Designing Decentralized Control Systems without Structural Fixed Modes: A Multilayer Approach ⋆

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Abstract: In this paper we propose a methodology to design decentralized controllers applied to large-scale systems. The key idea is to split the design into two control layers. The first layer, consists mainly in a pre-processing step, where an optimal subset of inputs and outputs are used for feedback to close the loop, such that the resulting associated state matrix of the closed-loop system has two desirable structural properties: it has no structural fixed modes and it is structurally controllable and observable thru any single state variable. After this first layer, we can select an arbitrary decentralization control scheme to achieve some specified performance, which is the second layer of the proposed approach. We illustrate this methodology for the following applications: a two zones temperature regulation and a frequency regulation of a 3-bus system.

Keywords: Structural Systems, Structural Fixed Modes, Decentralized Control

1. INTRODUCTION

In geographically distributed large-scale systems such as power systems, multi-agent networks, cyber-physical systems, to name a few, a decentralized control structure is often more desirable than a centralized one. This is due the fact that typically, it is not realistic to assume that each control input signal can be generated by using all the measurement signals of the system. In other words, some kind of constraint on the information structure is inevitable. Decentralized control theory has attracted several researchers in the past decades, and several methodologies have been proposed, see for instance Sandell et al. (1978); Bakule and Lunze (1988); Siljak (1991, 2007).

In this paper we consider a possible large scale plant modeled by

$$\dot{x} = Ax,$$

where $x \in \mathbb{R}^n$ is the state of the system. The goal is to design the input matrix structure $\bar{B}$, the output matrix structure $\bar{C}$ and a constrained information pattern $\bar{K}$ such that typical decentralized control methods hold. To this end, we propose a multilayer approach that is composed by two layers. The first layer of our approach consists in the following

L1 Consider the structural pattern $\bar{A}$ of A (i.e., the zero/non-zero pattern). In this first layer, the goal is to find the structural matrices $(\bar{B}_1, \bar{K}_1, \bar{C}_1)$ such that the associated state matrix of the closed-loop system given by

$$A_1 = A + B_1K_1C_1,$$

with a numerical realization [1] (B_1, K_1, C_1) with the same structural pattern as $(\bar{B}_1, \bar{K}_1, \bar{C}_1)$ satisfies the following: the closed-loop system A_1 composed by $(\bar{A}, \bar{B}_1, \bar{K}_1, \bar{C}_1)$ has no structural fixed modes (to be made precise in Section II) and is such that it is structurally controllable [2] (resp. observable) by manipulating (resp. measuring) any single state variable.

Note that at the end of this first control layer, we obtain a matrix A_1 that (by construction) satisfies the commonly required conditions to perform decentralized control once we consider an arbitrary set of inputs, outputs and feedback links. Hence, we can design a controller to achieve some performance. This is done in the next control layer.

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[1] Note that the parameters used by (B_1, C_1) are imposed by physical constraints or modelling assumptions, whereas K_1 is determined by a parametric choice of our interest.

[2] A pair (A, B) is said to be structurally controllable if there exists a pair (A', B') with the same structure as (A, B), i.e., same locations of zeroes and non-zeroes, such that (A', B') is controllable. By density arguments, it may be shown that if a pair (A, B) is structurally controllable, then almost all (with respect to the Lebesgue measure) pairs with the same structure as (A, B) are controllable. In essence, structural controllability is a property of the structure of the pair (A, B) and not the specific numerical values. A similar definition and characterization holds for structural observability (with obvious modifications).

L2 Consider some non-zero pair of matrices (B_2, C_2) (with no columns with all entries equal to zero) and an arbitrary desired (non-zero) information pattern $\bar{K}_2$. Then, find a realization K_2 with the same sparseness as $\bar{K}_2$ such that a pre-defined goal/performance is achieved.

Note that in many cases the performance criteria may depend on the number of actuators and sensors used in the decentralization process. Additionally, through the second layer we can easily ensure robustness properties, i.e., since the system is structurally controllable and observable thru any single state variable, any pairs sensor-actuator ensures that a decentralization scheme is still possible.

The proposed methodology overcomes some of the issues raised by the input-output (IO) decomposition described in Pichai et al. (1983) (see also Sezer and Siljak (1981b); Siljak (1991)), which is inherently a NP-complete problem. In the IO decomposition, first we need to find a collection of decompositions comprising inputs, outputs and state variables. Second, we need to verify if such decomposition leads to a structurally controllable and observable system. If such condition is not satisfied, then we need to redesign the system by experimenting different combinations of decompositions based on physical interpretation and experience. Once we have a set of structurally controllable/observable subsystems we need to design the feedback gain, by solving a set of linear equations, which imposes additional constraints in the feedback structure. Here, such steps are avoided and subsystems are ensured to be structurally controllable/observable by construction. Additionally, with the proposed method, other scenarios are avoided, for instance, the case where the information pattern is necessarily overlapping, and where there exists a lack of efficient and robust methods (as is the case of the expansion/contraction of the state space which is inherently uncontrollable, see Stankovic et al. (2000)), and the case where the coordination among controllers is required, see Witsenhausen (1968).

The main contribution of this paper is a methodology to design decentralized static output feedbacks and overcome issues raised by similar approaches. The present results follow by selection of particular instances of the results in Pequito et al. (2013a). A pre-processing efficient step (layer 1) is implemented to ensure desirable structural properties, the outcome system of the pre-processing step can be used in a subsequential step, where tools available in the literature are recovered.

This paper is organized as follows: In Section 2 we introduce some concepts and results from structural systems. In Section 3 we provide our main results (proofs were relegated to Appendix). Finally, in Section 4 we describe some simulations to illustrate the control methodology applied to some physical examples.

