Cooperative Path-Following of Underactuated Autonomous Marine Vehicles with Logic-based Communication

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Abstract: This paper addresses the problem of steering a group of underactuated marine vehicles along given spatial paths, while holding a desired inter-vehicle formation pattern (cooperative path-following). The CPF problem is divided into a motion control task of making each vehicle track a virtual target moving along the desired path and a dynamic assignment task of adjusting the speeds of the virtual targets so as to achieve vehicle coordination. At the path-following level, the controller derived exhibits a inner-outer-loop structure and includes an observer to estimate the ocean currents and the sway velocity. At the coordination level, the decentralized control algorithm addresses explicitly the case where the communications among the vehicles occur with non-homogeneous, possibly varying delays. Convergence and stability of the overall system are proved formally. Simulations results are presented and discussed.

Keywords: **Autonomous vehicles**, **Coordination**, **Cooperative control**, **Marine systems**, **Path-following**.

1. INTRODUCTION

The past few decades have witnessed considerable interest in the area of motion control of autonomous marine craft (Aguiar and Pascoal, 2007*b*; Alonge *et al.*, 2001; Encarnação and Pascoal, 2000; Fossen, 1994; Jiang, 2002; Lefeber *et al.*, 2003; Leonard, 1995). For underactuated vehicles in particular, the problem of control system design continues to pose considerable challenges. However, current research goes well beyond single vehicle control. In a great number of mission scenarios, multiple marine vehicles must work in cooperation. Considerable effort is being placed on the deployment of groups of networked autonomous marine vehicles (AMVs) which can interact autonomously with the environment and other vehicles, resulting in a significant improvement in efficiency, performance, reconfigurability and robustness, and in the emergence of new capabilities beyond the ones of individual vehicles. Motivated by the above considerations, the problem of coordinated or cooperative path-following (CPF) control has recently come to the forum (Aguiar and Hespanha, 2007; Aguiar *et al.*, 2007; Ghabcheloo *et al.*, 2007; Ihle *et al.*, 2006; Skjetne *et al.*, 2002, 2004). Different approaches to the solution of this and similar problems have been reported in the literature. They share a common strategy in that the problem of CFP is partially decoupled into two: *i) path-following*, where the objective is to find local closed loop control laws to steer each vehicle to its path at a reference speed, and *ii) multiple vehicle coordination*, where the goal is to adjust the reference speeds of the vehicles about the desired formation speed, so as to reach formation.

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In this paper we derive new nonlinear motion control strategies for single and multiple underactuated vehicles, building on the work reported in (Vanni, 2007). The solutions adopted are rooted in Lyapunov-based theory and address explicitly the vehicle dynamics as well as the constraints imposed by the topology of the inter-vehicle communications network. This is particularly relevant in the case of underwater applications, where vehicles exchange information over low bandwidth, short range communication channels that are plagued with intermittent failures, multi-path effects, and distance-dependent delays.

The path-following problem is solved individually for every vehicle, each having access to a set of local measurements, by introducing a "virtual target" that moves along the desired path. We control explicitly the rate of progression of the virtual target and derive inner-outer-loop control laws that make the vehicle track the virtual target. Coordination is then achieved by synchronizing the parametrization states that capture the positions of the virtual targets. To cope with asynchronous, discrete-time communications we exploit the techniques proposed in (Aguiar and Pascoal, 2007*a*), and expand the circle of ideas advanced in (Xu and Hespanha, 2006; Yook *et al.*, 2002), where decentralized controllers for a distributed system were derived by using, for each system, its local state information together with estimates of the states of the systems that it communicates with. To minimize the requirements of inter-vehicle data exchange we include a logic-based communication strategy that borrows from the results in (Aguiar and Pascoal, 2007*a*).

A subset of the results reported here was presented in (Vanni *et al.*, 2007). Due to space limitations, all the proofs are omitted. These can be found in (Vanni *et al.*, 2008).

In the following, we define $a \oplus b := \max\{a, b\}$, and use the standard definition of class $\mathcal{K}$, $\mathcal{K}_\infty$ and $\mathcal{KL}$ functions (see for example Khalil (2002)).

2. PROBLEM STATEMENT

In this section we describe the mathematical model of a class of AMVs used for motion control design and formulate the cooperative path-following problem.

