Temporally and Spatially Deconflicted Path Planning for Multiple Autonomous Marine Vehicles ⋆

Andreas J. Häusler * **Reza Ghabcheloo** **
António M. Pascoal * **A. Pedro Aguiar** * **Isaac I. Kaminer** ***
Vladimir N. Dobrokhodov ***

* *Institute for Systems and Robotics, Dept. of Electrical Engineering, Instituto Superior Técnico, Av. Rovisco Pais No. 1, 1049-001 Lisboa, Portugal. (e-mail: {ahaeusler,antonio,pedro}@isr.ist.utl.pt)*
** *Department of Intelligent Hydraulics and Automation, Tampere Univ. of Technology, P.O. Box 589, 33101 Tampere, Finland. (e-mail: reza.ghabcheloo@tut.fi)*
*** *Department of Mechanical and Astronautical Engineering, Naval Postgraduate School, Monterey, CA 93943, USA. (e-mail: {kaminer,vldobr}@nps.navy.edu)*

Abstract:
There is currently a surge of interest in the development of advanced systems for cooperative control of multiple autonomous marine vehicles. Central to the implementation of these systems is the availability of efficient algorithms for multiple vehicle path planning that can take explicitly into account the capabilities of each vehicle and existing environmental conditions. Multiple vehicle path planning methods build necessarily on key concepts and algorithms for single vehicle path following. However, they go one step further in that they must explicitly address inter-vehicle collision avoidance, together with a number of criteria that may include simultaneous times of arrival at the assigned target points and energy minimization, to name but a few. As such, they pose considerable challenges both from a theoretical and practical implementation standpoint. This paper is a short overview of multiple vehicle path planning techniques. The exposition is focused on specific algorithms developed in the scope of research work in which the authors have participated. Namely, algorithms that ensure that at no time will two vehicles get closer in space than a desired safety distance, that is, achieve trajectory *deconfliction*. The algorithms make ample use of *direct optimization methods* that lead to efficient and fast techniques for path generation using a polynomial-based approach. The paper affords the reader a fast paced presentation of key algorithms that had their genesis in the aircraft field, discusses the results of simulations, and suggests problems that warrant further consideration.

Keywords: Multiple Vehicle Missions, Path Planning, Spatial Deconfliction, Temporal Deconfliction, Autonomous Marine Vehicles.

1. INTRODUCTION

Space, land, and marine robots are becoming ubiquitous and hold promise to the development of networked systems to sample the environment at an unprecedented scale. This trend is clearly visible in the marine world, which harbors formidable challenges imposed by the extent of the areas to be surveyed, sea waves, currents, low visibility at depth, lack of global positioning systems underwater, and stringent acoustic communication constraints. Some of these difficulties can be partially overcome through the use of fleets of heterogeneous vehicles working in cooperation, under the supervision of advanced systems for cooperative control of multiple autonomous vehicles. Central to the implementation of these systems is the availability of efficient algorithms for multiple vehicle path planning that can take explicitly into account the capabilities of each vehicle and existing environmental conditions.

As an application example, consider the scenario where multiple autonomous marine vehicles (that have been launched from one or more support ships and are scattered in the ocean) are required to execute a cooperative mission underwater, adopting a desired geometrical formation pattern. To this effect, and while still at the surface, the vehicles must maneuver from their initial positions and reach formation at approximately the same speed, in a prescribed neighborhood of the diving site. Only then can the underwater mission segment start. Because the vehicles may be operating in a restricted area and in the vicinity of support ships, this initial Go-To-Formation maneuver must be executed in such a way as to avoid collisions. Furthermore, the vehicles must arrive at their target positions at approximately the same time. This scenario is depicted in Fig. 1, which shows the evolution of

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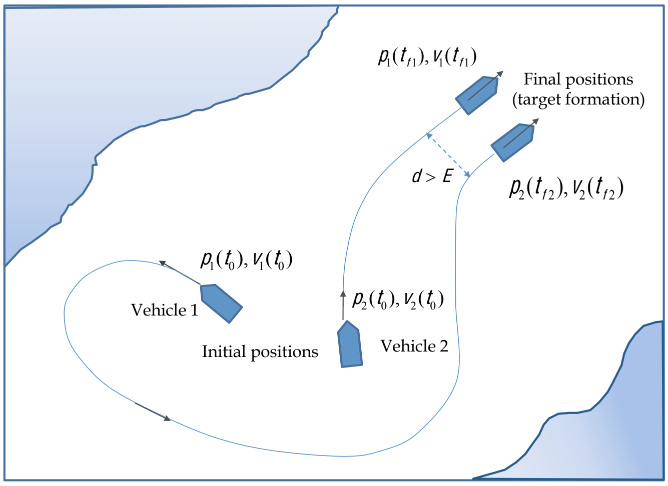


Fig. 1. Multiple Vehicle Path Planning: Go-To-Formation Maneuver with Spatial Deconfliction.

two vehicles that start from arbitrary positions and reach a simple side-by-side formation pattern prior to diving.