2. PRELIMINARIES AND TERMINOLOGY

The following standard terminology and notions from graph theory can be found in Pequito et al. (2013a). Structural system theory, provide us with an efficient representation and analysis of the system structure by using graphs. Therefore, let $\mathcal{D}(\bar{A}) = (\mathcal{X}, \mathcal{E}_{\mathcal{X},\mathcal{X}})$ be the digraph representation of $\bar{A}$, where the vertex set $\mathcal{X}$ represents the set of state variables (also referred to as state vertices) and $\mathcal{E}_{\mathcal{X},\mathcal{X}} = \{(x_i, x_j) : \ A_{ji} \neq 0\}$ denotes the set of edges. Similarly, we define the following digraphs: $\mathcal{D}(\bar{A}, \bar{B}) = (\mathcal{X} \cup \mathcal{U}, \mathcal{E}_{\mathcal{X},\mathcal{X}} \cup \mathcal{E}_{\mathcal{U},\mathcal{X}})$ where $\mathcal{U}$ represents the set of input vertices and $\mathcal{E}_{\mathcal{U},\mathcal{X}} = \{(u_i, x_j) : \ \bar{B}_{ji} \neq 0\}$; $\mathcal{D}(\bar{A}, \bar{C}) = (\mathcal{X} \cup \mathcal{Y}, \mathcal{E}_{\mathcal{X},\mathcal{X}} \cup \mathcal{E}_{\mathcal{X},\mathcal{Y}})$ where $\mathcal{Y}$ represents the set of output vertices and $\mathcal{E}_{\mathcal{X},\mathcal{Y}} = \{(x_i, y_j) : \ \bar{C}_{ji} \neq 0\}$; and $\mathcal{D}(\bar{A}, \bar{B}, \bar{K}, \bar{C}) = (\mathcal{X} \cup \mathcal{U} \cup \mathcal{Y}, \mathcal{E}_{\mathcal{X},\mathcal{X}} \cup \mathcal{E}_{\mathcal{X},\mathcal{Y}} \cup \mathcal{E}_{\mathcal{U},\mathcal{X}} \cup \mathcal{E}_{\mathcal{Y},\mathcal{U}})$ denotes the digraph associated with the closed-loop system and the set of feedback edges/links is given by $\mathcal{E}_{\mathcal{Y},\mathcal{U}} = \{(y_i, u_j) : \ \bar{K}_{ji} \neq 0\}$.

A digraph $\mathcal{D}_s = (\mathcal{V}_s, \mathcal{E}_s)$ with $\mathcal{V}_s \subset \mathcal{V}$ and $\mathcal{E}_s \subset \mathcal{E}$ is called a *subgraph* of $\mathcal{D}$. If $\mathcal{V}_s = \mathcal{V}$, $\mathcal{D}_s$ is said to *span* $\mathcal{D}$. A sequence of edges $\{(v_1, v_2), (v_2, v_3), \cdots, (v_{k-1}, v_k)\}$, in which all the vertices are distinct, is called *an elementary path* from v_1 to v_k. When v_k coincides with v_1, the sequence is called a *cycle.*

In addition, we will require the following graph theoretic notions (Cormen et al., 2001): A digraph $\mathcal{D}$ is said to be strongly connected if there exists a directed path between any two pairs of vertices. A strongly connected component (SCC) is a maximal subgraph $\mathcal{D}_S = (\mathcal{V}_S, \mathcal{E}_S)$ of $\mathcal{D}$ such that for every $v, w \in \mathcal{V}_S$ there exists a path from v to w and from w to v. Visualizing each SCC as a virtual node (or supernode), one may generate a *directed acyclic graph* (DAG), in which each node corresponds to a single SCC and a directed edge exists between two SCCs *iff* there exists a directed edge connecting the corresponding SCCs in the original digraph. The DAG associated with $\mathcal{D} = (\mathcal{V}, \mathcal{E})$ may be efficiently generated in $\mathcal{O}(|\mathcal{V}|+|\mathcal{E}|)$ (Cormen et al. (2001)), where $|\mathcal{V}|$ and $|\mathcal{E}|$ denote the number of vertices in $\mathcal{V}$ and the number of edges in $\mathcal{E}$, respectively. In the DAG representation, we refer to an SCC that has no incoming edge from any state in a different SCC as a *non-top linked SCC* and, similarly, we have a *non-bottom linked SCC* if the SCC does not have an edge from its states to the states of another SCC.

For any two vertex sets $\mathcal{S}_1, \mathcal{S}_2 \subset V$, we define the *bipartite graph* $\mathcal{B}(\mathcal{S}_1, \mathcal{S}_2, \mathcal{E}_{\mathcal{S}_1,\mathcal{S}_2})$ associated with $\mathcal{D} = (\mathcal{V}, \mathcal{E})$, to be a directed graph (bipartite), whose vertex set is given by $\mathcal{S}_1 \cup \mathcal{S}_2$ and the edge set $\mathcal{E}_{\mathcal{S}_1,\mathcal{S}_2}$ by $\mathcal{E}_{\mathcal{S}_1,\mathcal{S}_2} = \{(s_1, s_2) \in \mathcal{E} : s_1 \in \mathcal{S}_1, s_2 \in \mathcal{S}_2 \}$.

Given $\mathcal{B}(\mathcal{S}_1, \mathcal{S}_2, \mathcal{E}_{\mathcal{S}_1,\mathcal{S}_2})$, a matching M corresponds to a subset of edges in $\mathcal{E}_{\mathcal{S}_1,\mathcal{S}_2}$ that do not share vertices, i.e., given edges $e = (s_1, s_2)$ and $e' = (s_1', s_2')$ with $s_1, s_1' \in \mathcal{S}_1$ and $s_2, s_2' \in \mathcal{S}_2$, $e, e' \in M$ only if $s_1 \neq s_1'$ and $s_2 \neq s_2'$. A maximum matching M^* may then be defined as a matching M that has the largest number of edges among all possible matchings. The maximum matching problem may be solved efficiently in $\mathcal{O}(\sqrt{|\mathcal{S}_1 \cup \mathcal{S}_2|}|\mathcal{E}_{\mathcal{S}_1,\mathcal{S}_2}|)$ Cormen et al. (2001). Vertices in $\mathcal{S}_1$ and $\mathcal{S}_2$ are *matched vertices* if they belong to an edge in the maximum matching M^*, otherwise, we designate the vertices as *unmatched vertices.* If there are no unmatched vertices, we say that we have a *perfect match.* It is to be noted that a maximum matching M^* may not be unique.

For ease of referencing, in the sequel, the term *right-unmatched vertices* (w.r.t. $\mathcal{B}(\mathcal{S}_1, \mathcal{S}_2, \mathcal{E}_{\mathcal{S}_1,\mathcal{S}_2})$ and a maximum matching M^*) will refer to only those vertices in $\mathcal{S}_2$ that do not belong to a matched edge in M^*.

Similarly, we use the term *left-unmatched vertices* (w.r.t. $\mathcal{B}(\mathcal{S}_1, \mathcal{S}_2, \mathcal{E}_{\mathcal{S}_1,\mathcal{S}_2})$ and a maximum matching M^*) to refer to only those vertices in $\mathcal{S}_1$ that do not belong to a matched edge in M^*.

