Coordinated path-following control of multiple underactuated autonomous vehicles in the presence of communication failures

Reza Ghabcheloo, A. Pedro Aguiar, António Pascoal, Carlos Silvestre, Isaac Kaminer, and João Hespanha

Abstract— **This paper addresses the problem of steering a group of underactuated autonomous vehicles along given spatial paths, while holding a desired inter-vehicle formation pattern. For a general class of vehicles moving in either two or three-dimensional space, we show how Lyapunov-based techniques and graph theory can be brought together to yield a decentralized control structure where the dynamics of the cooperating vehicles and the constraints imposed by the topology of the inter-vehicle communications network are explicitly taken into account. *Path-following* for each vehicle amounts to reducing an appropriately defined geometric error to a small neighborhood of the origin. *Vehicle coordination* is achieved by adjusting the speed of each vehicle along its path according to information on the positions of a subset of the other vehicles, as determined by the communications topology adopted. The system obtained by putting together the path-following and vehicle coordination strategies adopted takes a cascade form, where the former subsystem is input-to-state stable (ISS) with the error variables of the latter as inputs. Convergence and stability of the overall system are proved formally. The results are also extended to solve the problem of temporary communication failures. Using the concept of "brief instabilities" we show that for a given maximum failure rate, the coordinated path following system is stable and the errors converge to a small neighborhood of the origin. We illustrate our design procedure for underwater vehicles moving in three-dimensional space. Simulations results are presented and discussed.**

I. INTRODUCTION

Increasingly challenging mission scenarios and the advent of powerful embedded systems and communication networks have spawned widespread interest in the problem of coordinated motion control of multiple autonomous vehicles. The types of applications envisioned are numerous and include aircraft and spacecraft formation flying control [5], [13], [19], coordinated control of land robots [7], [18], and control of multiple surface and underwater vehicles [8], [17], [21].

In spite of significant progress in the area, however, much work remains to be done to develop strategies capable of yielding robust performance of a fleet of vehicles in the presence of complex vehicle dynamics, severe communication constraints, and partial vehicle failures. These difficulties are specially challenging in the field of marine robotics for two main reasons: i) the dynamics of marine vehicles are often complex and cannot be simply ignored or drastically simplified for control design purposes, and ii) underwater communications and positioning rely heavily on acoustic systems, which are plagued with intermittent failures, latency, and multipath effects.

Inspired by the developments in the field, we consider the problem of *coordinated path-following* (CPF) where *multiple vehicles are required to follow pre-specified spatial paths while keeping a desired inter-vehicle formation pattern in time*. This problem arises, for example, in the operation of multiple autonomous underwater vehicles (AUV) for fast acoustic coverage of the seabed. In this application, two or more vehicles are required to fly above the seabed at the same or different depths, along geometrically similar spatial paths, and map the seabed using identical suites of acoustic sensors. Larger areas can be covered in a short period of time, by requiring that the vehicles traverse identical paths so that the projections of the acoustic beams on the seabed exhibit some overlapping. These objectives impose constraints on the inter-vehicle formation pattern. A number of other scenarios can also be envisioned that require coordinated motion control of marine vehicles.

We solve the coordinated path-following problem for a general class of underactuated vehicles moving in either two or three-dimensional space. The solution adopted is rooted in Lyapunov-based theory and addresses explicitly the vehicle dynamics as well as the constraints imposed by the topology of the inter-vehicle communications network. The latter are tackled in the framework of graph theory [14], which seems especially suitable to study the impact of communication topologies on the performance that can be achieved with coordination [9]. The class of vehicles to which the design procedure is applicable is quite general. In fact, it includes any vehicle modeled as a rigid-body subject to a controlled force and either one controlled torque if it is only moving on a planar surface, or two to three independent control torques for a vehicle moving in three-dimensional space. Furthermore, contrary to most of the approaches described in the literature, the controller proposed does not suffer from geometric singularities due to the parametrization of the vehicle's rotation matrix.

With the framework adopted, path-following (in space) and inter-vehicle coordination (in time) become essentially

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decoupled. Each vehicle is equipped with a controller that makes the vehicle follow a predefined path. The speed of each vehicle is then adapted so that the whole group of vehicles keeps the desired formation pattern. A supporting communications network provides the fleet of vehicles with the medium over which to exchange the information that is required to synchronize the so-called coordination states. Because of network faults, the problems brought about by temporary communication losses must be addressed explicitly. To this effect, this paper proposes a framework to study the effect of communication failures on the stability of the overall vehicle formation. Under this framework, the system that is obtained by putting together the path-following and vehicle coordination strategies adopted takes a cascade form, where the output of the latter enters to the former subsystem. Convergence and stability of the combined path-following / coordination system are proved formally. In the course of doing this, the concept of "brief instabilities" is exploited to model network failures and to show that, given a maximum failure rate, one can find control design parameters that ensure the stability of the formation. See for example [6] and [23], where coordination problems with switching communications are addressed.