The example above can be further detailed to show how multiple vehicle path planning yields an optimization problem subject to a number of critical constraints. For example, in the case of an energy-related cost criterion the function to be minimized may be an weighted sum of the energies spent during a Go-To-Formation maneuver. However, other criteria may be envisioned such as average maneuvering time. Vehicle related constraints are the total energy available for vehicle maneuvering and vehicle dynamic restrictions such as maximum vehicle accelerations. Environmental constraints include external disturbances caused by ocean currents and sea waves. It is also required that collisions be avoided among vehicles as well as between vehicles and stationary and moving obstacles (e.g. support ships, the coastline, and harbor structures). In particular, it is crucial that path planning algorithms yield feasible paths and that any two vehicles never come to close vicinity of each other. This property is often referred to as *deconfliction* in the area of multiple air vehicle control, for it ensures that at no time will two vehicles get closer in space than a desired safety distance E, see Figure 1.

Stated in such generality, path planning is obviously a problem with far reaching implications not only in robotics but also in control theory, computer science, artificial intelligence, and other related engineering subjects (LaValle [2006]). Figure 2 illustrates the problem at hand and shows how a cost criterion, initial and final vehicle conditions, and internal and external constraints are used to produce (if it exists) a trajectory that meets the constraints and minimizes the cost. The spatial and temporal coordinates of this trajectory yield a spatial path and a corresponding vehicle profile. This simple observation is at the root of the methodologies for path planning that are briefly summarized in the paper.

In practice, deconfliction can be spatial or temporal. In the first category, shown in Fig. 1 for the case of two vehicles, non-intersecting spatial paths are generated without explicit temporal constraints. In the second case, temporally deconflicted paths will give rise to nominal trajectories (defined in space and time) for the vehicles to track. Clearly, temporal deconfliction introduces an extra degree of freedom (time) that is not available in the case of spatial deconfliction. As such, it leads to solutions whereby paths are allowed to come to close vicinity or intersect in space, but the temporal scheduling of the vehicles involved separates these occurrences well in time. In summary, temporal deconfliction allows for the solution of a larger class of problems than those that can be tackled with spatial deconfliction algorithms.

Motivated by the above considerations, this paper addresses the problem of deconflicted path planning with applications to multiple autonomous marine vehicles. For simplicity of exposition, the main focus is on vehicles moving in 2D space. The problem formulation and the solutions proposed have been strongly influenced by several mission scenarios studied in the scope of the two EU research projects described in $FREE_{\mathrm{sub}}NET$ [2006—2010] and GREX [2006—2009]. The key objective is to *obtain path planning methods that are effective, computationally easy to implement, and lend themselves to real-time applications.*

The techniques that are the focus of this survey paper build upon and extend the work first reported for unmanned air vehicles in Yakimenko [2000] and later in Kaminer et al. [2006] and Kaminer et al. [2007]. See also Ghabcheloo et al. [2009b] for recent work on the subject. Explained in intuitive terms, the key idea exploited is to separate spatial and temporal specifications, effectively decoupling the process of spatial path computation from that of computing the desired speed profiles for the vehicles along those paths. The first step yields the vehicles' spatial profiles and takes into consideration geometrical constraints; the second addresses time related requirements that include, among others, initial and final speeds, deconfliction in time, and simultaneous times of arrival. Decoupling the spatial and temporal constraints can be done by parameterizing each path as a set of polynomials in terms of a generic variable τ and introducing a polynomial function $\eta(\tau)$that specifies the rate of evolution of τ with time, that is, $d\tau/dt = \eta(\tau)$, see Kaminer et al. [2007]. By restricting the polynomials to be of low degree, the number of parameters used during the computation of the optimal paths is kept to a minimum, a fact that stands at the root of the success of the direct method for rapid prototyping of near-optimal aircraft trajectories proposed in Kaminer et al. [2006]. Once the order of the polynomial parameterizations has been decided, it becomes possible to solve the multiple vehicle optimization problem of interest (e.g., simultaneous time of arrival under specified deconfliction and energy expenditure constraints) by resorting to any proven direct search method Kolda et al. [2003].