2.1 Vehicle model

The kinematic equations of motion of a vehicle moving in the horizontal plane can be developed using a global inertial coordinate frame $\{\mathcal{U}\}$ and a body-fixed coordinate frame $\{\mathcal{B}\}$, the origin of which coincides with the vehicle's center of mass, yielding

$$\dot{x} = u\cos(\psi) - v\sin(\psi), \tag{1a}$$
$$\dot{y} = u\sin(\psi) + v\cos(\psi), \tag{1b}$$
$$\dot{\psi} = r, \tag{1c}$$

where u (surge speed) and v (sway speed) are the body-fixed frame components of the vehicle's velocity, x and y are the inertial cartesian coordinates of its center of mass, ψ defines its orientation (heading angle), and r is the vehicle's angular speed. Defining $\boldsymbol{p} := [x\ y]'$ and $\boldsymbol{v} := [u\ v]'$, equations (1a)-(1b) can be written in compact form as

$$\dot{\boldsymbol{p}} = R(\psi)\boldsymbol{v}$$

where $R(\psi)$ is the orthonormal transformation matrix from $\{\mathcal{B}\}$ to $\{\mathcal{U}\}$. In the presence of a constant and irrotational ocean current, $\boldsymbol{v}$ is the sum of the vehicle's velocity with respect to the water $\boldsymbol{v}_r := [u_r\ v_r]'$ and the water current velocity $\boldsymbol{v}_c := [u_c\ v_c]'$, both expressed in the body-fixed reference frame.

Neglecting the motions in heave, roll, and pitch, the dynamic equations of motion of an underactuated marine vehicle for surge, sway and heading yield, under simplifying assumptions (Aguiar, 1996),

$$m_u\dot{u}_r - m_v v_r r + d_{u_r}u_r = \tau_u, \tag{2a}$$
$$m_v\dot{v}_r + m_u u_r r + d_{v_r}v_r = 0, \tag{2b}$$
$$m_r\dot{r} - m_{uv}u_r v_r + d_r r = \tau_r, \tag{2c}$$

where $m_u := m - X_{\dot{u}}$, $m_v := m - Y_{\dot{v}}$, $m_r := I_z - N_{\dot{r}}$, $m_{uv} := m_u - m_v$ are mass and hydrodynamic added mass terms, and $d_{u_r}(u_r) := -X_u - X_{|u|u}|u_r|$, $d_{v_r}(v_r) := -Y_v - Y_{|v|v}|v_r|$, $d_r(r) := -N_r - N_{|r|r}|r|$ capture hydrodynamic damping effects. The symbols τ_u and τ_r denote respectively the external force in surge and the external torque about the vertical axis of the vehicle.

2.2 Path-following

To solve the problem of driving a vehicle along a desired path, the key idea exploited here is to make the vehicle approach a virtual target that moves along the path with a desired timing law. Let $\boldsymbol{p}_d$ be the position of the target, and v_d its desired rate of progression. We decompose the motion-control problem into an inner-loop dynamic task, which consists of making the vehicle's actuated velocities u_r and r track a desired speed reference $\boldsymbol{u}_d := [u_d, r_d]'$, and an outer-loop kinematic task, which *i)* regulates the evolution of the virtual target and *ii)* assigns the reference speed so as to achieve convergence to the path. This decomposition is motivated by the fact that *it is common for autonomous vehicles to be equipped with an inner-loop controller for tracking a speed reference.* In principle, better results could be achieved with control laws designed directly for τ_u and τ_r, based on both the dynamics and the kinematics of the vehicle motion. However, the approach proposed here results in greater portability, since the same outer-loop controller could be run on a wide range of vehicles, regardless of the parameters that define their dynamics.

In what follows we assume that the inner-loop controller satisfies the following stability property:

Property 1. Let $\tilde{u} := u - u_d$ and $\tilde{r} := r - r_d$ be the speed errors and $\boldsymbol{x}_{il}^0$ the initial condition of the state of the inner-loop system. There exist functions $\beta^{\tilde{u}}, \beta^{\tilde{r}} \in \mathcal{KL}$ and positive constants $\epsilon_{\tilde{u}}, \epsilon_{\tilde{r}}$ such that

$$|\tilde{u}(t)| \leq \beta^{\tilde{u}}(\|\boldsymbol{x}_{il}^0\|, t) \oplus \epsilon_{\tilde{u}}\ \ |\tilde{r}(t)| \leq \beta^{\tilde{r}}(\|\boldsymbol{x}_{il}^0\|, t) \oplus \epsilon_{\tilde{r}}.$$

In addition, we also introduce the following constraint.

Property 2. The measurements of sway velocity v_r and the ocean current $\boldsymbol{v}_c$ are not available, as the sensors required are expensive.

The path-following problem for the outer loop can be formulated as follows:

Problem 1. (Path-following). Consider the AMV whose motion is described by (1) and (2), and let $\boldsymbol{p}_d(\gamma) \in \mathbb{R}^2$ be a desired path parameterized by a continuous variable $\gamma \in \mathbb{R}$ and $v_d(\gamma) \in \mathbb{R}$ a desired speed assignment. Suppose that $\boldsymbol{p}_d(\gamma)$ is sufficiently smooth and its derivatives with respect to γ are bounded. Derive control laws, subject to Properties 1 and 2, for $\boldsymbol{u}_d$ and $\dot{\gamma}$, such that the position error $\|\boldsymbol{p}(t) - \boldsymbol{p}_d(\gamma(t))\|$ and the speed error $|\dot{\gamma}(t) - v_d(\gamma(t))|$ converge to a small neighborhood of the origin as $t \to \infty$.

