Single Beacon Acoustic Navigation for an AUV in the presence of unknown ocean currents ⋆

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Abstract: This paper addresses the problem of underwater navigation, using a single beacon (or a transponder). We propose a solution to estimate the underwater vehicle position in the presence of unknown ocean currents. The main idea is to combine the dead-reckoning information with multiple range measurements taken at different instants of time from the vehicle to a single beacon. Then, applying a multilateration based algorithm and using a Kalman filter, the unknown velocity of ocean current and a more accurate estimation of vehicle's position is estimated. The stability and convergence of the position estimation error are analysed taken explicitly into account the presence of ocean currents, disturbances, measurement noise and discretization errors. Simulation results are presented and discussed. In particular, we show that the implementation of the Kalman filter to estimate the ocean current is crucial to achieve convergence of the estimated position.

Keywords: Single beacon; underwater navigation; Kalman filter; trilateration.

1. INTRODUCTION

Over the last decade, applications with ocean robotics have increased dramatically. The use of remotely operated vehicles and, more recently, autonomous underwater vehicles (AUVs) have shown to be extremely important tools to study and explore the oceans. A key enabling element for the use of such robotic vehicles is the availability of advance navigation and positioning systems. In this regard, often acoustic Long Baseline (LBL) navigation systems play an important role. Typically, these systems only provide range measurements and rely in multiple transponders to compute an estimative of the vehicle position. In order to obtain a desired precision, the location of each deployed transponder [1] must be precisely surveyed, prior to conducting autonomous vehicle operation. Deploying, surveying and prior recovering the transponders is a costly and a time consuming process. Instead, having a single transponder navigation system that guarantees the desired precision, may save money and time.

Motivated by the above considerations, this work addresses the single transponder underwater navigation problem. Inspired by previous works by [Larsen (2001)] and [LaPointe (2006)], we propose a solution to estimate the underwater vehicle position in the presence of unknown ocean currents. The main idea is to combine the dead-reckoning information with multiple ranges measurements taken at different instants of time from the vehicle to a single beacon. Then, applying a multilateration based algorithm and using a Kalman filter, the unknown velocity of ocean current and a more accurate estimation of the vehicle's position is estimated.

The stability and convergence of the position estimation error are analysed taken explicitly into account the presence of ocean currents, disturbances, measurement noise and discretization errors. In particular, we provide conditions under which the position error is (locally) Input to State Stable (ISS).

The organization of the paper is as follows: Section 2 describes the proposed navigation system. In Section 3, the stability and convergence of the position estimation error is analysed. Section 4 illustrates the performance of the navigation algorithm developed within several scenarios using computer simulations. Finally considerations are presented in Section 5.

2. POSITION NAVIGATION SYSTEM

To obtain the position of the vehicle using only a single beacon, a two stage algorithm is proposed. At first stage the position is computed using a multilateration-based scheme. At second stage a Kalman filter is implemented in order to refine the previous estimation and to estimate the

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[1] The transponder sends out an acoustic response only when interrogated, and the beacon sends out an acoustic signal at predetermined intervals.

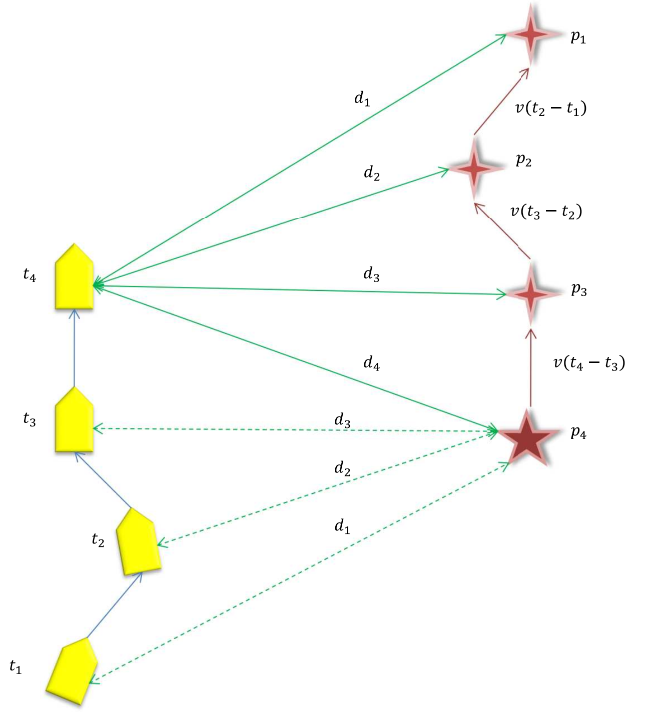


Fig. 1. Illustration of the creation of the virtual net of beacons.

ocean velocity current. As it will be shown, the integration of the first and the second stage of the algorithm is crucial to make position navigation possible, when ocean currents exists.