The paper is organized as follows. Section 2 offers a general description of the methodology adopted for deconflicted path generation and details its application to the generation of the Go-To-Formation manoeuver with an energy cost criterion and a simultaneous time of arrival constraint. Section 3 contains simulation examples that illustrate the efficacy of the methods developed. Finally, Section 4 overviews the main results obtained and summarizes theoretical and practical issues that warrant further research. *Due to space limitations, some important details are necessarily omitted. The reader is referred to Häusler et al. [2009] for a thorough treatment of the topic.*

2. MULTIPLE VEHICLE PATH PLANNING WITH SPATIAL AND TEMPORAL DECONFLICTION

This section describes two algorithms for multiple vehicle path planning with spatial and temporal deconfliction. In

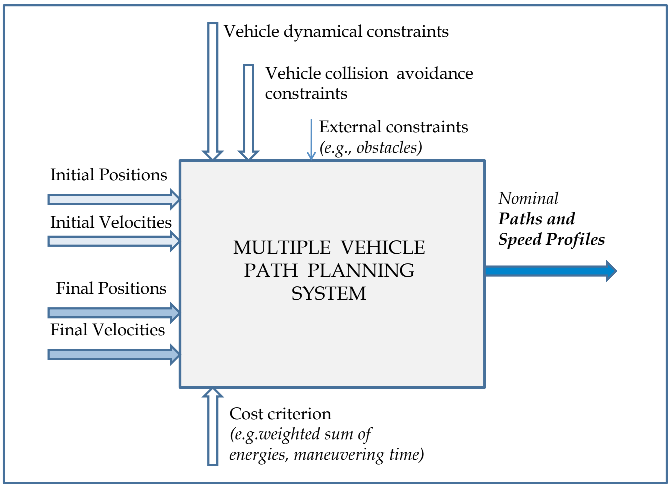


Fig. 2. Path planning system.

what follows, we let $\mathcal{V} := \{V_i; i = 1, .., n\}$ denote the set of $n \geq 2$ vehicles V_i involved in a maneuver. We start by recalling the difference between a path and a trajectory. A path is simply a curve $p : \tau \rightarrow \mathbb{R}^3$ parameterized by τ in a closed subset $[0, \tau_{f_i}], \tau_{f_i} > 0$ of $\mathbb{R}_+$. If τ is identified with time t or a function thereof then, with a slight abuse of notation, $p : t \rightarrow \mathbb{R}^3$ with $t \in [0; t_f], t_f > 0$ will be called a trajectory. *Path following* refers to the problem of making a vehicle converge to and following a path $p(\tau)$ with no explicit temporal schedule. However, the vehicle speed may be assigned as a function of parameter τ. *Trajectory tracking* is the problem of making the vehicle track a trajectory $p(t)$, that is, the vehicle must satisfy spatial and temporal schedules simultaneously. For the sake of clarity, and whenever one wishes to refer to a specific vehicle V_i, the variables of interest will be written with subscript index i. For example, $p_i(\tau); \tau \in [0, \tau_{f_i}]$ and $p_i(t); \tau \in [0, t_{f_i}]$ refer to a path and a trajectory for vehicle V_i, respectively.

Suppose the objective is to execute a multi-vehicle Go-To-Formation maneuver while avoiding inter-vehicle collisions, meeting dynamical constraints (e.g. bounds on maximum accelerations), and minimizing a weighted combination of vehicle energy expenditures. Further suppose that the vehicles are required to arrive at their final destination at the same final time t_f, that is, $t_{f_i} = t_f; i = 1, 2, .., n$. At first inspection, a possible solution to this problem would be to solve a constrained optimization problem that would yield (if at all possible) feasible trajectories $p_i(t), t \in [t_0, t_f]; i = 1, 2, ..., n$ for the vehicles, with t_0 and t_f denoting initial and final time, respectively. Trajectory tracking systems on-board the vehicles would then ensure precise tracking of the trajectories generated, thus meeting the mission objectives.