Notice that the speed v_d is not an actual velocity: it expresses the rate at which the parameter γ changes.

2.3 Coordination

Consider now a group of vehicles $\mathcal{I} := \{1, \ldots, n\}$, each with its own parametrized path $\boldsymbol{p}_{d_i}(\gamma_i)$, $i \in \mathcal{I}$. To achieve coordination among the elements of the group, the paths have to be designed conveniently and a common speed profile $\bar{v}_d(\gamma)$ has to be assigned to all the paths, so that the vehicles move along them while holding a desired, possibly varying, inter-vehicle formation pattern. The parameter γ of each vehicle can be seen as a coordination state such that coordination exists between two vehicles i and j if and only if $\gamma_i(t) = \gamma_j(t)$. The key idea in designing the coordination controller is to introduce a control variable in the form of a correction term $\tilde{v}_d$ that is added to the desired speed of each vehicle, yielding

$$v_d = \bar{v}_d(\gamma) + \tilde{v}_d. \tag{3}$$

The approach pursued here is a decentralized one, that takes into consideration the existing communication constraints: the correction speed $\tilde{v}_{d_i}$ is determined based on the information available to vehicle i only, that is, on the coordination states of the vehicles that communicate with i. Let $\boldsymbol{\gamma} := [\gamma_1, \ldots, \gamma_n]'$ be the vector containing the coordination states of the n vehicles, and let $\mathcal{N}_i$ denote the set of vehicles that vehicle i exchanges information with or, in the case of unidirectional communication, receives information from. The coordination problem can be formulated as follows (Aguiar and Pascoal, 2007*a*):

Problem 2. (Coordination). For each vehicle $i \in \mathcal{I}$ derive a control law for the correction speed $\tilde{v}_{d_i}$ as a function of γ_i and γ_j, with $j \in \mathcal{N}_i$, such that, as $t \to \infty$, for all $i, j \in \mathcal{I}$ the coordination error $\gamma_i - \gamma_j$ approaches zero, and the formation travels at an assigned speed $\bar{v}_d$, that is, $\gamma_i \to \bar{v}_d\ \forall i \in \mathcal{I}$.

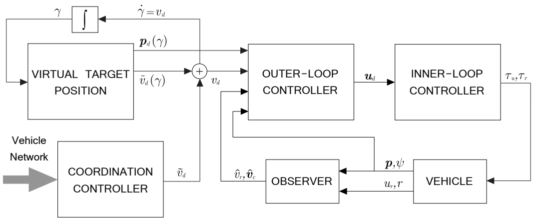

Fig. 1. Structure of the PF controller (Strategy I).

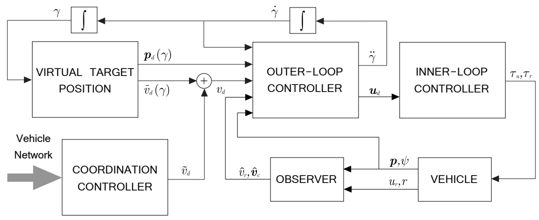

Fig. 2. Structure of the PF controller (Strategy II).

3. PATH-FOLLOWING CONTROL DESIGN

In this section we propose two approaches to the path-following problem stated in Section 2. The difference between the two strategies lies in the way the virtual target that the vehicle has to track follows the path. In the first strategy the speed of the virtual target depends only on its position along the trajectory; in the second strategy it also depends on the position of the vehicle, that is, the motion of the virtual target is regulated so as to "help" the vehicle to track it.

Define the position error $\boldsymbol{e}$ expressed in body-frame coordinates as the difference between the positions of the vehicle and of the virtual target,

$$\boldsymbol{e} := R'(\boldsymbol{p}(t) - \boldsymbol{p}_d(\gamma(t))).$$

Its dynamics are described by

$$\dot{\boldsymbol{e}} = -S(r)\boldsymbol{e} + \boldsymbol{v}_r + \boldsymbol{v}_c - R'\dot{\boldsymbol{p}}_d(\gamma), \qquad S(r) = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}. \quad (4)$$

We design an observer to take into account the fact that the sway velocity v_r and the water current speed are unknown signals. Let $\boldsymbol{v}_{c_{xy}}$ denote the ocean current expressed in the inertial reference frame $\{\mathcal{U}\}$, so that

$$\dot{\boldsymbol{p}} = R\boldsymbol{v}_r + \boldsymbol{v}_{c_{xy}} \quad (5)$$

Using (2b) and (5) we define the following estimation dynamics

$$\dot{\hat{\boldsymbol{p}}} = R\begin{bmatrix} u_r \\ \hat{v}_r \end{bmatrix} + \hat{\boldsymbol{v}}_{c_{xy}} + K_p(\boldsymbol{p} - \hat{\boldsymbol{p}}) \quad (6a)$$

$$\dot{\hat{\boldsymbol{v}}}_{c_{xy}} = K_c(\boldsymbol{p} - \hat{\boldsymbol{p}}) \quad (6b)$$

$$\dot{\hat{v}}_r = -\frac{m_u}{m_v}u_r r - \frac{d_v(\hat{v}_r)}{m_v}\hat{v}_r \quad (6c)$$

where K_p and K_c are the observer gain diagonal matrices. The water current velocity estimate, expressed in the body-fixed reference frame $\{\mathcal{B}\}$, is $\hat{\boldsymbol{v}}_c = R'\boldsymbol{v}_{c_{xy}}$.