To the best of our knowledge, previous work on coordinated path following control has mostly been restricted to the area of marine robotics. See for example [8], [17] , [20], and [21] and the references therein. However, the solutions developed so far for underactuated vehicles are restricted to two vehicles in a leader-follower type of formation and lead to complex control laws. Even in the case of fully actuated vehicles, the solutions presented do not address communication constraints explicitly. This paper builds upon and combines previous results on Path-Following (PF) control [2], [4], Coordination Control (CC) [11], [12] and brief instabilities [15].

The paper is organized as follows. Section II describes the model for a class of underactuated autonomous vehicles and formulates the path-following and vehicle coordination problems. Section III summarizes the solution to the single-vehicle path-following problem first introduced in [4]. A solution to the problem of multiple vehicle coordinated path-following is developed in Section IV when the communications topology is subjected to communication losses. Section V describes the results of simulation results. Finally, Section VI contains the main conclusions and describes problems that warrant further research.

II. Problem statement

Consider an underactuated vehicle modeled as a rigid body subject to external forces and torques. Let $\{\mathcal{I}\}$ be an inertial coordinate frame and $\{\mathcal{B}\}$ a body-fixed coordinate frame whose origin is located at the center of mass of the vehicle. The configuration $(R, \mathbf{p})$ of the vehicle is an element of the Special Euclidean group $SE(3) := SO(3) \times \mathbb{R}^3$, where $R \in SO(3) := \{R \in \mathbb{R}^{3\times 3} : RR^T = I_3, \det(R) = +1\}$ is a rotation matrix that describes the orientation of the vehicle and maps body coordinates into inertial coordinates, and $\mathbf{p} \in \mathbb{R}^3$ is the position of the origin of $\{\mathcal{B}\}$ in $\{\mathcal{I}\}$. Denoting by $v \in \mathbb{R}^3$ and $\omega \in \mathbb{R}^3$ the linear and angular velocities of the vehicle relative to $\{\mathcal{I}\}$ expressed in $\{\mathcal{B}\}$, respectively, the following kinematic relations apply:

$$\dot{\mathbf{p}} = Rv, \tag{1a}$$

$$\dot{R} = RS(\omega), \tag{1b}$$

where $S(x) := \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}, \forall x := (x_1, x_2, x_3)^T \in \mathbb{R}^3$.

We consider here underactuated vehicles with dynamic equations of motion of the following form:

$$\mathbf{M}\dot{v} = -S(\omega)\mathbf{M}v + f_v(v, \mathbf{p}, R) + g_1 u_1, \tag{2a}$$

$$\mathbf{J}\dot{\omega} = -S(v)\mathbf{M}v - S(\omega)\mathbf{J}\omega + f_\omega(v, \omega, \mathbf{p}, R) + G_2 u_2, \tag{2b}$$

where $\mathbf{M} \in \mathbb{R}^{3\times 3}$ and $\mathbf{J} \in \mathbb{R}^{3\times 3}$ denote constant symmetric positive definite mass and inertia matrices; $u_1 \in \mathbb{R}$ and $u_2 \in \mathbb{R}^3$ denote the control inputs, which act upon the system through a constant nonzero vector $g_1 \in \mathbb{R}^3$ and a constant nonsingular matrix[1] $G_2 \in \mathbb{R}^{3\times 3}$, respectively; and $f_v(\cdot)$, $f_\omega(\cdot)$ represent all the remaining forces and torques acting on the body. For the special case of an underwater vehicle, $\mathbf{M}$ and $\mathbf{J}$ also include the so-called hydrodynamic added-mass M_A and added-inertia J_A matrices, respectively, i.e., $\mathbf{M} = M_{RB} + M_A$, $\mathbf{J} = J_{RB} + J_A$, where M_{RB} and J_{RB} are the rigid-body mass and inertia matrices, respectively. See [10] for details. For each vehicle, the problem of following a predefined desired path is stated as follows:

Path-following problem: *Let $\mathbf{p}_{d_i}(\gamma_i) \in \mathbb{R}^3$ be a desired path parameterized by a continuous variable $\gamma_i \in \mathbb{R}$ and $v_{r_i}(t) \in \mathbb{R}$ a desired speed assignment for vehicle i with dynamics (1) and (2). Design feedback controller laws for u_1, u_2 (of vehicle i) and $\dot{\gamma}_i$ such that all the closed-loop signals are bounded, and the position of the vehicle i converges to and remains inside a tube centered around the desired path that can be made arbitrarily thin, i.e., $\|\mathbf{p}_i(t) - \mathbf{p}_{d_i}(\gamma_i(t))\|$ converges to a neighborhood of the origin that can be made arbitrarily small, and* ii) *satisfies a desired speed assignment v_{r_i} along the path, i.e., the speed error $|\dot{\gamma}_i(t) - v_{r_i}(t)|$ tends to zero.*

We now consider the problem of coordinated path-following (CPF) control. In the most general set-up, one is given a set of $n \geq 2$ autonomous underactuated vehicles and a set of n spatial paths $\mathbf{p}_{d_i}(\gamma_i)$; $i = 1, 2, ..., n$ and one requires that vehicle i follow path $\mathbf{p}_{d_i}$. As will become clear, the coordination problem will be solved by adjusting the speeds of the vehicles as functions of the "along-path" distances among them. Formally, the along-path distance between vehicle i and j at time t is defined as $\gamma_{ij}(t) := \gamma_i(t) - \gamma_j(t)$, that is coordination is achieved when $\gamma_{ij} = 0$ for all $i, j \in \{1, ..., n\}$ [12]. This will result in an in-line formation if, for example, the paths $\mathbf{p}_{d_i}$ are obtained as simple parallel translations of a "template" path. Next, we recall some properties from algebraic graph theory. See [14] for details.

Let N_i be the index set of the vehicles that vehicle i communicates with (the so called neighboring set of vehicle

[1] See [4] for the special case of $u_2 \in \mathbb{R}^2$ and $G_2 \in \mathbb{R}^{3\times 2}$.

i). We assume that the communication links are bidirectional, that is, $i \in N_j \Leftrightarrow j \in N_i$. Let $\mathcal{G}$ be the undirected graph induced by the underlying communication network and $L \in \mathbb{R}^{n\times n}$ its symmetric Laplacian matrix. The matrix L can be decomposed as $L = MM^T$, where $M \in \mathbb{R}^{n\times n-1}$ and $M^T\mathbf{1} = \mathbf{0}$. If the graph is connected, $\operatorname{Rank} M^T = \operatorname{Rank} L = n-1$ and consequently $M^TM > 0$; otherwise, $\operatorname{Rank} M^T < n-1$ and $M^TM \geq 0$.

Consider now the situation where the communication network changes in time so as to make the underlying communication graph $\mathcal{G}$ alternatively connected and disconnected. To study the impact of these temporary communication failures we explore the concept of "brief instabilities" developed in [15]. This concept will be instrumental in capturing the percentage of time that $\mathcal{G}$ may remain disconnected.

Consider the complete graph $\mathcal{G}$ defined on n vertices, with edges numbered $1, ..., \mathrm{m}$. Moreover, let p_i be a piecewise-continuous time-varying binary function which indicates the existence of edge i in the graph. Stack all p_i as $p = [p_i]_{\mathrm{m}\times 1}$. Denote by L_p, M_p, and $N_{i,p}$ the explicit dependence of the matrices L and M and neighboring set N_i on p, respectively. Further let P_c and P_{dc} denote the partitions of the set of parameters p, indicating connected graphs and disconnected graphs, respectively. That is, if $p \in P_c$, then the graph $\mathcal{G}(L_p)$ is connected, otherwise disconnected. Define the characteristic function of p as

$$\chi(p) := \begin{cases} 0 & p \in P_c \\ 1 & p \in P_{dc}. \end{cases} \tag{3}$$

For a given time-varying $p(t)$, let the connectivity loss time $T_p(\tau, t)$ over $[\tau, t]$ be defined as

$$T_p(\tau, t) := \int_\tau^t \chi(p(s))ds. \tag{4}$$

We will say that the communication network has *brief connectivity losses* if

$$T_p(\tau, t) \leq \alpha(t-\tau) + (1-\alpha)T_0, \ \forall t \geq \tau \geq 0 \tag{5}$$

for some $T_0 \geq 0$ and $0 \leq \alpha \leq 1$. According to this definition, α provides an asymptotic upper bound on the ratio $T_p(\tau, t)/(t-\tau)$, as $t - \tau \to \infty$ and is therefore called the *asymptotic connectivity loss rate*. When $p \in P_{dc}$ throughout an interval $[\tau, t]$, we have $T_p(\tau, t) = t - \tau$ and the above inequality requires that $t - \tau \leq T_0$. This justifies calling T_0 the *connectivity loss bound*.