Now, consider the linear time invariant (LTI) system

$$\dot{x} = Ax + Bu, \quad y = Cx. \tag{1}$$

Let $\bar{K}$ denote an *information pattern*, i.e., $\bar{K}_{ij} = 1$ if the sensor j is available to actuator i, and $\bar{K}_{ij} = 0$ otherwise. Additionally, denote by $[\bar{M}] = \{M : M_{ij} = 0 \text{ iff } \bar{M}_{ij} = 0\}$ an equivalence class of matrices of appropriate dimensions. The set of fixed modes of the closed-loop system (1) w.r.t. an information pattern $\bar{K}$ was introduced by Wang and Davison (1973) and is given by

$$\sigma_{\bar{K}} = \bigcap_{K \in [\bar{K}]} \sigma(A + BKC),$$

where $\sigma(M)$ denotes the set of eigenvalues of the square matrix M. It is known that (see Wang and Davison (1973)) if, for a non-empty symmetric open set $\mathcal{W} \subset \mathbb{C}$, $\sigma_{\bar{K}} \subset \mathcal{W}$, then there exists a gain $K \in [\bar{K}]$ such that all the eigenvalues (also known as the poles) of the closed-loop system $A + BKC$ are in $\mathcal{W}$.

The structural version of fixed modes (as in Papadimitriou and Tsitsiklis (1984)) which, essentially, are the fixed modes attributed to the structural pattern, i.e., location of zeroes and non-zeroes of a system, as opposed to fixed modes that originate from a *perfect cancelling* of the numerical parameters. Specifically, a structural linear system $(\bar{A}, \bar{B}, \bar{C})$ is said to have *structurally fixed modes* (SFMs) with respect to (w.r.t.) an information pattern $\bar{K}$ if, for all $A \in [\bar{A}]$, $B \in [\bar{B}]$, $C \in [\bar{C}]$, we have

$$\bigcap_{K \in [\bar{K}]} \sigma(A + BKC) \neq \emptyset.$$

Conversely, a structural system $(\bar{A}, \bar{B}, \bar{C})$ has no structurally fixed modes w.r.t. $\mathcal{K}$, if there exists at least one instantiation $A \in [\bar{A}]$, $B \in [\bar{B}]$, $C \in [\bar{C}]$ which has no fixed modes, i.e., $\cap_{K \in [\bar{K}]} \sigma(A + BKC) = \emptyset$. In this latter case, it may be shown (see Sezer and Siljak (1981a)) that almost all systems in the sparsity class $(\bar{A}, \bar{B}, \bar{C})$ have no fixed modes, and, hence, allow pole-placement arbitrarily close to any pre-specified set of eigenvalues. This also justifies our constraint of designing systems with no SFMs in L1.

The following graphical condition for a structured LTI system to be SFM free is provided in Pichai et al. (1984), where we assume that the system is both structurally controllable and observable.

Theorem 1 (Pichai et al. (1984)). *The structural system $(\bar{A}, \bar{B}, \bar{C})$ associated with (1) has no structurally fixed modes with respect to an information pattern $\bar{K}$, if and only if both of the following two conditions hold:*

a) in $\mathcal{D}(\bar{A}, \bar{B}, \bar{K}, \bar{C}) = (\mathcal{X} \cup \mathcal{U} \cup \mathcal{Y}, \mathcal{E}_{\mathcal{X},\mathcal{X}} \cup \mathcal{E}_{\mathcal{X},\mathcal{Y}} \cup \mathcal{E}_{\mathcal{U},\mathcal{X}} \cup \mathcal{E}_{\mathcal{Y},\mathcal{U}})$, each state vertex $x \in \mathcal{X}$ is contained in an SCC which includes an edge of $\mathcal{E}_{\mathcal{Y},\mathcal{U}}$;
b) there exists a finite disjoint union of cycles $\mathcal{C}_k = (\mathcal{V}_k, \mathcal{E}_k)$ (subgraph of $\mathcal{D}(\bar{A}, \bar{B}, \bar{C}, \bar{K})$) with $k \in \mathbb{N}$ such that

$$\mathcal{X} \subset \bigcup_k \mathcal{V}_k .$$

□

Now consider the following definition

Definition 1. *A* feasible dedicated input configuration *$\mathcal{S}_u \subset \mathcal{X}$ is a collection of state variables such that by assigning dedicated inputs (actuators) to the variables in $\mathcal{S}_u$, structural controllability is ensured. In other words, denoting by $\bar{B}$ the structural input matrix corresponding to the assignment of dedicated outputs to the state variables in $\mathcal{S}_u$, the configuration $\mathcal{S}_u$ is said to be a feasible dedicated input configuration if the pair $(\bar{A}, \bar{B})$ is structurally controllable. A feasible dedicated input configuration with the minimum number of state variables is said to be a* feasible minimal dedicated input configuration. □

Similarly, we can define the notion of a *feasible dedicated output configuration $\mathcal{S}_y$*. In addition, we also require the following general results on structural control design from Pequito et al. (2013a) (see also Pequito et al. (2013b)).

Theorem 2. *Let $\mathcal{D}(\bar{A}) = (\mathcal{X}, \mathcal{E}_{\mathcal{X},\mathcal{X}})$ denote the system digraph and $\mathcal{B} \equiv \mathcal{B}(\mathcal{X}, \mathcal{X}, \mathcal{E}_{\mathcal{X},\mathcal{X}})$ its bipartite representation. Let $S_u \subset \mathcal{X}$, then the following statements are equivalent:*

(1) The set $\mathcal{S}_u$ is a feasible dedicated input configuration;
(2) There exists a subset $\mathcal{U}_R \subset \mathcal{S}_u$ corresponding to the set of right-unmatched vertices of some maximum matching of $\mathcal{B}$, and a subset $\mathcal{A}_u \subset \mathcal{S}_u$ comprising one state variable from each non-top linked SCC of $\mathcal{D}(\bar{A})$.

□

By duality we have

Corollary 1. *Let $\mathcal{D}(\bar{A}) = (\mathcal{X}, \mathcal{E}_{\mathcal{X},\mathcal{X}})$ denote the system digraph and $\mathcal{B} \equiv \mathcal{B}(\mathcal{X}, \mathcal{X}, \mathcal{E}_{\mathcal{X},\mathcal{X}})$ its bipartite representation. Let $S_y \subset \mathcal{X}$, then the following statements are equivalent:*

(1) The set $\mathcal{S}_y$ is a feasible dedicated output configuration
(2) There exists a subset $\mathcal{U}_L \subset \mathcal{S}_y$ corresponding to the set of left-unmatched vertices of some maximum matching of $\mathcal{B}$, and a subset $\mathcal{A}_y \subset \mathcal{S}_y$ comprising one state variable from each non-bottom linked SCC of $\mathcal{D}(\bar{A})$. □

The following results on general non-dedicated structural input/output and control configuration design were also obtained in Pequito et al. (2013a,b). To ease the presentation, we denote by m the number of right/left-unmatched vertices in any maximum matching of $\mathcal{B}$, β, the number of non-top linked SCCs in $\mathcal{D}(\bar{A})$ and β', the number of non-bottom linked SCCs in $\mathcal{D}(\bar{A})$.