To make the desired speed $\boldsymbol{u}_d := [u_d \; r_d]'$ appear in the position error dynamics we let $\tilde{\boldsymbol{u}} := [\tilde{u} \; \tilde{r}]' := [u - u_d \;\; r - r_d]'$ be the speed error and introduce a constant design vector $\boldsymbol{\delta} := [\delta, 0]'$, $\delta < 0$. Following from (4), simple computations show that the position error dynamics are then given by

$$\dot{\boldsymbol{e}} = -S(r)(\boldsymbol{e}-\boldsymbol{\delta}) + \Delta(\boldsymbol{u}_d + \tilde{\boldsymbol{u}}) + \begin{bmatrix} 0 \\ \hat{v}_r + \tilde{v}_r \end{bmatrix} + \hat{\boldsymbol{v}}_c + \tilde{\boldsymbol{v}}_c - R'\dot{\boldsymbol{p}}_d(\gamma)$$

where $\Delta := \begin{bmatrix} 1 & 0 \\ 0 & -\delta \end{bmatrix}$.

3.1 PF Strategy I

The time derivative of the position of the virtual target is

$$\dot{\boldsymbol{p}}_d(\gamma) = \frac{\partial \boldsymbol{p}_d(\gamma)}{\partial \gamma}\dot{\gamma}.$$

The first strategy we adopt is to force the virtual target to move at the desired speed, by assigning

$$\dot{\gamma} = v_d(\gamma).$$

The resulting control scheme is depicted in Fig. 1. We are now ready to state the following result.

Proposition 3. Consider the vehicle model described by (1) and (2) in closed-loop with the output feedback control law composed by an inner loop that satisfies Property 1, the estimator (6), and the outer loop given by

$$\dot{\gamma} = \bar{v}_d(\gamma) + \tilde{v}_d \quad (7)$$

$$\boldsymbol{u}_d = \Delta^{-1}\left(-K_k \tanh(\boldsymbol{e} - \boldsymbol{\delta}) - \begin{bmatrix} 0 \\ \hat{v}_r \end{bmatrix} - \hat{\boldsymbol{v}}_c + R'\frac{\partial \boldsymbol{p}_d(\gamma)}{\partial \gamma}v_d\right) \quad (8)$$

where $K_k := \mathrm{diag}\{k_x, k_y\}$ satisfies $k_x > \varepsilon_{\tilde{u}}$, $k_y > |\delta|\varepsilon_{\tilde{r}}$. Then, the path-following error $\boldsymbol{e} - \boldsymbol{\delta}$ is input-to-output stable (IOS) with respect to $\tilde{\boldsymbol{u}}$ and $\tilde{\boldsymbol{v}}_c$, that is, there exist functions $\sigma^e_{\tilde{u}}, \sigma^e_{v_c} \in \mathcal{K}_\infty$, and $\beta^e \in \mathcal{KL}$ such that

$$\|\boldsymbol{e} - \boldsymbol{\delta}\| \le \beta^e(\|\chi^0_e\|, t) \oplus \sigma^e_{\tilde{u}}(\|\tilde{\boldsymbol{u}}\|_{[0,t]}) \oplus \sigma^e_{v_c}(\|\tilde{\boldsymbol{v}}_c\|_{[0,t]}). \quad \Box$$

The proof is based on the Lyapunov function $V = \|\boldsymbol{e}\|^2$. In (8) we have chosen to introduce the nonlinear term $\tanh(\cdot)$ in the outer-loop control law to enforce that the desired velocity should increase with the position error only up to a maximum value. If the distance between the positions of the vehicle and the virtual target is greater than this limit, the vehicle should approach the desired position at a constant velocity, until the distance becomes smaller than the limit and the velocity begins to decrease.

3.2 PF Strategy II

The second strategy we propose borrows from the technique of backstepping. Defining the virtual target speed error

$$z := \dot{\gamma} - v_d(\gamma) = \dot{\gamma} - \bar{v}_d - \tilde{v}_d \quad (9)$$

and computing its time derivative yields $\dot{z} = \ddot{\gamma} - \dot{\bar{v}}_d - \dot{\tilde{v}}_d$. By explicitly controlling $\ddot{\gamma}$ we introduce an additional control variable, as illustrated in Fig. 2.