This seemingly straightforward solution suffers from a major drawback: it does not allow for any "deviations from the plan". Absolute timing becomes crucial because the strategy described does not lend itself to on-line modification in the event that one or more of the vehicles cannot execute trajectory tracking accurately (e.g. due to adverse currents or lack of sufficient propulsion power). For this reason, it is far more practical to adopt a different solution where absolute time is not crucial and enough room is given to each vehicle to adjust its motion along the path in response to the motions of the other vehicles. The goal is that of reaching a terminal formation pattern and ensure simultaneous times of arrival. Dispensing with absolute time is key to the solution proposed. In this set-up, the optimization process should be viewed as a method to produce paths $p_i(\tau_i)$ without explicit time constraints, but with timing laws for $\tau_i(t)$ that effectively dictate how the nominal speed of each vehicle should evolve along the path. Using this set-up, spatial and temporal constraints are essentially decoupled and captured in the descriptions of $p_i(\tau_i)$ and $\eta_i(\tau) = d\tau_i/dt$, respectively, as will be seen later. Furthermore, adopting polynomial approximations for $p_i(\tau_i)$ and $\eta_i(\tau) = d\tau_i/dt$ keeps the number of optimization parameters small and makes real-time computational requirements easy to achieve. Intuitively, by making the path of a generic vehicle V_i a polynomial function of $\tau_i \in [0, \tau f_i]$, the shape of the path in space can be changed by increasing or decreasing τ_i - a single optimization parameter. This, coupled with a polynomial approximation for $\eta_i(\tau_i) = d\tau_i/dt$ makes it easy to shape the speed and acceleration profile of the vehicle along the path so as to meet desired dynamical constraints. The paths thus generated can then be used as "templates" for path following.

2.1 Path Planning: the single vehicle case

The approach adopted for path generation exploits a separation between spatial and temporal specifications. Let $p(\tau) = [x(\tau), y(\tau), z(\tau)]^\top$ denote the path of a single vehicle, parameterized by $\tau = [0; \tau_f]$. For computational efficiency, assume each coordinate $x(\tau), y(\tau), z(\tau)$ is represented by an algebraic polynomial of degree N. For example, $x(\tau)$ is of the form

$$x(\tau) = \sum_{k=0}^{N} a_{xk}\tau^k. \qquad i = 1, 2, 3, \tag{1}$$

The degree N of polynomials $x(\tau), y(\tau), z(\tau)$ is determined by the number of boundary conditions that must be satisfied. Notice that these conditions (that involve spatial derivatives) are computed with respect to the parameter τ. There is an obvious need to relate them to actual temporal derivatives, but this issue will only be addressed later. For the time being, let d_0 and d_f be the highest-order of the spatial derivatives of $x(\tau), y(\tau), z(\tau)$ that must meet specified boundary constraints at the initial and final points of the path, respectively. Then, the minimum degree N^* of each of the corresponding polynomials is $N^* = d_0 + d_f + 1$. For example, if the desired path includes constraints on initial and final positions, velocities, and accelerations (second-order derivatives), then the degree of each polynomial is $N^* = 2 + 2 + 1 = 5$. Explicit formulae for computing boundary conditions $p'0), p''(0)$ and $p'(\tau_f), p''(\tau_f)$ are given later. Additional degrees of freedom may be included by making $N > N^*$. As an illustrative example, consider the case where (1) is polynomial trajectory of 5$^{\text{th}}$ degree. In this case, the coefficients $a_{x,k}; k = 0, .., 5$ can be computed from

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 1 & \tau_f & \tau_f^2 & \tau_f^3 & \tau_f^4 & \tau_f^5 \\ 0 & 1 & 2\tau_f & 3\tau_f^2 & 4\tau_f^3 & 5\tau_f^4 \\ 0 & 0 & 2 & 6\tau_f & 12\tau_f^2 & 20\tau_f^3 \end{pmatrix} \cdot \begin{pmatrix} a_{x,0} \\ a_{x,1} \\ a_{x,2} \\ a_{x,3} \\ a_{x,4} \\ a_{x,5} \end{pmatrix} = \begin{pmatrix} x(0) \\ x'(0) \\ x''(0) \\ x(\tau_f) \\ x'(\tau_f) \\ x''(\tau_f) \end{pmatrix}$$

where τ_f is the terminal value of τ. Similar equations can be used to compute the coefficients $a_{y,k}$ and $a_{z,k}; k = 0, .., 5$. For 6$^{\text{th}}$ degree polynomial trajectories, an additional constraint on the fictitious initial jerk can be included, which increases the order of the resulting polynomial and affords extra (design) parameters $x'''(0), x'''(0), z'''(0)$.

It is now important to clarify how temporal constraints may be included in the feasible path computation process. A trivial solution would be to make $\tau = t$. In this case, however, little control exists over the speed profile along a path $x(t), y(t), z(t)$ that meets the required boundary conditions. In fact, once the path has been computed the speed v is inevitably given by

$$v(t) = \sqrt{\dot{x}^2(t) + \dot{y}^2(t) + \dot{z}^2(t)}, \qquad (2)$$

and not much freedom is available to meet temporal constraints unless one resorts to high order polynomials. We therefore turn our attention to a different procedure that will afford us the possibility of meeting strict boundary conditions and other constraints without increasing the complexity of the path generation process. To this effect, let $v_{\min}, v_{\max}$ and $a_{\max}$ denote predefined bounds on the vehicle's speed and acceleration, respectively. Let $\eta(\tau) = d\tau/dt$, yet to be determined, dictate how parameter τ evolves in time. A path $p(\tau)$ (with an underlying assignment $\eta(\tau)$) is said to constitute a *feasible* path if the resulting trajectory can be tracked by a vehicle without exceeding prespecified bounds on its velocity and total acceleration along that trajectory. With an obvious abuse of notation, we will later refer to a spatial path only, without the associated $\eta(\tau)$, as a feasible path.

