A Switched Based Control Strategy for Target Tracking of Autonomous Robotic Vehicles using Range-only Measurements *

Omid Namaki-Shoushtari,* A. Pedro Aguiar,** Ali Khaki Sedigh*

* Faculty of Electrical and Computer Engineering, K.N. Toosi
University of Technology, Tehran, Iran.

(e-mail: onamakis@dena.kntu.ac.ir, sedigh@kntu.ac.ir)

** Institute for Systems and Robotics (ISR), Instituto Superior Técnico
(IST), Lisbon, Portugal (e-mail: pedro@isr.ist.utl.pt)

Abstract: In this paper we address the pursuing or target tracking problem where an autonomous robotic vehicle is required to move towards a maneuvering target using range-only measurements. A new switched based control strategy is proposed to solve the pursuing problem that can be described as comprising a continuous cycle of two distinct phases: i) the determination of the bearing, and ii) following the direction computed in the previous step while the range is decreasing. We provide conditions under which the switched closed-loop system achieves convergence of the relative distance error to a small neighborhood around zero. Simulation results are presented and discussed.

Keywords: Target tracking, Range-only measurements, Switching control, Autonomous robotic vehicles

1. INTRODUCTION

The problem of tracking a moving target, which can be another robotic vehicle, by an autonomous robotic vehicle has received special attention in the literature. Particular examples can be found in the area of wheeled mobile robots (e.g., [D'Andrea-Novel et al. 1995, Dixon et al. 2001, Lawton et al. 2003]), aircraft vehicles (e.g., Hauser and Hindman [1997], Kaminer et al. [1998], Al-Hiddabi and McClamroch [2002]), and marine vehicles (Godhavn [1996], Encarnação and Pascoal [2001], Behal et al. [2002], Do et al. [2002], Aguiar and Hespanha [2003], Pettersen and Nijmeijer [2003], Aguiar and Hespanha [2007]).

In spite of the wide range of applications and the large number of control strategies developed, most of them rely on the assumption that both the bearing (line-of-sight angle) and the range (relative distance between the vehicle and the target) are known for navigation, guidance and control purposes.

In this paper, we are interested in the problem when the only information available about the target is the range. This type of problems appears in several application domains, e.g., wireless networks, surveillance, marine applications, localization (Arora et al. [2004], Crepaldi et al. [2006], Dil et al. [2006], Gadre and Stilwell [2004]). In our particular case, the practical motivation arises from applications to autonomous underwater vehicles (AUVs), where the range is obtained by measuring the time-of-flight of an acoustic pulse. An interesting and attractive scenario

is the case when a surface craft is performing a maneuver along a predefined path, while an AUV in a configuration master/slave is required to follow the surface craft. Note that the surface craft can use GPS for localization, but the AUV cannot because electromagnetic waves do not propagate well underwater. The idea to localize the AUV is that both vehicles carry an acoustic modem, and from these the AUV is able to compute the range and send/receive commands.

The target tracking problem using range-only measurements has been recently addressed in [Matveev et al. 2009], where the authors propose a sliding mode control law to steer a Dubins-like wheeled robot towards a target that moves with a constant speed while preserving a predefined range margin from the target. The proposed strategy ultimately makes the robot to move around the target along a circular trajectory. The radius of circle or the preserved margin from the target is predefined with a control parameter. Equiangular Navigation Guidance algorithms for approaching and following both steady and moving targets with range-only measurements are proposed in [Teimoori and Savkin 2010. With constant robot linear velocity, the robot-target range variation is used as a measure for the angle at which the robot approaches the target. The proposed guidance methods have the property that the trajectory of the controlled robot is close to a certain curve called an equiangular spiral. A different strategy is described in [Zhang et al. 2007, Cochran and Krstic 2009, Ghods and Krstic 2010] where the problem of seeking the source of a scalar signal (stopped target) using a nonholonomic vehicle with no position information, was solved using an extremum seeking approach.