2.1 Algorithm's first stage

The first part of the algorithm consists of creating a (dynamic) virtual network of beacons and then apply a similar navigation strategy as Long Baseline (LBL), but with virtual beacons, to obtain a rough estimation of the vehicle's position. The strategy to form the virtual network of beacons is described as follows (see Fig. 1): The vehicle records at pre-defined instants of time t_i, $i = 1, 2, \dots, N-1$, with $N \geq 3$ the distance d_i between the vehicle and the beacon by measuring the fly-time of the acoustic signal and using (an estimative) of the underwater speed of sound. Let p_N be the last beacon in the virtual net and set it to be the position of the real beacon, which is assumed to be known in some inertial frame $\{I\}$. Then, going backwards and using the vehicle's dead reckoning data, the next virtual beacons are created by applying the vehicle displacement to the beacon location, which simulates that the real beacon had moved with the same trajectory as the vehicle (see Fig. 1). Using this reasoning, it turns out that the distance measurements d_i taken during that process corresponds exactly to the ranges between the vehicle in the last position and the virtual beacons. Formally, the position of the i^{th} virtual beacon p_i, $i = 1, 2, \dots, N$ is given by

$$p_i = p_N + \int_{t_i}^{t_N} v(\tau) d\tau, \tag{1}$$

where p_N is the position of the real transponder, v is the total vehicle velocity vector expressed in the inertial frame $\{I\}$, and t_i is the instant of time decided to take a range measurement. After the virtual net of beacons is established, using a multilateration based algorithm, one can find the best estimative for the vehicle position $\bar{p}$. As an example, a simple way to determine $\bar{p}$ algorithm (but not at all the best) is to solve the straightforward optimization

$$\bar{p} = \arg\min_{p \in \mathbb{R}^2} \sum_{i=1}^{N} \left(\|p - p_i\|^2 - d_i^2 \right)^2. \tag{2}$$

To compute $\bar{p}$ it is implicitly assumed that the vehicle not only can measure the range from its location to the beacon but it can also measure the velocity v (see (1)). Note also that $\bar{p}$ can only be computed at the final time $t = t_N$, and then the process starts again by forming another virtual netwok of beacons to compute the next $\bar{p}$ (at time $t = 2t_N$).

2.2 Algorithm's second stage

It turns out that the previous step of the algorithm will only work when there is no ocean current or when one have a precise way of measure the ocean current's velocity v_c. Usually this is not the case. Since the velocity is measured through a Doppler velocity log. If the AUV is not near the seabed than the Doppler can only provide the velocity of the vehicle relative to the water v_r, which implies that the inertial velocity $v = v_r + v_c$ is not directly measured. Neglecting the effect of the ocean current may lead to divergence in the algorithm, as it is shown in the simulation results (see Section 4). To tackle this problem, we resort to a Kalman filter to estimate the ocean current and also filter out the vehicle position $\bar{p}$. The Kalman filter is implemented assuming that the unknown ocean current is constant and making use of a simple Kinematic relation between the velocity and the position. The equations that model the system are given by

$$\begin{cases} \dot{p} = v_r(t) + v_c + \boldsymbol{\omega}(t) \\ \dot{v}_c = 0 + \boldsymbol{\nu}(t) \end{cases} \tag{3}$$

where $\boldsymbol{\omega}_k(t)$ and $\boldsymbol{\nu}_k(t)$ are assumed to be stationary, Gaussian, zero mean white noise processes and mutually independent, with covariances $E[\boldsymbol{\omega}_k(t)\boldsymbol{\omega}_k(s)]$ and $E[\boldsymbol{\nu}_k(t)\boldsymbol{\nu}_k(s)]$ given by $Q_\omega \delta(t-s)$ and $Q_\nu \delta(t-s)$ respectively. Applying a Euler approximation to the continuous model (3), we obtain the discrete-time process model