R1 Construction of feasible $\bar{B}$ and $\bar{C}$:
A pair $(\bar{A}, \bar{B})$ is structurally controllable if and only if there exists a maximum matching of $\mathcal{B}$ with a set of right-unmatched vertices $\mathcal{U}_R$, such that, $\bar{B}$ has (at least) m non-zero entries, one in each of the rows corresponds to the different state variables in $\mathcal{U}_R$ and located at different columns, and (at least) β non-zero entries, each of which belongs to a row (state variable) corresponding to a distinct non-top linked SCC and located in arbitrary columns. ⋄

We emphasize that the second set of β non-zero entries may share columns with the first set of m non-zero entries. Also, as a direct consequence of R1, we obtain that any $\bar{B}$, such that $(\bar{A}, \bar{B})$ is structurally controllable, must have at least m distinct non-zero columns (or m distinct control inputs). Similarly, a pair $(\bar{A}, \bar{C})$ is structurally observable if and only if there exists a maximum matching of $\mathcal{B}$ with a set of left-unmatched vertices $\mathcal{U}_L$, such that, $\bar{C}$ has (at least) m non-zero entries, one in each of the columns corresponds to the different state variables in $\mathcal{U}_L$ and located at different rows, and (at least) β' non-zero entries, each of which belongs to a column (state variable) corresponding to a distinct non-bottom linked SCC and located in arbitrary rows.

R2 For any two distinct maximum matchings M^* and $M^{*\prime}$. Then, there exists a (common) maximum matching M^o such that the set of right-unmatched vertices of M^o coincide with those of M^* and the set of left-unmatched vertices of M^o coincide with those of $M^{*\prime}$. ⋄

As will be shown later, R2 allows us to design $\bar{B}$ and $\bar{C}$ *independently*. Finally, we list the following two results on the feasibility of information patterns:

R3 Let $\bar{B}$ and $\bar{C}$ be feasible input and output matrices (i.e., $(\bar{A}, \bar{B})$ is structural controllable and $(\bar{A}, \bar{C})$ is structural observable), and by R1-R2, let M^* be a (common) maximum matching, such that $\bar{B}$ and $\bar{C}$ have a correspondence (in the sense of R1) with the set $\mathcal{U}_R$ of right and the set $\mathcal{U}_L$ of left-unmatched vertices of M^*, respectively. Then, for any information pattern $\bar{K}$, which consists of non-zero entries corresponding to a disjoint (but arbitrary) pairing of the state variables in $\mathcal{U}_R$ and $\mathcal{U}_L$, the digraph $\mathcal{D}(\bar{A}, \bar{B}, \bar{C}, \bar{K})$ satisfies condition b) of Theorem 1. ⋄

Results established in R1-R3 will play a fundamental role in proving designing a system such that the properties of layer L1 hold. The description of such design is provided in the next section.

3. MAIN RESULTS

In this section we provide the main results of this paper: the design of $\bar{B}$, $\bar{C}$ and $\bar{K}$ such that L1 is ensured. Now consider the following Lemma that plays a central role in showing that the desirable structural properties stated in L1 are ensured.

Lemma 1. *Let $\mathcal{D}(\bar{A}) = (\mathcal{X}, \mathcal{E}_{\mathcal{X},\mathcal{X}})$ be an SCC. If $\mathcal{D}(\bar{A})$ is spanned by a disjoint union of cycles, then $(\bar{A}, \bar{B})$ is structurally controllable with any $\bar{B} \neq 0$ and $(\bar{A}, \bar{C})$ is structurally observable with any $\bar{C} \neq 0$. In particular, it is structurally controllable/observable, by manipulating/measuring any single state variable.* □

Now, we state the main result of this paper, which ensures that all desirable properties enunciated in L1 hold.

Theorem 3. *Let $\mathcal{D}(\bar{A}) = (\mathcal{X}, \mathcal{E}_{\mathcal{X},\mathcal{X}})$ be a single digraph with $|\mathcal{X}| = n$ and $\mathcal{B}(\mathcal{X}, \mathcal{X}, \mathcal{E}_{\mathcal{X},\mathcal{X}})$ be its bipartite representation. In addition, let $\mathcal{S}_u$ and $\mathcal{S}_y$ be minimal feasible dedicated input and output configurations, respectively, and consider $\mathcal{U}_R \subset S_u$ and $\mathcal{U}_L \subset \mathcal{S}_y$ to be the set of right/left-unmatched vertices with respect to some maximum matching of $\mathcal{B}(\mathcal{X}, \mathcal{X}, \mathcal{E}_{\mathcal{X},\mathcal{X}})$ (that exist by Theorem 2, Corollary 1 and R3). Suppose that $(\bar{B}_1, \bar{K}_1, \bar{C}_1)$ are designed such that the corresponding conditions hold:*

(i) independently inputs are assigned to each of the state variable in $\mathcal{U}_R$;
(ii) independently outputs are assigned to each state variable in $\mathcal{U}_L$;
(iii) there exist inputs assigned to the state variables in $\mathcal{S}_u - \mathcal{U}_R$ and there exist outputs assigned to the state variables in $\mathcal{S}_y - \mathcal{U}_L$;
(iv) there exist a set of feedback links, from the outputs in (ii) to the inputs in (i), such that together with a subset of edges of a maximum matching (common to $\mathcal{U}_R, \mathcal{U}_L$) we have a single cycle;
(v) there exist a feedback link from the some output in (ii) to each input in (iii) and some feedback from each output in (iii) to the inputs in (i);

Then, the associated state matrix of the closed-loop system using $(\bar{A}, \bar{B}_1, \bar{K}_1, \bar{C}_1)$ has no structural fixed modes. In addition, considering a realization (B_1, K_1, C_1) with the same sparseness of $(\bar{B}_1, \bar{K}_1, \bar{C}_1)$, the associated state matrix of the closed-loop system

$$A_1 = A + B_1 K_1 C_1$$

has the following structural properties:

P1 *$(\bar{A}_1, \bar{B}_2)$ is structurally controllable for any $\bar{B}_2 \neq 0$;*
P2 *$(\bar{A}_1, \bar{C}_2)$ is structurally observable for any $\bar{C}_2 \neq 0$;*
P3 *the associated state matrix of the closed-loop system using $(\bar{A}_1, \bar{B}_2, \bar{K}_2, \bar{C}_2)$ has no structural fixed modes for any $\bar{K}_2 \neq 0$.* □

From the result in Theorem 3 we can now do decentralization of the resulting system (i.e., original system in closed-loop with the first control layer) with respect to any information structure. This methodology is applied to two different physical systems, presented in the next section.