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This paper addresses the pursuing or target tracking problem in 2D, where an autonomous robotic vehicle is required to move towards a steady or maneuvering target. The robotic vehicle does not have the capability of sensing its position or the position of the target. The only available information about the target is the range (relative distance) between the pursuer and the target. We propose a switched based control strategy that can be described as comprising a continuous cycle of two distinct phases: i) the determination of the bearing, and ii) following the direction computed in the previous step while the range is decreasing. In the first phase, inspired by the extremum seeking approach, the control strategy is to keep the forward velocity constant and actuate on the angular velocity so that the vehicle will move in a "persistence of excitation" mode to obtain the bearing of the target. The bearing is obtained by averaging the measurements of vehicle heading. In the second phase, the main idea is to actuate on the linear and angular velocities so that the vehicle will follow the direction of the bearing computed in the first phase and converge to a neighborhood of the target. A supervisor switching control law coordinates which phase, and when, of these two modes are enabled. The stability of the overall closedloop switched system is analyzed and conditions for the convergence of the relative distance error are described. To illustrate the effectiveness and performance of the proposed control scheme, we present simulation results where the target is required to move in a straight line and also to perform an arc maneuver.

This paper is organized as follows: Section 2 formulates the target tracking problem and describes the vehicle model. Section 3 presents the switched control algorithm and in Section 4, the stability of the overall closed-loop system is analyzed. Simulation results and conclusions are included in Sections 5 and 6, respectively. Due to space limitations, some of the proofs are omitted. These can be found in Namaki-Shoushtari and Aguiar [2011].

2. PROBLEM FORMULATION

Consider an autonomous robotic vehicle moving in horizontal plane (see Figure 1) and let $(x,y,\theta) \in SE(2)$ denote the configuration of an inertial coordinate frame $\{U\}$ with respect to a body-fixed frame $\{B\}$ that satisfies

$$\dot{x} = u_1 \cos \theta, \tag{1a}$$

$$\dot{y} = u_1 \sin \theta, \tag{1b}$$

$$\dot{\theta} = u_2.$$
 (1c)

where $(x, y)^T$ is assumed to be the position of the center of mass of the vehicle, θ is its orientation, and u_1 and u_2 are the body-fixed linear and angular velocities, respectively.

Consider also a second vehicle (the target vehicle) with the following associated equations of motion

$$\dot{x}_t = V_t \cos \theta_t, \tag{2a}$$

$$\dot{y}_t = V_t \sin \theta_t, \tag{2b}$$

$$\theta_t = \omega_t,$$
 (2c)

where the position $(x_t, y_t)^T$, the orientation θ_t , and the velocities V_t and ω_t are unknown to the first vehicle.

Suppose that the first vehicle is equipped with a set of sensors that provide a measurement of the angle θ and

the distance (range) from the sensor position located at R away from the center of mass

$$\begin{pmatrix} x_s \\ y_s \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + R \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \tag{3}$$

to the position of the target vehicle, that is, $r := \sqrt{(x_t - x_s)^2 + (y_t - y_s)^2}$. See Figure 1.

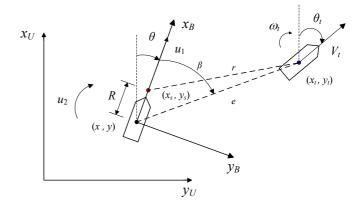


Fig. 1. Setup for the pursuing control problem.

The control problem considered in this paper can be formulated as follows:

Using as only measurements the orientation θ and the range r, derive a feedback law for $u = (u_1, u_2)$ such that for any initial condition in a predefined set $J \subset SE(2)$ the vehicle's center of mass (x, y) converges to a predefined ball $B_{\epsilon}(x_t, y_t)$ with center (x_t, y_t) and radius $\epsilon > 0$.

3. SWITCHED CONTROL DESIGN

This section presents a control strategy to solve the challenging pursuing problem stated above. The proposed solution consists of a continuous cycle of two distinct phases: *Phase 1:* The determination of the bearing, so that the vehicle will be able to point to the target. Note that in this stage, the vehicle will have to move in a "persistence of excitation" mode.

Phase 2: Follow the direction computed in the previous step while the (estimated) rate of change of the range r is greater than some acceptable value. Switch to step (1) when this does not hold.