$$\begin{bmatrix} x \\ y \\ v_{c_x} \\ v_{c_y} \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & \Delta t \end{bmatrix} \begin{bmatrix} x \\ y \\ v_{c_x} \\ v_{c_y} \end{bmatrix}_k + \begin{bmatrix} \Delta t & 0 \\ 0 & \Delta t \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_{r_x} \\ v_{r_y} \end{bmatrix}_k + \boldsymbol{\omega}_k \tag{4}$$

$$\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix}_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ v_{c_x} \\ v_{c_y} \end{bmatrix}_k + \boldsymbol{\nu}_k \tag{5}$$

where the state is composed by the vehicle's position (x, y) and the ocean current velocity vector $v_c = (v_{c_x}, v_{c_y})$. The input is the water relative vehicle's velocity vector $v_r = (v_{r_x}, v_{r_y})$ provided by the Dopler, and the measurement is the position $\bar{p} = (\bar{x}, \bar{y})$ computed in (2) replacing v in (1) by $v_r + \hat{v}_c$, where $\hat{v}_c$ denotes the estimative of v_c provided by the Kalman filter. The symbol Δt denotes the time step $t_{k+1} - t_k$ and $\{\mathbf{w}_k\}$ and $\{\mathbf{v}_k\}$ are mutually independent,

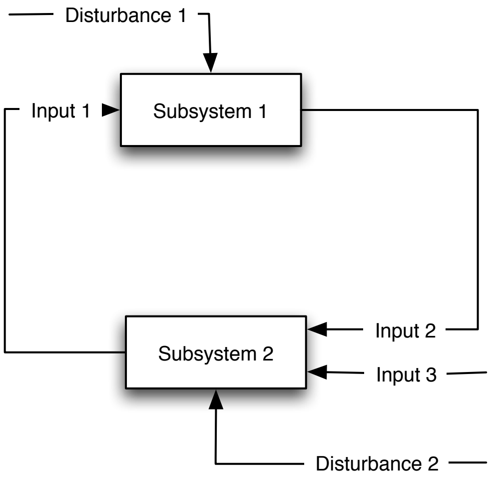


Fig. 2. Interconnection of the algorithm subsystems. Subsystem 1 represents equation (11). Subsystem 2 is the dynamic system (14). Input 1 is $\tilde{\zeta}$. Input 2 is the error vector $\tilde{p}$. Input 3 is the relative velocity error $\tilde{v}_r$. Disturbance 1 represents the errors in the range measurements, $\tilde{d}_i$, and Disturbance 2 represents the signals $\boldsymbol{\omega}(t)$ and $\boldsymbol{\nu}(t)$.

stationary, zero mean white Gaussian sequences of random variables, with covariances given by [Mao et al. (2007)]

$$\begin{cases} \boldsymbol{\omega}_k = \sqrt{\Delta t}\boldsymbol{\gamma}_{\omega,k} \\ \boldsymbol{\nu}_k = \sqrt{\Delta t}\boldsymbol{\gamma}_{\nu,k} \end{cases} \tag{6}$$

where the vector sequences $\{\boldsymbol{\gamma}_{\omega,k}\}$ and $\{\boldsymbol{\gamma}_{\nu,k}\}$ are mutually independents, stationary zero mean, white Gaussian sequences of vector random variables, with covariances Q_ω and Q_ν. For convenience of notation we re-write (4) and (5) in the compact notation

$$\begin{cases} \zeta(k+1) = A\zeta(k) + Bu(k) + \boldsymbol{\omega}(k) \\ z(k+1) = C\zeta(k) + \boldsymbol{\nu}(k) \end{cases} \tag{7}$$

where

$$\zeta = \begin{bmatrix} x \\ y \\ v_{c_x} \\ v_{c_y} \end{bmatrix}, \quad u = \begin{bmatrix} v_{r_x} \\ v_{r_y}. \end{bmatrix} \tag{8}$$

Then the Kalman filter is given by

$$\begin{cases} \hat{\zeta}(k+1) = A\hat{\zeta}(k) + L[z(k) - \hat{z}(k)] + Bu(k) + \boldsymbol{\omega}(k) \\ \hat{z}(k) = C\hat{\zeta}(k) + \boldsymbol{\nu}(k) \end{cases} \tag{9}$$

where L is the Kalman gain. Equation (9) together with the algorithm to compute $\bar{p}$ (equations (1) and (2)) forms the Position Navigation System.

The use of the Kalman filter in a integrated fashion with the algorithm to compute $\bar{p}$ improves the estimative of the vehicle position and allows to predict the velocity of the ocean current. These results are discussed in Section 4.