4. AN ILLUSTRATIVE EXAMPLE

We now provide a couple of applications with simulation results to illustrate the proposed multilayer approach.

4.1 Two Zone Building

Consider a building with two thermally coupled zones and an air handling unit (AHU). The building contains one internal wall that separates both zones and two boundary walls that separate the indoors from the outdoors. Zone temperatures (T_1, T_2) are controlled by local heaters (I_1, I_2) and heat delivered from the AHU (I_1^{hc}, I_2^{hc}). Outdoor air T_o is brought into the AHU at a mass flow rate $f_1 + f_2$ where f_i $(i = 1, 2)$ corresponds to the flow rate associated with each zone, and heated to temperature T_{hc} with a heat exchanger, I_b. The net heat received from the AHU in zone i is $I_i^{hc} = f_i(T_{hc} - T_i)$ for $i = 1, 2$.

Building thermodynamics can be modeled as a circuit of resistors and capacitors (Andersen et al., 2000). Fig. 1 depicts the RC model of the two zone building scenario described above. Nodes T_1, T_2, and T_{hc} represent the mean temperatures of zone 1, 2, and the AHU, respectively.

Given Fig. 1, the building thermodynamics are modeled as follows:

$$\dot{x} = \begin{bmatrix} -\frac{f_1}{C_1} - \frac{1}{\tau_{1N}} - \frac{1}{\tau_{12}} & \frac{1}{\tau_{12}} & \frac{f_1}{C_1} \\ \frac{1}{\tau_{21}} & -\frac{f_2}{C_2} - \frac{1}{\tau_{2S}} - \frac{1}{\tau_{21}} & \frac{f_2}{C_2} \\ 0 & 0 & -\frac{(f_1+f_2)}{C_{hc}} \end{bmatrix} x + \begin{bmatrix} \frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 \\ 0 & 0 & \frac{1}{C_{hc}} \end{bmatrix} u + \begin{bmatrix} \frac{1}{\tau_{1N}} \\ \frac{1}{\tau_{2S}} \\ \frac{f_1+f_2}{C_{hc}} \end{bmatrix} T_o, \quad y = x, \tag{2}$$

with $\tau_{12} = R_{12}C_1$, $\tau_{21} = R_{12}C_2$, $\tau_{1N} = R_{1N}C_1$, $\tau_{2S} = R_{2S}C_2$, where C_1, C_2, and C_{hc} represent heat capacitance of the air in zone 1, 2, and the AHU, respectively, and

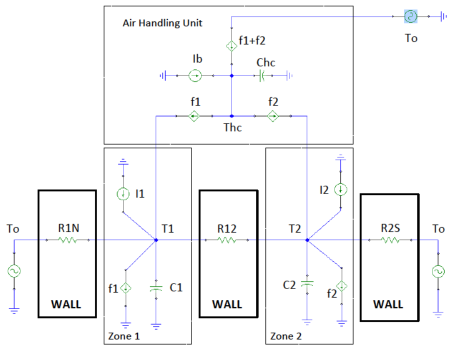


Fig. 1. An illustration of the RC Model for the two zone building. C_1, C_2, and C_{hc} represent heat capacitance of the air in zone 1, 2, and the AHU, respectively. R_{1N}, R_{12}, and R_{2S} represent resistance to heat flowing through the walls in the building.

R_{1N}, R_{12}, and R_{2S} represent resistance to heat flowing through the walls in the building. The system state is given by $x = [T_1, T_2, T_{hc}]$, its input by $u = [I_1, I_2, I_b]$ and y corresponds to the measured output.

In the sequel we assume that: mass flow rates f_1, f_2 are assumed constant, and T_o is the outdoor temperature, taken to be a disturbance that follows a normal distribution with an expected value of $273°K$ and standard deviation of $2°K$. The parametric values of (2) are omitted here due to space constrains, but can be found in Andersen et al. (2000).

Layer 1 A digraph representation of (2) is depicted in Fig. 3(a). Although the system is controllable given all inputs, the system cannot be controlled with a single input. For instance, consider constant heat input $I_1 = 300W, I_2 = I_b = 0W$. In this case, it can seen from Fig. 2(a) that the zone temperatures rise, while the AHU temperature does not change and maintains the outdoor temperature, $T_{hc} = T_o$.

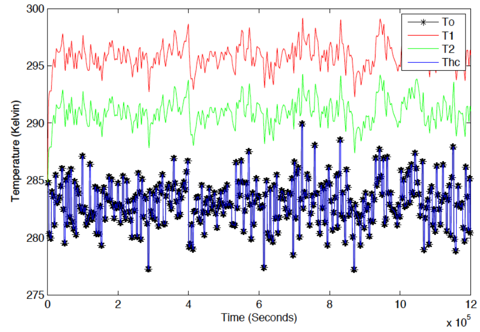
Invoking Theorem 3, we can now obtain $A_1 = A + B_1K_1C_1$ depicted in Fig. 3(b), with $B_1 = [0, 0, 1]^T$, $C_1 = [1, 0, 0]$ and K_1 any non-zero scalar. Intuitively, the measurement T_1 is fed back to the AHU controller such that the new input is $\tilde{I}_b = I_b + K_1T_1$. Subsequently, from Theorem 3, any of the inputs I_1, I_2, or $\tilde{I}_b$ can control the entire system. Fig. 2(b) depicts the time evolution of temperatures in the building when $K_1 = 0.5$, $I_1 = 300W$, and $I_2 = I_b = 0$. We see that all temperatures are now controlled with only I_1.

Layer 2 In this (second) control layer, the goal is to find K_2 such that the poles of $A_1 + BK_2C$ improve the performance of the system. Notice that now, temperatures can be controlled to meet set points using only local information, as consequence of Theorem 3.