3.1 Phase 1: The determination of the bearing

In this stage the goal is to compute, using as measurements the range r and the heading θ , the (correct) bearing so that the vehicle will be able to point to the target. To accomplish this task, we propose a feedback law that acts on the angular velocity u_2 while keeping the forward velocity u_1 constant. Inspired by the Extremum Seeking (ES) approach (Cochran and Krstic [2009]) we consider the control law

$$u_1 = V_c, (4a)$$

$$u_2 = k_1 \omega \cos(\omega t) + k_2 \xi \sin(\omega t), \tag{4b}$$

where ξ satisfies the dynamics,

$$\dot{\chi} = -\lambda \left(\chi + r^2\right),\tag{5a}$$

$$\xi = -(\chi + r^2),\tag{5b}$$

and $k_1,\ k_2$, $\lambda,\ V_c$ and ω are parameters to be tuned. To compute the bearing at time t=T, we propose to average $\theta(t)$, that is,

$$\bar{\theta} = \frac{1}{T} \int_0^T \theta(s) \, ds. \tag{6}$$

Note that in (4b) we are forcing the angular velocity of the vehicle to have an oscillation of frequency ω . This excitation signal will consequently lead to an oscillation on the vehicle's heading. In Section 4 we show that after a few oscillations, the average heading can be used as a desired orientation to steer the vehicle towards the target.

3.2 Phase 2: Following a constant direction

In this phase the feedback law is given by

$$u_1 = k_3 r \tag{7a}$$

$$u_2 = -k_4(\theta - \bar{\theta}). \tag{7b}$$

where $\bar{\theta}$ is the desired direction computed at the end of phase 1. Notice that, since $\bar{\theta}$ is constant in this stage the vehicle is following "blindly" the target. It may be possible that during this stage the target may change its direction so that $\bar{\theta}$ may not be a good measure of the bearing. To detect this situation, we estimate the rate of change of r and compare with a given threshold that depends on the linear velocity u_1 . To estimate \dot{r} , we propose a simple Kalman filter designed according to the linear model

$$\dot{r} = V_r + w_1, \tag{8a}$$

$$\dot{V}_r = w_2, \tag{8b}$$

$$y = r + v \tag{8c}$$

where (r, V_r) is the state, y is the measured output, and w_1 , w_2 and v are assumed to be mutually independent stationary, Gaussian, zero mean white noise processes.

Remark 1: For simplicity, the feedback laws are derived without taking into account the problem of input saturation. However, it can be shown that the saturated feedback laws

$$u_1 = k_3^a \tanh(k_3^b r)$$

$$u_2 = -k_4^a \tanh(k_4^b (\theta - \bar{\theta}))$$

instead of (7) would also work by selecting the proper gains.

3.3 Switched Controller

Let $\sigma:[t_0,\infty)\to\{1,2\}$ be a piecewise constant switching signal that is continuous from the right and evolves according to

$$\sigma(t) = \begin{cases} 1, & t \in [t_{i-1}, t_i), i \text{ odd }, i \in \mathbb{N} \\ 2, & t \in [t_{i-1}, t_i), i \text{ even, } i \in \mathbb{N} \end{cases}$$
(9)

In (9), $\{t_0, t_1, t_2, t_3, ...\}$ is a sequence of strictly increasing infinite switching times in $[t_0, \infty)$ and $t_0 = 0$ is the initial time. The switching controller is given by

$$u = \alpha_{\sigma}(\theta, r),$$

where $\alpha_1(\cdot)$ corresponds to the control law (4)-(5) and $\alpha_2(\cdot)$ to the control law (7). In (9), when i is odd ($\sigma = 1$) we set

$$t_i = t_{i-1} + nT$$
, $T = \frac{2\pi}{\omega}$.

for some given $n \in \mathbb{N}$. For i even $(\sigma = 2)$ we set t_i as the time t such that

$$t_i = \max \left\{ t_{i-1} + \Delta, \min_{t} \left\{ t \ge t_{i-1} : \hat{V}_r(t) \ge -\delta \right\} \right\}$$
 (10)

where $\delta > 0$ is a given threshold, \hat{V}_r is the estimate of \dot{r} using the Kalman filter described in Section 3.2, and $\Delta > 0$ is a dwell time to enforce that the second controller will be enabled at least Δ seconds. It is important to stress that the only input signals of the overall control law (as it can be seen from (4-5) and (7)) are only the range r and the heading of the vehicle θ . The controller does not need the rate of r neither the bearing angle.