3. STABILITY ANALYSIS

In the present section we discuss the convergence of the algorithm proposed. The analysis is done in a deterministic setting where it is assumed that the disturbance and noise signals are deterministic and bounded but unknown. We start to analyze the algorithm's first stage, then the second stage, and finally the influence of the interconnection between these two stages. Note that the computation of $\bar{p}$ (done at the first stage), where $\bar{p}$ is an input signal of the Kalman filter (implemented at the second stage), depends on the output of the Kalman filter (more precisely from the estimative of the ocean current).

To analyse the influence of the range measurement errors, dead-reckoning errors, discretization errors, and others, in the computation of $\bar{p}$ we resort to the results of the work presented in [Thomas and Ros (2005)]. In that work, a closed-form formulation for trilateration based on the Cayley-Menger determinants and geometric concepts was derived. Using that fact, selecting three arbitrarily points from the virtual network of beacons, say q_1, q_2, and q_3, the position $\bar{p}$ determined by 3D trilateration can be expressed as

$$\bar{p} = q_1 + k_1 a_1 + k_2 a_2 \pm k_3(a_1 \times a_2), \tag{10}$$

where

$$k_1 = -\frac{D(q_1, q_2, q_3; q_1, q_3, p)}{D(q_1, q_2, q_3)},$$
$$k_2 = \frac{D(q_1, q_2, q_3; q_1, q_2, p)}{D(q_1, q_2, q_3)},$$
$$k_3 = \frac{+\sqrt{D(q_1, q_2, q_3, p)}}{D(q_1, q_2, q_3)},$$

$a_1 = q_2 - q_1$, $a_2 = q_3 - q_1$, $D(\cdot,\cdot,\cdot)$ are the Cayley-Menger determinants, $D(\cdot,\cdot,\cdot;\cdot,\cdot,\cdot)$ are Cayley-Menger bi-determinants, and the pm sign accounts for the two mirror symmetric locations of $\bar{p}$ with respect to the base plane defined by q_1, q_2, and q_3. In the sequel, given a vector a, we denote by a^0 its nominal value (no errors) and $\tilde{a}$ additive error such that $a = a^0 + \tilde{a}$. Using (10) it can be concluded that

$$\begin{aligned} \tilde{p} = \bar{p} - p &= \tilde{q}_1 + k_1\tilde{a}_1 + k_2\tilde{a}_2 \\ &\pm k_3[(a_1^0 \times \tilde{a}_2) + (\tilde{a}_1 \times a_2^0) + (\tilde{a}_1 \times \tilde{a}_2)]. \end{aligned} \tag{11}$$

We now analyze each term in (11) with the objective of obtaining an upper bound of the position error $\tilde{p}$. From (1) it follows that

$$\tilde{q}_i = \tilde{q}_N + \int_{t_i}^{t_N} (\tilde{v}_r(\tau) + \tilde{v}_c(\tau))\, d\tau.$$

Thus, for all $i = 1, 2, \ldots, N$

$$\|\tilde{q}_i\| \leq \|\tilde{q}_N\| + \max_{t_1 \leq \tau \leq t_N} \|\tilde{v}_r(\tau) + \tilde{v}_c(\tau)\|(t_N - t_1),$$

$$\|\tilde{a}_i\| \leq 2 \max_{t_1 \leq \tau \leq t_N} \|\tilde{v}_r(\tau) + \tilde{v}_c(\tau)\|(t_N - t_1).$$

In addition,

$$\begin{aligned} \|a_i^0 \times \tilde{a}_j\| &\leq \|a_i^0\|\|\tilde{a}_j\| \\ &\leq 4 \max_{t_1 \leq \tau \leq t_N} \|v(\tau)\| \max_{t_1 \leq \tau \leq t_N} \|\tilde{v}_r(\tau) + \tilde{v}_c(\tau)\|(t_N - t_1)^2 \end{aligned}$$

$$\|\tilde{a}_i^0 \times \tilde{a}_j\| \leq 4 \max_{t_1 \leq \tau \leq t_N} \|\tilde{v}_r(\tau) + \tilde{v}_c(\tau)\|^2(t_N - t_1)^2$$

for $i, j = 1, 2, \ldots, N$. Therefore, using the above bounds in (11) we obtain