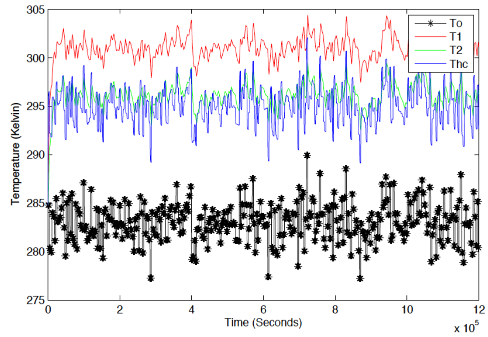
To improve the performance of the system we propose a full decentralized information pattern, given by

$$K_2 = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}. \quad (3)$$

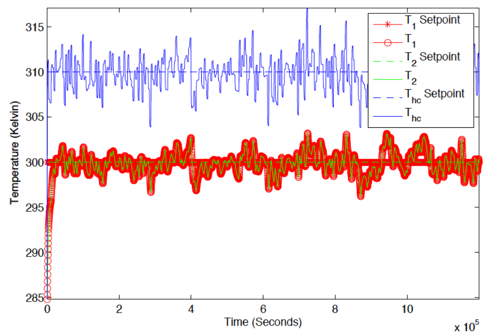
Fig. 2(c) depicts a tracking scenario where the control law, $u = K_2(x_{set} - x)$, with set point given by $x_{set} = [T_{1,set}\ T_{2,set}\ T_{hc,set}] = [300\ 300\ 310]$, is applied to the altered system A_1 to regulate building temperatures. The gains were set to set $k_1 = k_2 = .125$, $k_3 = .3$, such that,



(a) Original System (A)

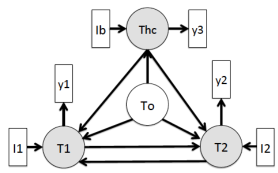


(b) Original System with Layer 1 ($A_1 = A + B_3K_1C_1$)

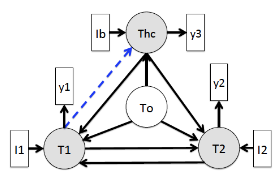


(c) Original System with Layers 1 & 2 ($A_2 = A_1 + BK_2C$)

Fig. 2. Illustration of the two layer method applied to the two zone building



(a) Original System, A



(b) Altered System, A_1

Fig. 3. Directed graphs of the 2 zone building

together with the choice to K_1 we satisfy the constraints of the system, given by

$$u_{min} \le \tilde{K}x \le u_{max}, \qquad \tilde{K} = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ K_1 & 0 & k_3 \end{bmatrix}.$$

Discussion of results As shown in Fig. 2, the first layer removes fixed modes in the original system, which allows the second layer to apply a decentralized temperature controller. However, design constraints on K_1 and K_2 introduces performance tradeoffs between set point error and time to meet temperature set points.

4.2 3-Bus Power System

Consider the 3-bus power system depicted in Fig. 4, that consists of three dynamical subsystems: two generators and one aggregate load, which constitute dynamical subsystems interconnected through the transmission lines.

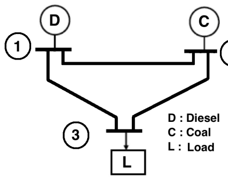


Fig. 4. 3-bus system (*1,2,3* represents the bus number)

The generation technologies considered in this example are diesel and coal which are modeled as linearized governor control[3]. We now provide a brief description of the component model used hereafter.

Diesel Generator: At bus-1, the linearized governor model for the diesel generator is represented as:

$$\begin{bmatrix} \dot{W}_f \\ \dot{\omega}_1 \\ \dot{v}_1 \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{1}{T_f} & 0 & 0 \\ \frac{1}{M_D} & -\frac{D_D}{M_D} & 0 \\ 0 & -\frac{k_D}{T_v} & -\frac{1}{T_v} \end{bmatrix}}_{A_D} \begin{bmatrix} W_f \\ \omega_1 \\ v_1 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{k_D}{T_v} \end{bmatrix}}_{B_D} {\omega_1}^r + \underbrace{\begin{bmatrix} 0 \\ -\frac{1}{M_D} \\ 0 \end{bmatrix}}_{N_D} P_{G_1},$$

where M_D represents the inertia of the prime mover, D_D the damping coefficient, and T_f, T_v represents the time constants of the turbine fuel flow and the valve, respectively. The state is denoted by $x_D = \left[W_f \ \omega_1 \ v_1 \right]^T$ compring the following variables: the turbine fuel flow W_f, prime mover frequency ω_1 and governor valve position v_1. The governor reference set point is denoted by ω_1^r. In addition, the combustion turbine valve responds to the locally sensed frequency deviations from the governor reference set point and the network coupling matrix N_D connects the diesel generator to the network through bus-1, in particular, the generation output P_{G_1}.

Coal Generator: At bus-2, the linearized governor model for the coal generator is represented as:

$$\begin{bmatrix} \dot{\omega}_2 \\ \dot{P}_t \\ \dot{v}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{D_C}{M_C} & \frac{1}{M_C} & \frac{e_t}{M_C} \\ 0 & -\frac{1}{T_u} & \frac{k_t}{T_u} \\ -\frac{1}{T_v} & 0 & -\frac{r}{T_v} \end{bmatrix}}_{A_C} \begin{bmatrix} \omega_1 \\ P_t \\ v_1 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_v} \end{bmatrix}}_{B_C} \omega_2^r + \underbrace{\begin{bmatrix} -\frac{1}{M_C} \\ 0 \\ 0 \end{bmatrix}}_{N_C} P_{G_2}$$

where parameters k_t, e_t are some constants due to governor linearization, M_C denoted the moment of inertia of the prime mover, D_C the damping coefficient and T_u, T_v are the time constants of the turbine and valve, respectively. The state $x_C = \left[\omega_2 \ P_t \ v_2 \right]^T$ consists of: the prime mover frequency ω_2, the steam turbine power output P_t and the governor valve position v_2. Similarly to the diesel generator, the governor reference set point is denoted by ω_2^r. At last, the steam valve responds to the locally sensed frequency deviations from the governor reference set point and the network coupling matrix N_C connects the coal generator to the network through bus-2, in particular, the generation output P_{G_2}.

Aggregate Load: At bus-3, the physical load is represented as a cyber model:

$$\dot{\omega_3} = -\underbrace{\left(\frac{D_L}{M_L}\right)}_{A_L} \omega_3 + \underbrace{\left(\frac{1}{M_L}\right)}_{N_L} P_{L_3} - \left(\frac{1}{M_L}\right) P_m$$

where M_L and D_L refer to the effective moment of inertia and the damping coefficient of the aggregate load[4]. The postulated cyber model of the load is based on a Newton-like representation of the load dynamics. The dynamics is governed by the instantaneous mismatch between the power delivered to the load P_{L_3} and the power taken by the load P_m. The aggregate load at bus-3 is coupled to the network through its electrical power consumption P_{L_3}.