From (2), and for a given choice of $\eta(\tau)$, the temporal speed $v(\tau(t))$ and acceleration $a(\tau(t))$ of a vehicle along the path are given by

$$\begin{aligned} &v(\eta) = \eta(\tau)\sqrt{x'^2(\tau) + y'^2(\tau) + z'^2(\tau)} = \eta(\tau)||p'(\tau)|| \\ &a(\tau) = ||p''(\tau)\eta^2(\tau) + p'(\tau)\eta'(\tau)\eta(\tau)|| \end{aligned} \qquad (3)$$

At this point, a choice for $\eta(\tau)$ must be made. A particular choice is simply $\eta(\tau) = \eta(0) + \frac{\tau}{\tau_f}(\eta(f) - \eta(0))$ with $\eta(0) = v(0)$ and $\eta(\tau_f) = v(t_f)$, where t_f is the terminal time yet to be determined. This polynomial is of degree sufficiently high to satisfy boundary conditions on speed and acceleration because the boundary conditions $p'(0), p'', p'(\tau_f), p''(\tau_f)$ can be easily obtained from given $\dot{p}(0), \ddot{p}(0), \dot{p}(t_f), \ddot{p}(t_f)$ using the definition of $\eta(\tau)$. In fact, since $\dot{p}(t) = p'(\tau)\eta(\tau)$, it is easy to see that

$$p'(0) = \frac{\dot{p}(0)}{\eta(0)}, \qquad p''(0) = \frac{\ddot{p}(0) - p'(0)\eta'(0)\eta(0)}{\eta^2(0)},$$
$$p'(\tau_f) = \frac{\dot{p}(t_f)}{\eta(\tau_f)}, \quad p''(\tau_f) = \frac{\ddot{p}(t_f) - p'(t_f)\eta'(\tau_f)\eta(\tau_f)}{\eta^2(\tau_f)},$$

where $\eta'(0) = \eta'(\tau_f) = \frac{\eta(\tau_f)-\eta(0)}{\tau_f}$. Furthermore, the choice of boundary conditions on $\eta(\tau)$ guarantees that $||p'(0)|| = ||p'(t_f)|| = 1$. It now follows from (3) that a path $p(\tau)$ is feasible if all boundary conditions are met, together with the additional speed and acceleration constraints

$$\begin{aligned} &v_{\min} \le \eta(\tau)\ ||p'(\tau)|| \le v_{\max}, \\ &||p''(\tau)\eta^2(\tau) + p'(\tau)\eta'(\tau)\eta(\tau)|| \le a_{\max},\ \forall \tau \in [0, \tau_f]. \end{aligned} \qquad (4)$$

A feasible trajectory can be obtained by solving, for example, the optimization problem

$$F1 : \min_{\Xi} J \quad \textit{subject to } (4)$$

and to the boundary conditions at initial and final points, where Ξ is the vector of optimization parameters that may include, for example, τ_f together with $x'''(0), y'''(0), z'''(0)$. The latter definition of Ξ corresponds to the case where the degree of the polynomial path selected is 6 . The cost function J may be defined to be the total energy consumption of the vehicle, given by

$$J = \int_0^{t_f} c_f c_D \rho v_c^3(t) dt \;=\; \int_0^{\tau_f} c_f c_D \rho \eta^3(\tau)\ ||p'(\tau)||^3\ d\tau,$$

where ρ is dynamic pressure, c_f is a propulsion efficiency factor, and c_D is the total drag coefficient of the vehicle. Other choices of J can be made to address time optimal or minimum length paths.