$$
\begin{aligned}
\|\tilde{p}\| \leq\ & \|\tilde{p}_N\| + \max_{t_1 \leq \tau \leq t_N} \|\tilde{v}_r(\tau) + \tilde{v}_c(\tau)\|(t_N - t_1) \\
& + 2(k_1 + k_2) \max_{t_1 \leq \tau \leq t_N} \|\tilde{v}_r(\tau) + \tilde{v}_c(\tau)\|(t_N - t_1) \\
& + 8k_3 \max_{t_1 \leq \tau \leq t_N} \|v(\tau)\| \max_{t_1 \leq \tau \leq t_N} \|\tilde{v}_r(\tau) + \tilde{v}_c(\tau)\|(t_N - t_1)^2 \\
& + 4k_3 \max_{t_1 \leq \tau \leq t_N} \|\tilde{v}_r(\tau) + \tilde{v}_c(\tau)\|^2 (t_N - t_1)^2 \\
\leq\ & \|\tilde{p}_N\| + \alpha_1 \max_{t_1 \leq \tau \leq t_N} \|\tilde{v}_r(\tau)\| + \alpha_2 \max_{t_1 \leq \tau \leq t_N} \|\tilde{v}_r(\tau)\|^2 \\
& + \alpha_3 \max_{t_1 \leq \tau \leq t_N} \|\tilde{v}_c(\tau)\| + \alpha_4 \max_{t_1 \leq \tau \leq t_N} \|\tilde{v}_c(\tau)\|^2 \qquad (12)
\end{aligned}
$$

where α_i are given by

$$
\begin{aligned}
\alpha_1 &= (t_N - t_1)(1 + 2(k_1 + k_2) \\
&\quad + 8k_3(t_N - t_1) \max_{t_1 \leq \tau \leq t_N} \|v(\tau)\|) \\
\alpha_2 &= (4k_3(t_N - t_1)^2) \\
\alpha_3 &= (t_N - t_1)(1 + 2(k_1 + k_2) \\
&\quad + 8k_3(t_N - t_1) \max_{t_1 \leq \tau \leq t_N} \|v(\tau)\| \\
&\quad + 4k_3(t_N - t_1) \max_{t_1 \leq \tau \leq t_N} \|\tilde{v}_r(\tau)\|) \\
\alpha_4 &= 4k_3(t_N - t_1)^2.
\end{aligned}
$$

Note that the bound of the error $\tilde{p}$ turns out to be a function of the position error of the real beacon $\tilde{p}_N$, the relative velocity measurement error $\tilde{v}_r$, and the ocean estimation error $\tilde{v}_c$ with a linear and a quadratic term, which will prevent from yielding a global stability result (only local). These dependences with exception of the first one, are also function of the discretization time interval $\Delta t = t_N - t_1$.

We now analyze the second part of the algorithm. Let $\tilde{\zeta}(k) = \zeta(k) - \hat{\zeta}(k)$ be the state estimation error. Then, from (7) and (9) we obtain

$$
\tilde{\zeta}(k+1) = (A - LC)\tilde{\zeta}(k) = A_0\tilde{\zeta}(k), \qquad (13)
$$

for $\boldsymbol{\omega}(k) = \boldsymbol{\nu}_k = 0$. Since system (7) is linear and time-invariant with (A,C) detectable, then there exists a matrix L such all the eigenvalues of $A_0 = A - LC$ have magnitudes (strictly) less than one. Therefore, the error dynamics (13) is globally asymptotical stable.

Consider now system (13) with input signals (due to errors)

$$
\tilde{\zeta}(k+1) = A_0\tilde{\zeta}(k) + S\tilde{p}(k) + \boldsymbol{\omega}(k) + LC\boldsymbol{\nu}(k). \qquad (14)
$$

Since A_0 is a Schur matrix, i.e., all his eigenvalues are located strictly inside the unit ball, then the Lyapunov equation

$$
A_0^T P - A = -I
$$

holds with $P > 0$. Consider the positive-definite Lyapunov function

$$
V = \tilde{\zeta}^T P \tilde{\zeta}. \qquad (15)
$$

Straightforward computations shows

$$
\begin{aligned}
V_{k+1} - V_k =\ & \tilde{\zeta}(k)^T[A_0 P A_0 - P]\tilde{\zeta} \\
& + (S\tilde{p} + \boldsymbol{\omega} + LC\boldsymbol{\nu})^T P(S\tilde{p} + \boldsymbol{\omega} + LC\boldsymbol{\nu}(k)) \\
& + 2\tilde{\zeta}^T A_0 P(S\tilde{p} + \boldsymbol{\omega} + LC\boldsymbol{\nu}) \\
\leq\ & \sigma_0(\|\zeta\|) + \sigma_1(\|\tilde{p}\|) + \sigma_2(\|\boldsymbol{\omega}\|) + \sigma_1(\|\boldsymbol{\nu}\|)
\end{aligned}
$$