4. STABILITY ANALYSIS

In this section, we analyze the stability of the closed-loop system. To this effect, we first introduce the following variables. Let e (see Figure 1) denote the position error between (x, y) and (x_t, y_t) , that is,

$$e = \sqrt{(x - x_t)^2 + (y - y_t)^2}$$

and β the angle between the vector x_B and the vector defined by (x, y) and (x_t, y_t) . From Figure 1, it follows that

$$x - x_t = -e \cos(\theta + \beta)$$
$$y - y_t = -e \sin(\theta + \beta)$$

where $\theta + \beta = \tan^{-1}\left(\frac{-(y-y_t)}{-(x-x_t)}\right)$. From the above equations and computing the time derivative of e and β it can be shown that e and β satisfy the dynamics

$$\dot{e} = -u_1 \cos \beta + V_t \cos(\beta + \theta - \theta_t) \tag{11}$$

$$\dot{\beta} = \frac{u_1}{e} \sin \beta - u_2 - \frac{V_t}{e} \sin(\beta + \theta - \theta_t) \tag{12}$$

We now provide conditions for the convergence of $\bar{\theta}$ defined in (6) to the correct bearing $\bar{\theta}^*$, which satisfies (see Fig. 1)

$$\bar{\theta}^* = \theta + \beta$$
.

Theorem 1. Let Σ_1 denote the closed-loop system that results from the feedback interconnection of (1) with (4). Let $e^* > 0$ and $\epsilon > 0$ be given allowable tolerant position and orientation errors, respectively, and $D_{[0,t_f]}$ the largest set in \mathbb{R}^3 such that for every initial condition $(x,y,\theta)(0) \in D_{[0,t_f]}$, the solution of Σ_1 is well defined for all $t \in [0,t_f]$ with $e(t) \geq e^*$. Let $k_2 > 0$ be a sufficiently large gain such that the following holds

$$V_c \le \frac{J_1(k_1)}{J_0(k_1)} k_2 R e^* - \gamma \tag{13}$$

with $V_c > 0$ and $\gamma > V_t$. In (13), $J_0(k_1)$ and $J_1(k_1)$ denote the Bessel integral equalities (Abramowitz and Stegun [1964])

$$J_0(k_1) = \frac{1}{2\pi} \int_0^{2\pi} e^{jk_1 \sin(t)} dt$$
$$J_1(k_1) = \frac{-j}{2\pi} \int_0^{2\pi} e^{jk_1 \sin(t)} \sin(t) dt$$

and $k_1 > 0$ is selected such that $J_0(k_1)$ and $J_1(k_1)$ are positive.

Then, there exist a sufficiently large $\omega>0$ and a positive natural number n such that for every initial condition in $D_{[0,t_f]}$ with $t_f=nT=n\frac{2\pi}{\omega}$,

$$|\bar{\theta}(t_f) - \bar{\theta}^*(t_f)| \le \epsilon. \tag{14}$$

Now we examine the stability of the closed loop system when the second stage (following a constant direction) is enable.

Theorem 2. Let Σ_2 denote the closed-loop system that results from the interconnection of (1), (3) with the control law (7). Let $e^* > R$ be a given allowable tolerant position error, and $D_{[0,t_f]}$ the largest set in \mathbb{R}^3 such that for every initial condition $(x,y,\theta)(0) \in D$, the solution of Σ_2 is well defined for all $t \in [0,t_f]$ with $e(t) \geq e^*$ and the error between the real bearing $\bar{\theta}^*$ and $\bar{\theta}$ defined as $\tilde{\bar{\theta}} = \bar{\theta}^* - \bar{\theta}$ remains bounded by some $\epsilon > 0$, that is, $\sup_{0 \leq t \leq t_f} |\tilde{\bar{\theta}}| \leq \epsilon$. Let k_4 be a positive gain such that the following holds:

$$2\frac{k_3}{k_4} + \epsilon + \frac{V_t}{e^* k_4} < \frac{\pi}{2}.\tag{15}$$

Then, for sufficiently large gain k_3 the position error e converges to a neighborhood around zero with ultimate bound $e \leq e^*$.