Transmission Lines: The transmission lines electrically couple the generators and the load through their electrical power outputs and consumption respectively. The coupling constraints are the linearized power flow equations. For the purpose of real-power balancing, decoupling between real and reactive power flow is assumed, see Ilić and Zaborszky (2000). The decoupled linearized real-power flow equations are differentiated to obtain the relation $\dot{x}_{net} = J\omega$, see Ilic et al. (2008), where $x_{net} = \left[P_{G_1} \ P_{G_2} \ -P_{L_3} \right]^T$, the J matrix represents the jacobian corresponding to the linearization of the network constraints and the local frequencies are given by $\omega = \left[\omega_1 \ \omega_2 \ \omega_3 \right]^T$.

System Model: The coupled dynamical subsystems, i.e. the generators and load can be compactly represented as (Ilic et al., 2008):

$$\dot{x}_s = A_s x_s + B_s u_s + C_s x_{net} \qquad (4)$$

with $x_s = \left[x_D \ x_C \ x_L \right]^T$, $A_s = diag\left(A_D, A_C, A_L\right)$, $B_s = diag\left(B_D, B_C, B_L\right)$, $C_s = diag\left(C_D, C_C, C_L\right)$. Therefore, (4) can be re-written (by augmenting the state) as an LTI system, given by

$$\dot{x} = A_{3bus} x + B_{3bus} u \qquad (5)$$

where $x = \left[x_s \ x_{net} \right]^T$, see Fig. 5 for its digraph representation.

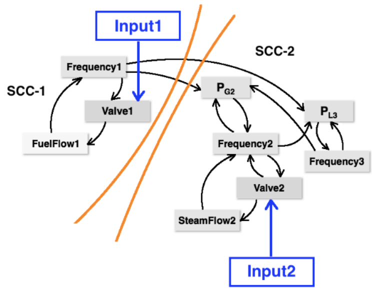


Fig. 5. Open-loop digraph of 3-bus system (*1,2,3* represents the bus number)

The 3-bus network is modeled as a tenth order LTI system. To avoid the structural singularity of the decoupled linearized power flow equations[5], the diesel plant D is set

[3] Their description and numerical parameters can be found in Appendix-A of Cardell (1997), where as the aggregate load is modeled as a large induction machine, see Ilic et al. (2008, 2011)

[4] Their values can be obtained through systematic model identification methods

[5] In a network with n nodes, the power injections at only $(n-1)$ nodes can be controlled. The net power injection must sum up to zero within the system.

as a reference bus. Therefore, its power injection P_{G_1} is not being controlled, given the structural singularity of the system matrix A_{3bus}. The state space model is therefore reduced to a ninth order LTI system.

The balancing of a power system is a two-step approach. At first, the generator's output is scheduled for the predicted load. Subsequently, a feedback control action is required to correct real-time supply and demand imbalance due to the prediction error. The balancing of the intermittent wind power output requires additional feedback control. The real-time control comes at a high cost, provided primarily by the fast combustion turbines or the stored steam in the boilers of the coal plants. Before placing additional actuators it is critical to ensure that the new system configuration is devoid of structural fixed modes.

We now illustrate the use of the proposed multi-layer approach, by invoking Theorem 3.

Layer 1 The aim is to avoid any decentralized structural fixed modes by ensuring that the properties of **L1** holds. The digraph of the open loop system consists of two interconnected SCC (Fig. 5). Structurally the system matrix A_{3bus} can be modified to $A_1 = A_{3bus} + B_1K_1C_1$, where $B_1 = [B_D^T\ 0, \cdots,\ 0]^T$ and $C_1 = [0,\ \cdots\ 0,\ 1]$. Intuitively, it corresponds to the feedback where the diesel generator's valve actuator is closed with the measurement of the power P_{L_3} delivered to the load, shown in Fig. 6. The scalar gain $K_1 = -0.06784$ alters the system matrix A_{3bus} so that the properties P1-P3 of Theorem 3 holds. This option is motivated by two reasons: firstly, for the purpose of feedback control it is expensive to maintain large volume of high-pressure steam in the boiler of the coal plant. It is economically prudent to inject diesel in real-time to balance the system. Secondly, the accuracy of the phasor measurement (i.e., a commonly used sensor) unit is much higher for the active power measurement than frequency, see (IEEE Standard, 2006). Therefore, compared to the local frequency ω_3, the local power consumption of the load at bus-3 can be measured much more accurately. For the first layer control, alternatively we could have considered feeding the measurement of the power injection at bus-2 (P_{G_2}) to the valve actuator of the diesel generator (v_1).

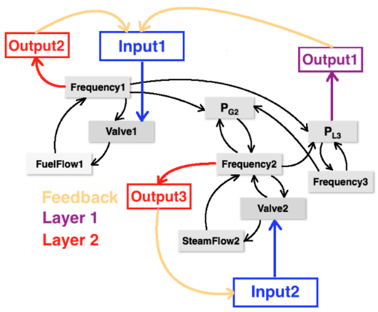


Fig. 6. Layer 1 & 2 control-loop digraph of 3-bus system

Layer 2 In addition to the limited information structure and the central computation complexity, the application of classical control theory to power systems is also restricted by the deregulated market structure. The generator owners have no obligation to share the information with each other. As the first layer guarantees a system without structurally decentralized fixed modes, the purpose of the second layer is to improve the system response through local feedback controllers. In power systems, the frequency offset is an indicator of supply-demand mismatch. The local controllers respond to the local frequency measurements. As shown in Fig. 6, frequency measurements of the generators are fed to their respective local actuators to improve the system performance.

Now, let $B_2 = [b_{v_1}, b_{v2}]$ denote the input matrix that consists of the real $n \times 1$ input vectors b_{v_1}, b_{v_2} that manipulate the valve openning of the diesel and coal generator, respectively. In addition, let $C_2 = [c_1, c_2]^T$ denote the output matrix that consists of the real $1 \times n$ vectors c_1, c_2 that measures the frequency of the diesel and coal generator, respectively. Then, we obtain

$$A_2 = A_1 + B_2K_2C_2, \text{ with } K_2 = \begin{bmatrix} -4.70327 & 0 \\ 0 & -1.81628 \end{bmatrix}.$$

where the structure of K_2 was determined by considering the local available information and its values were tunned (by trial and error) to achieve a reasonable overall transient behavior.