where

$$
\begin{aligned}
\sigma_0(r,k) &= \frac{1}{2}[1 - \|P\|(\alpha_3 + \alpha_4\rho), \\
&\quad - 2\|A_0\|\|P\|(\alpha_3 + \alpha_4\rho)]r^2, \\
\sigma_1(r) = \sigma_2(r) &= \left(\frac{2\|A_0^T P\|}{\gamma_1} + \|P\|\right)r^2, \\
\sigma_3(r) &= \left(\frac{2\|A_0^T PLC\|}{\gamma_2} + \|(LC)^T P(LC)\|\right)r^2,
\end{aligned}
$$

$S = [1,1,0,0]$, $\gamma_1 = 1 - (\|P\|(\alpha_3 + \alpha_4\rho) - 2\|A_0\|\|P\|(\alpha_3 + \alpha_4\rho))$ and $\gamma_2 = 1 - (\|LC\|^2\|P\|(\alpha_3 + \alpha_4\rho) - 2\|A_0\|\|P\|\|LC\|(\alpha_3 + \alpha_4\rho))$, and $\tilde{\zeta}$ is assumed to satisfies $\|\tilde{\zeta}\| \leq \rho$, and σ_i to be K_∞-function [2], which it is true for sufficiently small sampling time $t_N - t_1$ and measurement noise and disturbances. Therefore, it can be concluded that V is an ISS-Lyapunov function (c.f. [Jiang and Wang (2001)]), and consequently ζ is (local) ISS.

4. SIMULATION RESULTS

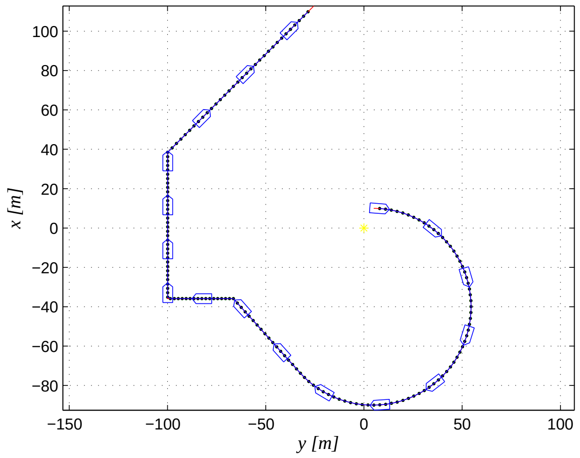
This section illustrates the single beacon navigation algorithm through computer simulations. The simulations were performed with $N = 4$ (number of virtual beacons), which means that for each 4 measurements, one estimated point of the trajectory is computed.

Figure 3 shows the estimated trajectory performed by the vehicle with and without ocean current. The yellow star represents the Beacon location (which is at the origin), the red line represents the true path of the vehicle, the black dots represents the estimative $\bar{p}$ ignoring the presence of ocean current (that is, in (3) we set $v = v_r$), the green dots represents the estimative $\bar{p}$ using the Kalman filter (in (3) we set $v = v_r + \hat{v}_c$), and the blue line represents the estimative of the vehicle's position provided by the Kalman filter. This simulation was performed without sensor noises. As expected, (see Fig. 3(a)) without ocean current, there is no estimation error even using only the algorithm described in the first step (black dots). However, in the presence of ocean currents (see Fig. 3(b)), it can be seen that the algorithm without the Kalman filter diverges (black dots).

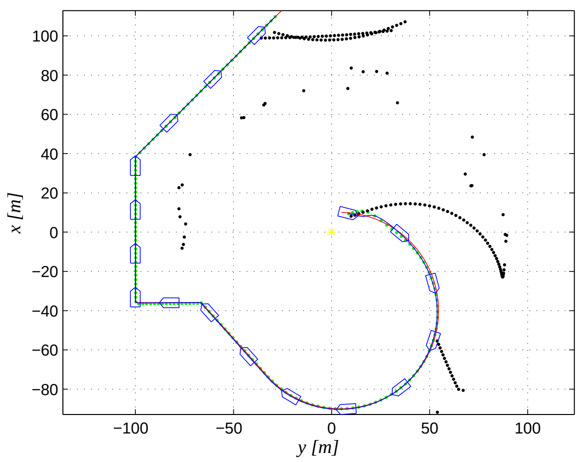
Figure 4 illustrates the same set of simulations but including Gaussian noise in all sensors with a $(\sigma)^2 = 0.8(m.s^{-1})^2$, which is more than 10% of the vehicle velocity. It can be seen that there is a significantly improvement in using the Kalman filter compared with only applying the first step of the algorithm (see green dots). The postion estimation error with water current is greater than without, but is still small when compared with the distance of the vehicle to the beacon, that is around $100m$.