Proof. The proof is organized as follows. First, it will be shown that β is ultimately bounded with the ultimate bound less than $\frac{\pi}{2}$. Using this fact, we then prove the convergence of e. Consider the dynamics of β (see (12)), which in closed-loop satisfies

$$\dot{\beta} = \frac{k_3 r}{e} \sin \beta - k_4 (\theta - \bar{\theta}) - \frac{V_t}{e} \sin(\beta + \theta - \theta_t)$$
 (16)

From Figure 1 it follows that

$$r^2 = e^2 + R^2 - 2Re\cos\beta.$$

which implies that r can be written in the form

$$r = e + f(e, R, \beta), \tag{17}$$

where $f(e, R, \beta)$ is a bounded function with $|f| \leq R$. Now, consider the positive definite function $V_1(\beta) := \frac{1}{2}\beta^2$. Computing its time derivative along the trajectories of (16), using (17) and the fact that $\theta - \bar{\theta} = -(\beta + \tilde{\bar{\theta}})$, one obtains

$$\dot{V}_1 \le -k_4(1-\alpha)\beta^2, \quad \forall \, |\beta| \ge \frac{2k_3 + k_4 \,\epsilon + V_t/e^*}{k_4 \alpha}$$

where α is any scalar that satisfies $0 \le \alpha < 1$. It is clear that if the gain k_4 is selected as in (15), β will be ultimately bounded with the ultimate bound less than $\frac{\pi}{2}$.

We are now ready to prove the convergence of e. From (11), with $u_1 = k_3 r$ (7a), and using (17) the dynamics of e satisfies

$$\dot{e} = -k_3 (e+f) \cos \beta + V_t \cos(\beta + \theta - \theta_t).$$

Let $V_2(e) := \frac{1}{2}e^2$. Its time-derivative satisfies

$$\dot{V}_2 \le -k_3(1-\alpha)e^2\cos\beta, \forall |e| \ge \frac{V_t}{k_3\alpha\cos\beta} + \frac{R}{\alpha\cos\beta}$$
 (18)

for $0 \le \alpha < 1$. From (18) we can conclude that for any finite time $t \ge 0$, e(t) is bounded (that is, there is no finite escape) and furthermore, after a finite time t_1 with $|\beta(t_1)| < \frac{\pi}{2}$, e(t) will converge to a neighborhood around

zero of size less than e^* , provided that k_3 is sufficiently large and $e^* > R$.

We can now conclude that according to the switching rule described in Section 3.3 and resorting to Theorems 1 and 2, if the switching between these two phases is slowed down by a sufficiently large dwell time τ_D such that $\min(nT, \Delta) \geq \tau_D$, based on the results in (Liberzon [2003] and Hespanha and Morse [1999]) the switched closed-loop system is bounded (that is all the states are bounded) and there exists an $\epsilon > 0$ such that while $e(t) > \epsilon$ the vehicle will converge towards the target and will remain afterwards around the target. This behavior will be illustrated in the next section.

5. SIMULATION RESULTS

This section illustrates the performance of the proposed control scheme through computer simulations. In the following simulations, the initial configurations of the pursuer and the target vehicles are respectively $(x,y,\theta)(0)=(0,0,0)$ and $(x_t,y_t,\theta_t)(0)=(10m,10m,0)$ (for the 1st simulation) and $(x_t,y_t,\theta_t)(0)=(20m,20m,0)$ (for the 2nd simulation). The target linear velocity was set to $V_t=0.2m/s$. The control parameters were selected as follows: $\omega=0.5rad/sec$, R=0.5m, $k_1=1$, $k_2=1$, $k_3=0.2$, $k_4=10$, $\lambda=1$, $e^\star=1m$, and $V_c=0.2m/s$.

5.1 The target moves along a straight line

Figures 2- 5 show the simulation results when the target vehicle moves along a straight line.