Discussion of results The 3-bus network was subjected to near zero-mean load disturbance at bus-3 (load). The frequency transients of the generators were evaluated for two scenarios, first with only the *Layer 1* and second with both *Layer 1, 2*. Although the system could be stabilized using only *Layer 1*, , one may not be able to always guarantee a numerically stable system. Notice also, as shown in Fig. 8 the initial transients in the frequency of the coal generator can be damaging for the prime-mover. Such transient are commonly observed in decentralized controllers. In contrast, the two layer approach as shown in Fig. 7,8. improves significantly the transient performance by adding of decentralized control at each generator. Thus, it effectively circumvents the need of implementing a centralized control with a full information structure.

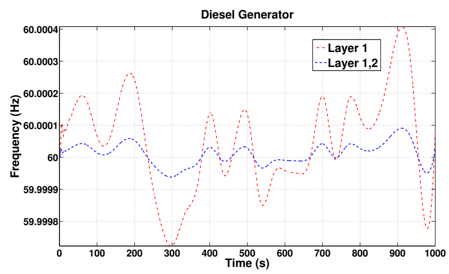


Fig. 7. Frequency of the diesel generator in response to load deviations

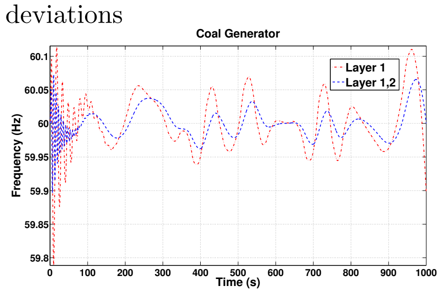


Fig. 8. Frequency of the coal generator in response to load deviations

5. CONCLUSIONS AND FURTHER RESEARCH

In this paper we introduced a methodology based in a multi-layer approach to design a decentralized controller. Mainly, the first layer is a pre-processing step that ensures several structural desirable properties, such as, structural controllability (resp. observability) by actuating (resp. measuring) any single state variable. In addition, once the first layer is done, by considering to fed back any possible output to an input we achieve a system without structurally decentralized fixed modes. Therefore, common tools in decentralized control (that require, as assumption, an information pattern that incurs in no structurally fixed modes) can be recovered and used in the second layer. Although, in this paper we have tuned the gains associated with constrained information pattern, a possible venue to future research consists of exploring the relation between the structural approach provided here and optimization tools, in order to design systematically the gains (associated with constrained information patterns).

APPENDIX

Proof of Lemma 1 First, since $\mathcal{D}(\bar{A})$ is spanned by a disjoint union of cycles it follows that there exists a maximum matching of $\mathcal{B}(\mathcal{X}, \mathcal{X}, \mathcal{E}_{\mathcal{X},\mathcal{X}})$ that is a perfect match (Pequito et al., 2013b,a), which means that the set of right/left-unmatched vertices $\mathcal{U}_R, \mathcal{U}_L$ are empty. Second, because $\mathcal{D}(\bar{A})$ is an SCC, this implies that this SCC is a non-top linked and a non-bottom linked SCC. Therefore, any $\bar{B}, \bar{C} \neq 0$ (i.e., by assigning any inputs/outputs to the state variables) fulfils R2, and the result follows. In particular, it holds for the case where $\bar{B}, \bar{C}$ is a canonical vector, i.e., any single state variable that is manipulated/measured.

■

Proof of Theorem 3 Condition (i)-(iii) ensure that R1 holds, which implies that $\bar{B}_1$ and $\bar{C}_1$ are such that $(\bar{A}, \bar{B}_1)$ is structurally controllable and $(\bar{A}, \bar{C}_1)$ is structurally observable, which is a necessary condition to obtain a system without structurally fixed modes under information pattern constraints. To show that $\mathcal{D}(\bar{A}, \bar{B}_1, \bar{K}_1, \bar{C}_1)$ has no structurally fixed modes, we just need to fullfil the conditions in Theorem 1. First, remark that condition (iv) implies, by recalling R3, that $\mathcal{D}(\bar{A}, \bar{B}_1, \bar{K}_1, \bar{C}_1)$ satisfies condition b) of Theorem 1. To show that condition a) in Theorem 1 holds, remark that $\mathcal{D}(\bar{A}, \bar{B}_1, \bar{K}_1, \bar{C}_1)$ is an SCC, by noticing that from the non-top linked SCCs (in particular its state variables) we can reach all the other state variables in the digraph. In particular we can reach those state variables in the non-bottom linked SCCs. Hence, from (v) we ensure the existence of a feedback link between the outputs measuring state variables in the non-bottom linked SCCs and the inputs manipulating state variables in the non-top linked SCCs, which implies that $\mathcal{D}(\bar{A}, \bar{B}_1, \bar{K}_1, \bar{C}_1)$ is an SCC. Hence, condition a) in Theorem 1 holds, which together with the satisfiability of condition b) leads to the result that $\mathcal{D}(\bar{A}, \bar{B}_1, \bar{K}_1, \bar{C}_1)$ has no structurally fixed modes holds.

To verify that properties P1-P3 hold, remark that the entry $[\bar{B}_1\bar{K}_1\bar{C}_1]_{ij} \neq 0$ if a state variable x_i is actuated by some input u (corresponding to a column of $\bar{B}_1$ with non-zero entry in row i), x_j is measured by some output y (corresponding to a row of $\bar{C}_1$ with non-zero entry in column j), and there exists a feedback link between y and u. In particular, it follows from (iv) that Theorem 1 b) holds (see R3) and so $A_1 + B_1K_1C_1$ is spanned by a disjoint union of cycles comprising only state variables. In addition with the fact that $\mathcal{D}(\bar{A}, \bar{B}_1, \bar{K}_1, \bar{C}_1)$ is an SCC, follows from Lemma 1 that $P1$ and $P2$ hold. Finally, P3 holds by recalling that $\mathcal{D}(\bar{A}_1)$ is an SCC spanned by a disjoint union of cycles comprising state variables and by allowing $\bar{K}_2 \neq 0$ we have that $\mathcal{D}(\bar{A}_1, \bar{B}_2, \bar{K}_2, \bar{C}_2)$ with $\bar{B}_2, \bar{C}_2$ (with no column and row with all entries equal to zero), satisfies both conditions on Theorem 1, hence it has no structurally fixed modes.

■

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