Figures 5 and 6 display the time evolution of the estimative of the ocean current and the vehicle position estimation error, respectively. Note that the first part of the trajectory is worst estimated. This happens because the ocean current is still in the process of being determined. Once it is estimated, it can be seen almost a perfect match between the real and the estimated trajectory. In the horizontal part of the trajectory one see again another worst performance of the estimator because the trajectory becomes almost in

[2] $\sigma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a K_∞-function if it is continuous, strictly increasing, $\sigma(0) = 0$, and $\sigma(r) \to \infty$ as $r \to \infty$.

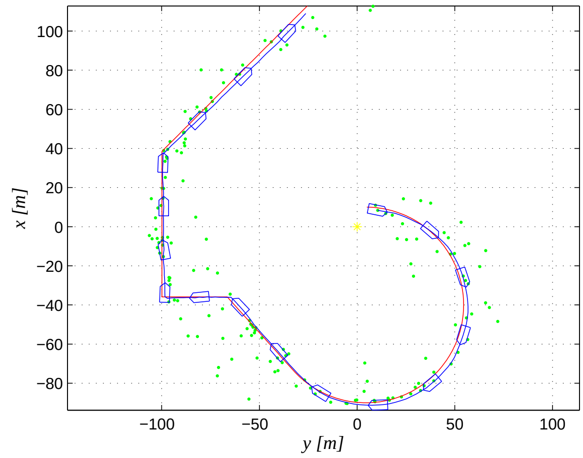


(a) Simulation without noise on the sensors, and no ocean current.

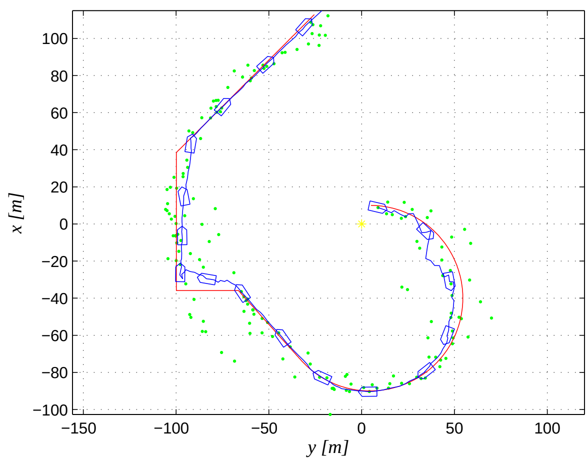


(b) Simulation without sensors noise, but with unknown constant ocean current.

Fig. 3. The yellow star at $(0, 0)$ represents the Beacon, the red line represents the true path of the vehicle, the green dots represents the estimative using only the first step of the algorithm (with ocean current estimative obtained from the Kalman filter), the doted blue line represents the estimative from the Kalman filter, and the black dots represent the vehicle's position estimative using the first step of the algorithm, but without any ocean current's velocity estimation.



(a) Simulation without ocean current.



(b) Simulation with unknown constant ocean current.

Fig. 4. The yellow star represents the Beacon, the red line represents the true path of the vehicle, the green dots represents the estimative using only the first step of the algorithm, and the doted blue line represents the estimative using the Kalman filter.

the same line as the beacon. In all simulations it was not used an outliers rejection algorithm. The results could be improved by an implementation of a such algorithm.

The position navigation system works even in the presence of variable water current's velocity, both in magnitude and direction. This is showed in figure 7.