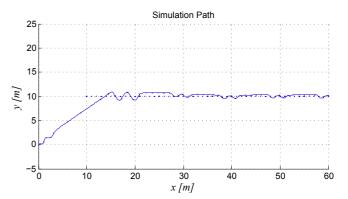


Fig. 2. Simulation paths for the pursuer and the target vehicles. The target moves on a straight line.

Figure 2 displays the resulting trajectories of the vehicles where it can be seen that the pursuer moves towards the target. Notice also that the pursuer vehicle is constantly switching between the two modes. This behavior is clearly seen in Fig. 3 that shows the "steady-state" trajectories of the last 20 seconds of the simulation. It is important to point out that the pursuer cannot perform for all time a straight-line motion because in this mode the system is not observable.

5.2 The target performs a lawn mowing maneuver

Figures 6-8 show the results of another simulation where the vehicle is required to track a target vehicle that is performing a lawn mowing maneuver composed by straight lines and arcs.

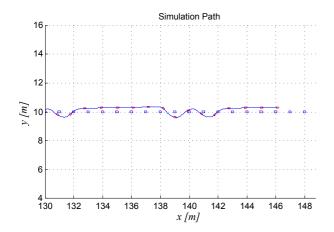


Fig. 3. Zoom in of the last 20 seconds of the pursu target vehicles maneuvers shown in Fig. 2 (steady-state trajectories).

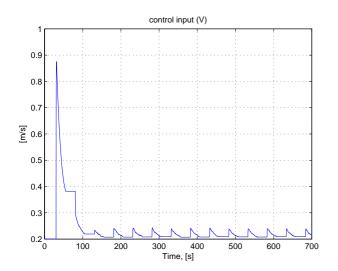


Fig. 4. Time evolution of the pursuer vehicle forward velocity.

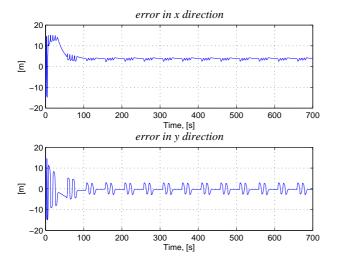


Fig. 5. Time evolution of the components of the error vector $e = (e_x, e_y)'$ expressed in the body fixed frame. The target vehicle moves on a straight line.

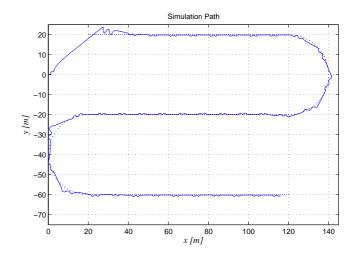


Fig. 6. Simulation paths for the pursuer and the target vehicles. The target is performing a lawn mowing maneuver.

From the figures, it can be seen that the vehicle is capable to converge to the target and follow it. However, as it is expected, the distance error increases when the target is on the arc phase.

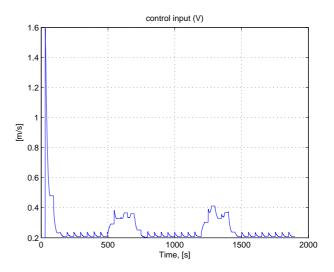


Fig. 7. Time evolution of the pursuer vehicle forward velocity.

6. CONCLUSION

In this paper, we addressed the target tracking problem where an autonomous robotic vehicle is required to move towards a maneuvering target using range-only measurements. We proposed a switched based control strategy to solve the pursuing problem that unfolds in two distinct phases: i) the determination of the bearing, and ii) following the direction computed in the previous step, while the range is decreasing. We provided conditions under which the switched closed-loop system achieves convergence of the relative distance error to a small neighborhood around zero. The simulation results showed the good performance of the proposed solution.

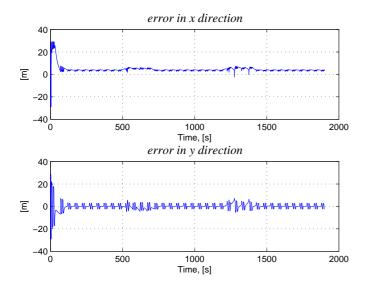


Fig. 8. Time evolution of the components of the error vector $e = (e_x, e_y)'$ expressed in the body fixed frame. The target is performing a lawn mowing maneuver.

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