2.2 Path Planning: the multiple vehicle case

The above methodology is now extended to deal with multiple vehicles. In particular, we address the problem of time-coordinated control where all vehicles must arrive at their respective final destinations at the same time. The dimension of the corresponding optimization problem increases and the time coordination requirement introduces additional constraints on parameters $\tau_{fi}; i = 1, 2, .., n$. To achieve simultaneous times of arrival we adopt the simplified functions

$$\eta_i(\tau_i) = \eta_i(0) + \frac{\tau_i}{\tau_{fi}}(\eta_i(\tau_{fi}) - \eta_i(0)).$$

Integrating $\dot{\tau}_i = \eta_i(\tau_i)$ yields

$$\tau_{f_i} = \tau_i(t_f) = \begin{cases} \eta_i(0)t_f, & \eta_i(\tau_{fi}) = \eta_i(0) \\ \dfrac{\eta_i(\tau_{fi}) - \eta_i(0)}{\ln(\eta_i(\tau_{fi})/\eta_i(0))} t_f & \eta_i(\tau_{fi}) \ne \eta_i(0) \end{cases} \qquad (5)$$

and

$$\frac{t}{t_f} = \begin{cases} \dfrac{\tau_i}{\tau f_i}, & \eta_i(\tau_{fi}) = \eta_i(0) \\ \dfrac{\ln\left(1 + \left(\frac{\eta_f}{\eta_0} - 1\right)\frac{\tau}{\tau_f}\right)}{\ln\frac{\eta_f}{\eta_0}} & \eta_i(\tau_{fi}) \ne \eta_i(0) \end{cases} \qquad (6)$$

Thus, given a value of the final time t_f, the final values $\tau_{fi}; i = 1, 2, ..., n$ of the path parameters τ_i are uniquely defined by (5). This set-up can now be exploited to achiever either spatial or temporal deconfliction by viewing t_f, in some specified interval $[t_1, t_2]$, as the key search parameter in an optimization problem.

Spatial deconfliction. In the case of spatial deconfliction, feasible trajectories for all the vehicles are obtained by solving an optimization problem of the form

$$F2 : \begin{cases} \min\limits_{t_f \in [t_1, t_2]} \sum\limits_{i=1}^{n} w_i J_i \\ \text{subject to geometric boundary conditions and (4)} \\ \text{for any } i \in [1, n], \text{ and} \\ \\ \min\limits_{j,k=1,\ldots,n, j\ne k} ||p_{c_j}(\tau_j) - p_{c_k}(\tau_k)||^2 \ge E^2 \\ \text{for any } \tau_j, \tau_k \in [0, \tau_{fj}] \times [0, \tau_{fk}], \\ \text{with } \tau_{f_j}, \tau_{f_k} \text{ obtained from Eq. (5)} \end{cases}$$

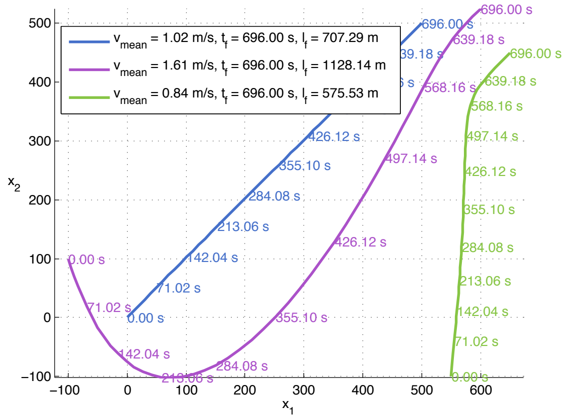
where J_i represents total energy consumption of vehicle V_i and the weights $w_i > 0$ penalize the energy consumptions of all vehicles. Note that in contrast to $F1$, in $F2$ an additional constraint $\min_{j,k=1,\ldots,n,j\ne k} ||p_j(\tau_j) - p_k(\tau_k)||^2 \ge E^2$

for any $\tau_j, \tau_k \in [0, \tau_{fj}] \times [0, \tau_{fk}]$ was added to guarantee spatially deconflicted trajectories separated by a minimum spatial clearance distance E. In summary, we seek to minimize the simultaneous time of arrival, subject to constraints that include minimum and maximum vehicle speeds, maximum vehicle accelerations, the allowed window of times of arrival, and spatial clearance requirements for deconfliction.

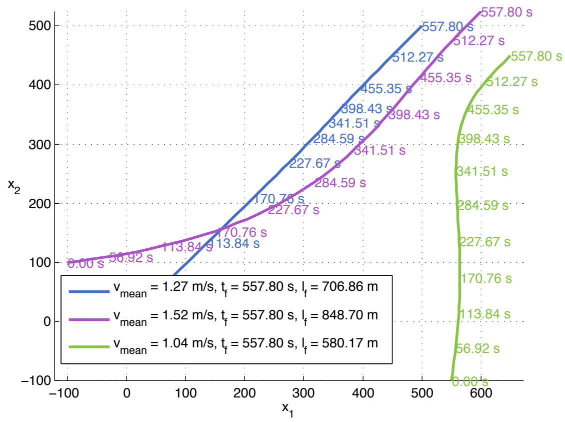
Temporal deconfliction. We now address the problem of multiple vehicle path planning with temporal deconfliction. As argued before, temporal deconfliction introduces an extra degree of freedom (time) that is not available in the case of spatial deconfliction. As such, it yields solutions whereby paths are allowed to come to close vicinity or intersect in space. However, the temporal scheduling of the vehicles involved separates these occurrences well in time. The crucial point is therefore to guarantee that the vehicles maneuver along the assigned paths, in a synchronized manner.