Proposition 4. Consider the vehicle model described by (1) and (2) in closed-loop with the output feedback control law composed by an inner loop that satisfies Property 1, the estimator (6), and the outer loop given by

$$\ddot{\gamma} = -k_z(\dot{\gamma} - \bar{v}_d) + \dot{\bar{v}}_d(\gamma) + (\boldsymbol{e} - \boldsymbol{\delta})'R'\frac{\partial \boldsymbol{p}_d(\gamma)}{\partial \gamma} + k_z\tilde{v}_d \quad (10)$$

$$\boldsymbol{u}_d = \Delta^{-1}\left(-K_k \tanh(\boldsymbol{e} - \boldsymbol{\delta}) - \begin{bmatrix} 0 \\ \hat{v}_r \end{bmatrix} - \hat{\boldsymbol{v}}_c + R'\frac{\partial \boldsymbol{p}_d(\gamma)}{\partial \gamma}v_d\right) \quad (11)$$

where $k_z > 0$ and $K_k := \mathrm{diag}\{k_x, k_y\}$ satisfies $k_x > \varepsilon_{\tilde{u}}$, $k_y > |\delta|\varepsilon_{\tilde{r}}$. Then, the path-following error $\boldsymbol{e} - \boldsymbol{\delta}$ is input-to-output stable (IOS) with respect to $\tilde{\boldsymbol{u}}$, $\tilde{\boldsymbol{v}}_c$ and $\dot{\tilde{v}}_d$, that is, there exist functions $\sigma^e_{\tilde{u}}, \sigma^e_{v_c}, \sigma^e_{\tilde{v}_d} \in \mathcal{K}_\infty$ and $\beta^e \in \mathcal{KL}$ such that

$$\|\boldsymbol{e}-\boldsymbol{\delta}\| \le \beta^e(\|\chi_e^0\|, t) \oplus \sigma_{\tilde{u}}^e(\|\tilde{\boldsymbol{u}}\|_{[0,t]}) \oplus \sigma_{v_c}^e(\|\tilde{\boldsymbol{v}}_c\|_{[0,t]}) \oplus \sigma_{v_d}^e(|\dot{\tilde{v}}_d|_{[0,t]}). \square$$

The proof is based on the Lyapunov function $V = \|\boldsymbol{e}\|^2 + z^2$. The difference from the first path-following strategy is that the evolution of the position of the virtual target $\boldsymbol{p}_d$ also depends on the position error $(\boldsymbol{e}-\boldsymbol{\delta})$, so if the vehicle is ahead/behind the desired position the virtual target moves faster/slower.

4. COORDINATION CONTROLLER

In this section we develop a decentralized control law to solve the coordination problem. We start by assuming that the communications take place continuously, and then tackle the situation where they take place at discrete instants of time and are affected by time delays.

To describe the communication topology it is a natural choice to resort to graph theory. The vehicles in a formation are the vertices of a graph of which the existing communication links are the edges (undirected, as we assume that communications are bidirectional). Letting A and D denote respectively the adjacency matrix and the degree matrix associated with the graph that describes the communication network, we can define an error vector

$$\boldsymbol{\xi} = L\boldsymbol{\gamma}, \tag{12}$$

where $L = D^{-1}(D-A)$ is the normalized graph Laplacian. The i-th element of vector $\boldsymbol{\xi}$ is

$$\xi_i = \gamma_i - \frac{1}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i}\gamma_j,$$

that is, the sum of the coordination errors between vehicle i and the vehicles that communicate with it. The single variable ξ_i captures the communication constraints of the network and can be used for control purposes. The assumption is made that the communication topology does not change in time, *i.e.*, the Laplacian is constant. By construction, $L \ge 0$, $\|L\| \le 2$ and $L\boldsymbol{x} = \boldsymbol{0}$ iff $\boldsymbol{x} = \boldsymbol{1}$ (Godsil and Royle, 2001).

4.1 Continuous communication

Let $\bar{\boldsymbol{v}}_d$, $\tilde{\boldsymbol{v}}_d$ and $\boldsymbol{z}$ denote the vectors containing the desired speed, the correction term, and the virtual target speed error, respectively, for each vehicle in the formation.

Proposition 5. Consider a formation of n vehicles and let L be the normalized Laplacian of a graph that describes the inter-vehicle communications network, with L_2 its smallest non-null eigenvalue. Let the desired formation speed $\bar{v}_d(\gamma)$ be a Lipschitz function with Lipschitz constant κ, and assume that any two neighboring vehicles communicate in a continuous manner. The correction terms for the vehicles in the formation are assigned through the decentralized control law

$$\tilde{\boldsymbol{v}}_d = -k_\xi L\boldsymbol{\gamma} \tag{13}$$

where $k_\xi > \frac{\kappa}{L_2}$. Then, for strategy I the coordination error $\boldsymbol{\xi}$ converges to $\boldsymbol{0}$ and $\boldsymbol{v}_d \to \bar{\boldsymbol{v}}_d$ as $t \to \infty$. For strategy II, $\boldsymbol{\xi}$ is ISS with respect to $\boldsymbol{z}$ and $\|\dot{\boldsymbol{\gamma}} - \bar{\boldsymbol{v}}_d\|$ is detectable through $\boldsymbol{z}$, that is, there exist functions $\beta^\xi \in \mathcal{KL}$ and $\sigma_z^\xi, \sigma_z^\gamma, \in \mathcal{K}_\infty$ such that

$$\|\boldsymbol{\xi}\| \le \beta^\xi(\|\chi_\xi^0\|, t) \oplus \sigma_z^\xi(\|\boldsymbol{z}\|_{[0,t]})$$
$$\|\dot{\boldsymbol{\gamma}} - \bar{\boldsymbol{v}}_d\| \le \sigma_z^\gamma(\|\boldsymbol{z}\|_{[0,t]}). \square$$

