Proc. of UUST'09*.
- Bar-Shalom, Y., Kirubarajan, T., and Li, X.R. (2002). *Estimation with Applications to Tracking and Navigation*. John Wiley & Sons, Inc., New York, NY, USA.
- Brown, H.C., Kim, A., and Eustice, R.M. (2008). Development of a multi-AUV SLAM testbed at the University of Michigan. In *Proc. of IEEE/MTS OCEANS Conference and Exhibition*, 1–6. Quebec, Canada.
- Burschka, D. and Hager, G. (2004). V-GPS (SLAM): Vision-based inertial system for mobile robots. In *Proc. of ICRA*, 409–415.
- Caccia, M. (2007). Vision-based ROV horizontal motion control: Near-seafloor experimental results. *Control Engineering Practice*, 15(6), 703 – 714. Special Section on Control Applications in Marine Systems - CAMS2004, Control Applications in Marine Systems.
- Davison, A., Cid, Y., and Kita, N. (2004). Real-time 3D SLAM with wide-angle vision. In *Proc. of IFAC Symposium on Intelligent Autonomous Vehicles, Lisbon*.
- Davison, A. and Murray, D. (2002). Simultaneous Localization and Map-Building Using Active Vision. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 24(7), 865–880.
- Davison, A., Reid, I., Molton, N., and Stasse, O. (2007). MonoSLAM: Real-time single camera SLAM. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 29(6), 1052.
- Folkesson, J., Leonard, J., Leederkerken, J., and Williams, R. (2007). Feature tracking for underwater navigation using sonar. In *Proc. of IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS*, 3678–3684.
- Fossen, T.I. (1998). *Guidance and control of ocean vehicles*. John Wiley & Sons, Inc., New York, NY, USA.
- Garcia, M. and Solanas, A. (2004). 3D simultaneous localization and modeling from stereo vision. In *Proc. of IEEE International Conference on Robotics & Automation*, volume 853.
- Karras, G. and Kyriakopoulos, K. (2007). Localization of an underwater vehicle using an IMU and a laser-based vision system. In *Proc. of MED '07. Mediterranean Conference on Control & Automation*, 1–6.
- Ma, Y., Soatto, S., Kosecka, J., and Sastry, S.S. (2003). *An Invitation to 3-D Vision: From Images to Geometric Models*. SpringerVerlag.
- Mallet, A., Lacroix, S., and Gallo, L. (2000). Position estimation in outdoor environments using pixel tracking and stereovision. In *Proc. of IEEE International Conference on Robotics and Automation*, volume 4, 3519–3524.
- Nygren, I. and Jansson, M. (2004). Terrain navigation for underwater vehicles using the correlator method. *IEEE Journal of Oceanic Engineering*, 29(3), 906–915.
- Perez, J., Castellanos, J., Montiel, J., Neira, J., and Tardos, J. (1999). Continuous mobile robot localization: vision vs. laser. In *Proc. of IEEE International Conference on Robotics and Automation*, volume 4, 2917–2923 vol.4.
- Ribas, D., Ridao, P., Neira, J., and Tardos, J. (2006). SLAM using an imaging sonar for partially structured underwater environments. In *Proc. of IEEE/RSJ International Conference on Intelligent Robots and Systems*, 5040–5045.
- Teixeira, F. and Pascoal, A. (2005). AUV terrain aided navigation using Particle Filters. In *Proc. of International Workshop on Underwater Robotics, Genoa, Italy*, 207–216.
- Thomas, H. (1998). Gib buoys: An interface between space and depths of the oceans. In *Proc. of IEEE Autonomous Underwater Vehicles, Cambridge, MA, USA*, 181–184.
- Thrun, S., Burgard, W., and Fox, D. (2005). *Probabilistic Robotics (Intelligent Robotics and Autonomous Agents)*. The MIT Press.
- Williams, S., Newman, P., Dissanayake, G., and Durrant-Whyte, H. (2000). Autonomous underwater simultaneous localisation and map building. In *Proc. of IEEE International Conference on Robotics and Automation, ICRA '00*, volume 2, 1793–1798 vol.2.
- Williams, S., Pizarro, O., Jakuba, M., and Barrett, N. (2009). AUV benthic habitat mapping in south eastern tasmania. In *Proc. of Field and Service Robotics*.