Stated in these terms, it appears as if one is led inevitably to the situation where each vehicle must track a pre-assigned trajectory with great precision, that is, to execute a trajectory tracking maneuver. This strategy meets with considerable problems. In fact, in the event that one of the vehicles will deviate considerably from its planned spatial and temporal schedule (due to environmental disturbances or temporary failures), the original plan can no longer be executed and replanning will become necessary. However, this strategy does not lend itself to formal analysis. For this reason, it is important to adopt a new strategy where the vehicles cooperate and adjust their motion in reaction to deviations from the original plan, so as to keep maintain the spatial formation that is naturally imposed by that plan. In this set-up absolute time ceases to play an important role, and all that is relevant is for the vehicles to arrive simultaneously at their target positions, the final times of arrival being left unspecified (but within defined bounds).

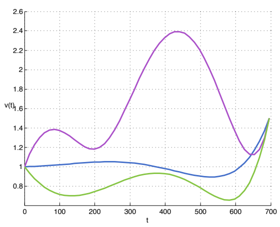
This circle of ideas, originally proposed in Ghabcheloo et al. [2009b], leads to an integrated strategy for multiple vehicle path planning and control that is referred to as Time-Coordinated Path Following (TC-PF). The methodology proposed unfolds in three steps. First, extending the methods exposed in Kaminer et al. [2007], *temporally deconflicted trajectories are generated for a group of vehicles.* At the end of this step, the trajectories obtained are conveniently re-parametrized by a variable that we call *virtual time*, leading to a set of spatial paths, together with the corresponding nominal vehicle speed profiles along them. The second step involves the design of *path following algorithms* to steer each vehicle along its assigned path, while tracking the corresponding speed profile. Here, absolute time does not play any role. Finally, the last step aims to *coordinate the relative motion of the vehicles along their paths*, so as to guarantee deconfliction and meet desired temporal constraints such as equal times of arrival. This is done by varying the speed of each vehicle about the nominal speed profile computed in the first step, based on the exchange of information with its neighbors. The information exchanged is related to the virtual time referred to above. The resulting scheme lends itself to a rigorous formulation and avoids replanning except for the situation where, due to strong disturbances, the vehicles deviate considerably from the paths or fail to meet required temporal constraints. In this paper we focus on the first step described above. For an introduction to the



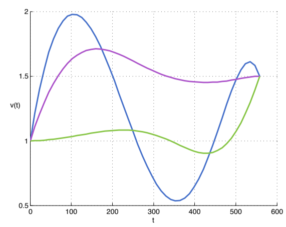
(a) Spatial deconfliction.



(b) Temporal deconfliction.



(c) Velocity profiles.

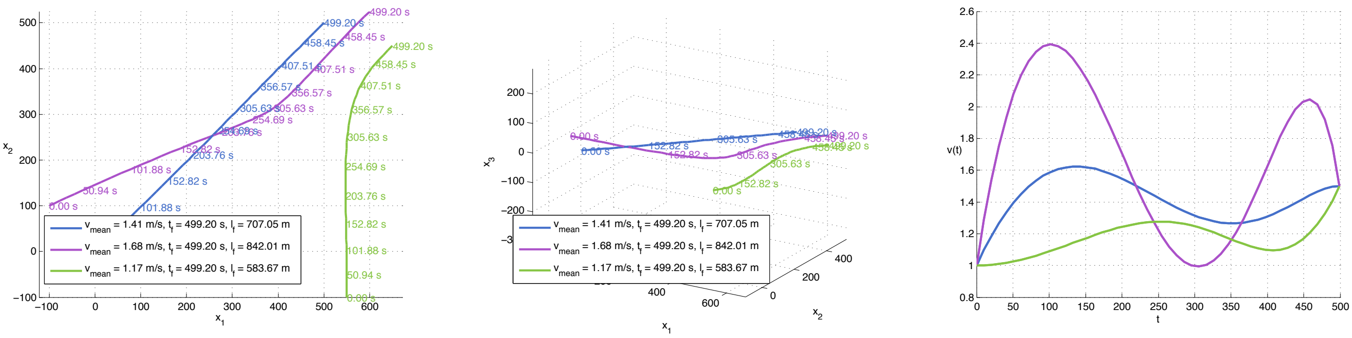


(d) Velocity profiles.