The proof is based on the Lyapunov function $V = \boldsymbol{\xi}' L \boldsymbol{\xi}$.

4.2 Discrete communication

The coordination controller designed in Section 4.1 relies on the continuous exchange of information among the vehicles in the formation. This assumption is unrealistic because underwater communication systems are characterized by low bandwidths and require that the exchange of data take place at discrete instants of time. In (Aguiar and Pascoal, 2007*a*), a logic-based communications strategy is proposed that takes into account both the fact that communications do not occur in a continuous manner and the cost of exchanging information. In between communications, that are regulated by a supervisory logic, each vehicle runs estimations of the coordination states of the rest of the formation. This is done through synchronized estimation blocks, identical for every vehicle, that admit the following dynamics, based on (3) and (13):

$$\dot{\hat{\boldsymbol{\gamma}}} = \bar{\boldsymbol{v}}_d(\hat{\boldsymbol{\gamma}}) - k_\xi\left(L\hat{\boldsymbol{\gamma}}\right). \tag{14}$$

In particular, every vehicle runs an estimate of its own state. It is by comparing the actual value of its state with this estimate that a vehicle decides when to communicate with the vehicles in its neighborhood. If, at a certain instant t_k, $|\gamma_i - \hat{\gamma}_i| \ge \epsilon^2$, then vehicle i broadcasts the value of γ_i. Assuming that no delays affect the communication links, each vehicle updates its estimate instantly, so that

$$\hat{\boldsymbol{\gamma}}_i(t_k) = \boldsymbol{\gamma}_i(t_k).$$

Remembering the expression of the normalized Laplacian, the control law (13) becomes

$$\tilde{\boldsymbol{v}}_d = -k_\xi\left(\boldsymbol{\gamma} - D^{-1}A\hat{\boldsymbol{\gamma}}\right), \tag{15}$$

where it has been explicited that the correction term for every vehicle is the sum of a term that depends on the coordination state of the vehicle itself, which is available at every instant, and a term built on the estimates of the states of the other vehicles. The control law (15) can be rewritten as

$$\tilde{\boldsymbol{v}}_d = -k_\xi L\gamma - k_\xi D^{-1}A\tilde{\boldsymbol{\gamma}}.$$

In the instants between communication, the estimation error $\tilde{\boldsymbol{\gamma}} = \boldsymbol{\gamma} - \hat{\boldsymbol{\gamma}}$ acts as a perturbation input, so $\boldsymbol{\xi}$ is ISS with respect to $\boldsymbol{z}$ and $\tilde{\boldsymbol{\gamma}}$, which is bounded by ϵ^2. Selecting a lower tolerance ϵ^2 reduces the neighborhood of the origin to which $\boldsymbol{\xi}$ converges but increases the number of messages exchanged among vehicles.

4.3 Time-delays

Assume that at time t vehicle i broadcasts its coordination state. Vehicles j and k will receive the message at $t+t_j$ and $t+t_k$ respectively. The communication strategy must be modified, taking into account the network topology, and the update must take place so as to keep the estimators synchronized. We consider the case of time-varying and nonhomogeneous delays that are not known a priori, and assume that all the agents have synchronized clocks. Thus, each agent can compute the time-delay when the timetagged data arrives.

A solution, proposed in (Aguiar and Pascoal, 2007*a*), requires for each vehicle to be equipped with as many independent estimation blocks as the number of its neighbors. A single vehicle runs $|\mathcal{N}_i|$ different estimates of $\boldsymbol{\gamma}$. Let $\boldsymbol{\gamma}_i^{jk}$ denote the estimate of $\boldsymbol{\gamma}_i$ run by vehicle j on the estimator associated with the link between j and k. If, at a certain instant t_k, $|\boldsymbol{\gamma}_i - \hat{\boldsymbol{\gamma}}_i^{ij}| \ge \epsilon^2$, then vehicle i sends a message containing the actual value $\boldsymbol{\gamma}_i$ and the time t_k to vehicle j. Vehicle j receives the message at $t_k+\tau$ but does not update its estimate of $\boldsymbol{\gamma}_i$ instantly. Instead, it sends a "received" message back to i, and only updates its estimate at $t_k+2\tau$, while i does the same upon reception of the reply. This strategy is based on the assumption that the delay on a communication channel is the same in both directions. A small difference $\tilde{\tau}$ however will always be present, so an error exists between the estimates of the two vehicles over the same link.