5. CONCLUSIONS

The problem of underwater navigation, using a single beacon (or a transponder) was formalized. We proposed a solution to estimate the underwater vehicle position in the presence of unknown ocean currents. The algorithm unfolds in two basic steps: First, combining the dead-reckoning information with multiple range measurements taken at different instants of time from the vehicle to a single beacon a virtual network of beacons is created, which then permits to apply a multilateration based algorithm. In the second step a Kalman filter is implemented to obtain an estimative of the unknown velocity of ocean current and a more accurate estimation of vehicle's position. The stability and convergence of the position estimation error were analyzed taken explicitly into account the presence of ocean currents, disturbances, measurement noise and discretization errors. We have shown that the implementation of the Kalman filter to estimate the ocean current is crucial to achieve convergence of the estimated position. The computer simulations results show that the navigation algorithm works well even in presence of relatively large amount of noise in the sensors.

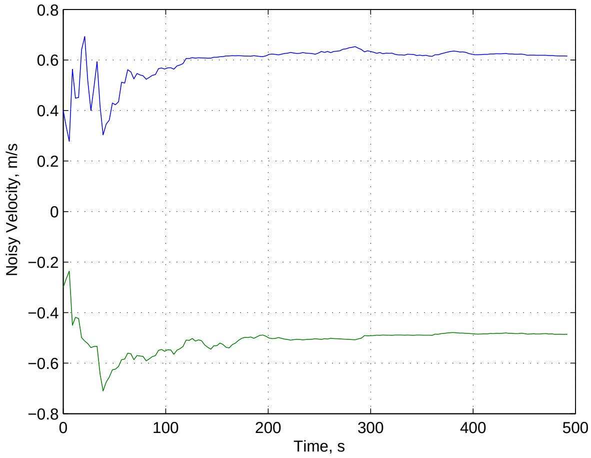
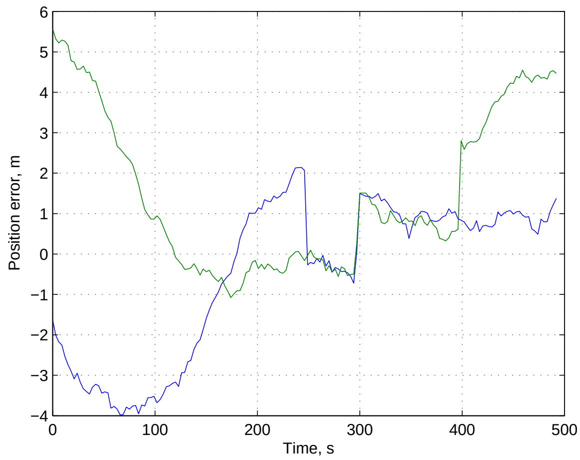
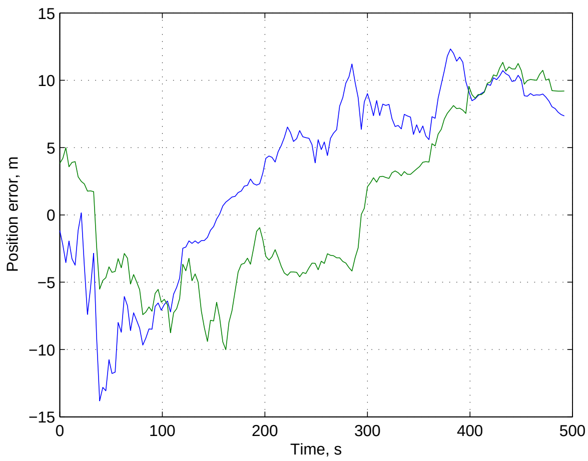


Fig. 5. The blue line represents the v_{c_x} component of the ocean current velocity and the green one the v_{c_y} component of the ocean current.



(a) Time evolution of the position error without ocean current.



(b) Time evolution of the position error with ocean current.

Fig. 6. The blue line represents the x component of the vehicle position error and the green one the y component.

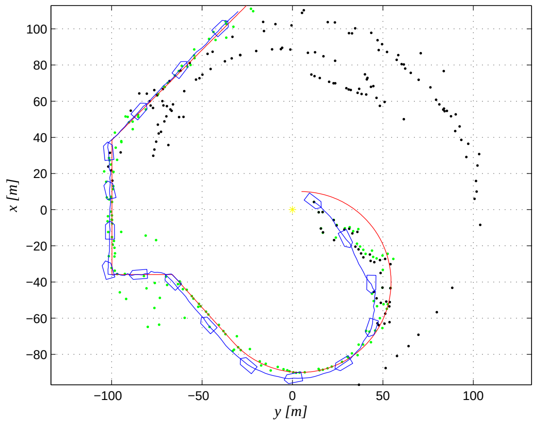


Fig. 7. Estimated vehicle's trajectory in presence of variable underwater currents.

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