Fig. 3. Path planning using spatial and temporal deconfliction. Boundary conditions are as follows: V_1 - $[x_{1_0}, y_{1_0}] = [0,0]$m, $[x_{1_f}, y_{1_f}] = [500, 500]$m, $\psi_{1_f} = 45°$; V_2 - $[x_{2_0}, y_{2_0}] = [-100, 100]$m, $[x_{2_f}, y_{2_f}] = [600, 525]$m, $\psi_{2_f} = 45°$; V_3 - $[x_{3_0}, y_{3_0}] = [550, -100]$m, $[x_{3_f}, y_{3_f}] = [650, 450]$m, $\psi_{3_f} = 45°$. For all vehicles, the initial and final speed were set to $1m/s$ and $1.5m/s$, respectively. The initial headings as well as the initial and final accelerations were determined during the optimization process.

theoretical machinery that supports steps two and three above, the reader is referred to Ghabcheloo et al. [2009a].

It is against this backdrop of ideas that we now describe a solution to the problem of multiple vehicle path planning with temporal deconfliction. The solution borrows from the concepts previously introduced in the section on spatial deconfliction. Namely, the optimization problem to be



(a) Spatial deconfliction in 3D, top view. (b) The same solution in isometric view. (c) Velocity profiles.

Fig. 4. A spatially deconflicted solution to the problem depicted in Fig. 3, in three dimensional space. The (x, y) projection of the path path resembles that obtained with the algorithm for temporal deconfliction in Fig. 3b; notice, however (isometric view) that one of the vehicles dives under the nominal path of another vehicle in order to ensure spatial deconfliction.

solved is (F2), except that the avoidance constraint is now expressed as

$$||p_i(t) - p_j(t)||^2 \geq E^2 \quad \forall\ i, j = 1, \ldots, n; i \neq j \text{ and } t \in [0, t_f],$$

where t_f is the optimization parameter and t is is related to the τ_i via (5). We again stress that multiple vehicle path planning with temporal deconfliction is the first step in the general methodology of Time-Coordinated Path Following introduced in Ghabcheloo et al. [2009b]. At the end of this step, the temporal coordinate used in the computations becomes a path parameter that is hidden in the remaining two steps. This is simply done by reparameterizing the paths produced as $p_i(\gamma)$, with $\gamma = t/t_i \in [0, 1]$ defined as

$$\gamma(\tau_i) = \begin{cases} \dfrac{\tau_i}{\tau f_i}, & \eta_i(\tau_{fi}) = \eta_i(0) \\ \dfrac{\ln\left(1 + \left(\frac{\eta_f}{\eta_0} - 1\right)\frac{\tau}{\tau_f}\right)}{\ln \frac{\eta_f}{\eta_0}} & \eta_i(\tau_{fi}) \neq \eta_i(0) \end{cases} \qquad (7)$$

3. SIMULATION RESULTS

This section contains the results of simulations aimed at illustrating the efficacy of the multiple vehicle path planning systems developed. The zero-order optimization method adopted is described in Hooke and Jeeves [1961].

Fig. 3 shows the results of running the algorithms for spatial and temporal deconfliction in 2D. The initial and final conditions for the three vehicles are the same. In the case of spatial deconfliction, vehicle V_1 (purple) must perform a large turn (in a direction that points away from its final position) to avoid coming close the the path of vehicle V_2 (blue), see Fig. 3a. The same situation is solved with temporal deconflicted path planning in Fig. 3b. The solution is far simpler in terms of path shapes and the saving in maneuvering time is more than 2 minutes. The velocity profiles are also much smoother. Notice that because the paths are temporally deconflicted, spatial intersections are now allowed. However, collisions among vehicles will not occur because the actual spatial clearance will never fall below its limit E at any time t.

It is intuitive that by adding a third depth coordinate, spatial deconfliction problems that are hard to solve in 2D may have straightforward solutions in 3D. This is illustrated in Fig. 4, which should be compared against Fig. 3.

4. CONCLUSIONS

The paper addressed the problem of multiple vehicle path planning with spatial and temporal constraints. Solutions were described that build on polynomial-based techniques and *direct optimization methods*. The path planning methods derived are computationally easy to implement, thus lending themselves to real-time applications. The efficacy of the solutions developed was shown with selected simulation examples. Future work will include the development of path planning methods to avoid fixed obstacles or to meet nonconventional constraints that arise when the directivity of the inter-vehicle communication links must be taken into account.

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