An alternative update logic is based on a statistical evaluation of the time τ_{max} that is required for the message to be sent and for the answer to be received. Assume that at time t_k vehicle i communicates $\boldsymbol{\gamma}_i$ to j. If i receives an answer from j before $t_k+\tau_{max}$ then both vehicles update their estimates at the instant $t_k+\tau_{max}$. If instead the answer is not received within the time limit, the message is considered lost and a new message is sent, with the up-to-date value of $\boldsymbol{\gamma}_i$. In the event that j receives the message but i does not receive the reply, only j makes an update. Vehicle i however sends immediately a new message. The limit τ_{max} has to be choosen so that the probability of a message being lost is lower than a defined margin.

5. COORDINATED PATH-FOLLOWING

We now state the main result of this work, that shows that the proposed decentralized control law architecture solves the coordinated path-following problem and exhibits some strongly desirable stability properties.

Theorem 6. Consider the overall closed-loop system composed by

(1) a formation of n vehicles, whose motion is described by (1) and (2), guided along paths $\boldsymbol{p}_{d_i}(\gamma_i)$, $i=1,\ldots,n$, where each vehicle uses an inner loop that satisfies Property 1, the estimator (6), and the motion control laws (7) and (8) (or (10) and (11));
(2) the coordination law (15) with the estimator (14), and
(3) the logic-based communication system described in Section 4.

Suppose that in the presence of time-delays these are bounded, so that the post-reset value of $\tilde{\gamma}_i$ is less than ϵ^2. Then, the control system proposed solves the coordinated path-following problem, that is, Problem 1 and Problem 2. More precisely, all the states of the closed-loop system with the exception of γ are bounded, the control signals for the inner-loop are bounded, the coordination error $\boldsymbol{\xi}$ and the speed error $\dot{\boldsymbol{\gamma}}-\bar{\boldsymbol{v}}_d$ are IOS with respect to ϵ^2, and for each vehicle i the path-following error $\boldsymbol{e}_i-\boldsymbol{\delta}$ is IOS with respect to $\tilde{\boldsymbol{u}}_i$ and $\tilde{\boldsymbol{v}}_{c_i}$, that is, there exist functions $\sigma^{\xi}_{\epsilon},\sigma^{\gamma}_{\epsilon},\sigma^{e}_{\tilde{u}},\sigma^{e}_{v_c}\in\mathcal{K}_\infty$, and $\beta^{\xi},\beta^{\gamma},\beta^{e}\in\mathcal{KL}$ such that

$$\|\boldsymbol{\xi}\|\le\beta^{\xi}(\|\chi^0_{\xi}\|,t)\oplus\sigma^{\xi}_{\epsilon}(\epsilon^2),$$
$$\|\dot{\boldsymbol{\gamma}}-\bar{\boldsymbol{v}}_d\|\le\beta^{\gamma}(\|\chi^0_{\gamma}\|,t)\oplus\sigma^{\gamma}_{\epsilon}(\epsilon^2),$$
$$\|\boldsymbol{e}-\boldsymbol{\delta}\|\le\beta^{e}(\|\chi^0_{e}\|,t)\oplus\sigma^{e}_{\tilde{u}}(\|\tilde{\boldsymbol{u}}\|_{[0,t]})\oplus\sigma^{e}_{v_c}(\|\tilde{\boldsymbol{v}}_c\|_{[0,t]}).\quad\square$$

6. SIMULATION RESULTS

To illustrate the performance of the CPF control strategies proposed, we consider a group of three underactuated autonomous underwater vehicles (AUVs) that are required to follow a lawn mower path (a typical trajectory in ocean exploration scenarios) while maintaining a formation pattern that consists of having them aligned along a horizontal line perpendicular to the paths. AUV 1 is allowed to communicate with AUVs 2 and 3, but the latter two do not communicate between themselves directly. We adopted the following inner-loop control laws for surge speed and yaw rate,

$$\tau_u=-k_u(u_r-u_d)+d_{u_r}(u_d)u_d-m_v\hat{v}_r r+m_u\dot{u}_d,$$
$$\tau_r=-k_r(r-r_d)+d_r(r_d)r_d-m_{uv}u_r\hat{v}_r+m_r\dot{r}_d,$$

which make the inner-loop error $\tilde{\boldsymbol{u}}$ converge to zero. The numerical values used for the physical parameters match those of the Sirene underwater shuttle described in (Aguiar, 1996). To test the robustness of the proposed control algorithm we introduced noise in every sensed signal (the x and y positions, the orientation angle ψ, the linear velocity u and the angular velocity r) and thruster saturation. Figure 3 shows the trajectories of the AUVs in the presence of a constant ocean current (which is unknown from the point of view of the controller). When strategy I is adopted coordination between the three vehicles is achieved when, at the end of the first straight line, each of them reaches its virtual target (Fig. 3a), the speed of which is statically assigned *a priori*. With strategy II, however, the movement of each virtual target is regulated by the path-following controller of the corresponding vehicle, that makes it slow down if the vehicle is behind it, and by the coordination controller, that tries to maintain the virtual targets synchronized. For this reason, the virtual targets move slower at the beginning of the trajectory, to allow the vehicles to reach them, and then reach the desired speed. The coordination between the true vehicles is thus achieved earlier along the trajectory, as illustrated in Fig. 3b. Furthermore, when the path is curved and the AUVs are required to move at different speeds to maintain formation, strategy II ensures that the virtual targets move so that all the true vehicles, including the one that has to move faster, are able to follow the desired trajectory while maintaining formation.

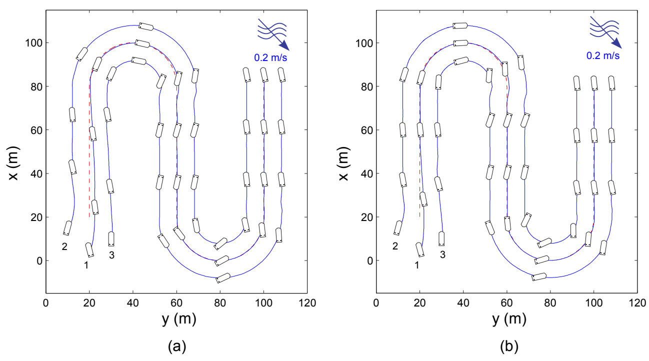

Fig. 3. Trajectories of the three vehicles in the xy plane with strategy I (a) and strategy II (b).

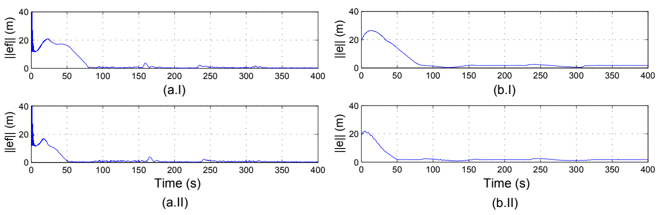

Fig. 4. Evolution of the formation error (a) and of the position error (b) for the two path-following strategies.

To measure of how well the vehicles maintain the desired configuration, independently of how they follow the as-

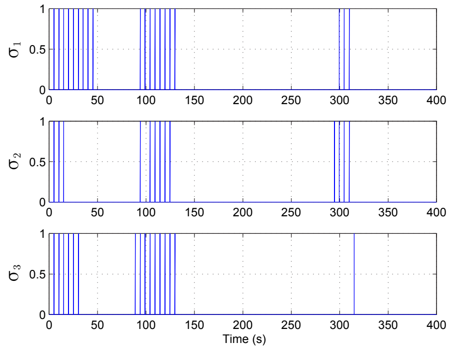

Fig. 5. Communication signal for the three vehicles along the paths (strategy II). Each impulse corresponds to a transmission.

signed path, we introduce a measure called formation error (e_{f_i}), defined as the distance of the position of vehicle i from a virtual point $\boldsymbol{p}_{v_i} = [p_{v_{yi}},\ p_{v_{yi}}]'$, that is,

$$e_{f_i} = \boldsymbol{p}_i - \boldsymbol{p}_{v_i}.$$

These virtual points represent the desired formation (in this case a line) that moves with the center of mass of the vehicles. Figure 4a clearly shows that strategy II (a.II) makes this error smaller than strategy I (a.I). In Figure 4b we compare the sum of the position error (the distance between the vehicle and the virtual target) of the three vehicles, for strategy I (b.I) and strategy II (b.II). The position errors are smaller with strategy II, since the movement of the virtual targets takes into account their distance from the vehicles. Figure 5 shows the communications among the vehicles when strategy II is adopted. Signal $\sigma = \{0, 1\}$ indicates, by switching its value, when a vehicle communicates with the others. Notice the reduced number of communications in the overall period. The need for communication arises, in order to keep the coordination error bounded, at the beginning of the trajectory (when the distances of the vehicles from the virtual targets are large), and along the curved portions of the paths.

7. CONCLUSIONS

The paper addressed the problem of cooperative path-following for a group of underactuated autonomous marine vehicles, moving in the horizontal plane and in the presence of constant unknown ocean current disturbances. The solutions proposed are valid for a large class of underactuated marine vehicles.

The path-following strategies are based on an inner-outer structure. The output feedback control laws designed for the outer loop embody in themselves an observer for the current and for the sway velocity. Coordination is obtained through a decentralized control law that takes into account the constraints imposed by the topology of the inter-vehicle communications network and the presence of time-delays, and requires reduced exchange of data among the vehicles.

Simulations with a nonlinear model of a representative AUV showed the efficacy of the control laws proposed. The simulations also indicate that the control laws yield good performance in the presence of actuator saturation and measurement noise. The impact of sensor noise on system performance can be further alleviated by using state filter estimators. A rigorous analysis of this issue is a topic for future research.

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