The following lemma plays a key role in deriving the vehicle coordination dynamics with switching topologies.

Lemma 1: Let $\bar{M} \in \mathbb{R}^{n\times n-1}$ such that $\operatorname{Rank} \bar{M}^T = n-1$ and $\bar{M}^T\mathbf{1} = \mathbf{0}$, and $\bar{M}^T\bar{M} = I_{n-1}$. Define $U_p := M_p^T\bar{M}$ with $M_p \in \mathbb{R}^{n\times n-1}$ and $M_pM_p^T = L_p$, where the latter is the graph Laplacian. Then

1) $M_p^T = U_p\bar{M}^T$,
2) $\sigma(U_p^TU_p) = \sigma(L_p) \setminus \{0\}$, where $\sigma(.)$ denotes the spectrum of the matrix in the argument.

Proof: We first show that $\bar{M}\bar{M}^TM_p = M_p$. Since $\bar{M}^T\bar{M} = I$, then $\bar{M}\bar{M}^T$ has $n-1$ eigenvalues at 1 and one eigenvalue at 0. Thus, $\operatorname{Rank}(I - \bar{M}\bar{M}^T) = 1$ and using the fact that $(I - \bar{M}\bar{M}^T)\mathbf{1} = \mathbf{1}$, then $(I - \bar{M}\bar{M}^T)\nu = \mathbf{0}$ if $\nu \in \mathbf{1}^\perp$ (the orthogonal space). On the other hand, $M_p^T\mathbf{1} = \mathbf{0}$, that is, M_p has $n-1$ columns orthogonal to $\mathbf{1}$. Therefore $(I - \bar{M}\bar{M}^T)M_p = 0$, or $\bar{M}\bar{M}^TM_p = M_p$. Thus $M_p^T = M_p^T\bar{M}\bar{M}^T = U_p\bar{M}^T$. To prove the second part of the Lemma, notice that

$$\begin{aligned}\sigma(U_p^TU_p) = \sigma(U_pU_p^T) &= \sigma(M_p^T\bar{M}\bar{M}^TM_p) \\ = \sigma(M_p^TM_p) &= \sigma(M_pM_p^T) \setminus \{0\}.\end{aligned}$$

■

Coordination problem: *For vehicle $i = 1, ..., n$ derive a control law for $\dot{\gamma}_i$ as a function of local states and the variables γ_j, $j \in N_i$ such that $\gamma_i - \gamma_j, \forall i, j$ approach a small neighborhood of zero as $t \to \infty$ and the formation travels at the speed $v_L(t)$, that is, $\dot{\gamma}_i \to v_L$ $\forall i$.*

III. Path-following

This section provides a brief summary of the techniques derived in [2], [4] to solve the single vehicle Path-Following (PF) problem stated in Section II.

Let $e_i := R_i^T\big[\mathbf{p}_i(t) - \mathbf{p}_{d_i}(\gamma_i(t))\big]$ be the PF error of vehicle i expressed in its body-fixed frame. Borrowing from the techniques of backstepping, in [2], [4] a feedback law for u_{1_i}, u_{2_i} was derived that makes the time-derivative of the Lyapunov function

$$V_i := \frac{1}{2}e_i^Te_i + \frac{1}{2}\varphi_i^T\mathbf{M}_i^2\varphi_i + \frac{1}{2}z_{2_i}^T\mathbf{J}_iz_{2_i} \tag{6}$$

along the solutions of (1) and (2) take the form

$$\dot{V}_i = -k_{e_i}\, e_i^T\mathbf{M}_i^{-1}e_i + e_i^T\delta_i - \varphi_i^TK_{\varphi_i}\varphi_i - z_{2_i}^TK_{z_2i}z_{2_i} + \mu_i\eta_i \tag{7}$$

where φ_i and z_{2_i} are linear and angular velocity errors, k_{e_i} is a positive scalar, K_{φ_i}, K_{z_2i} are positive definite matrices, and δ_i is a small constant vector to be chosen. Further, η_i denotes the speed tracking error defined as $\eta_i := \dot{\gamma}_i - v_{r_i}$ and μ_i is a known function of the states that admits the bound

$$|\mu_i| \leq \beta_{1i}||e_i|| + (\beta_{2i}k_{e_i} + \beta_{3i})||\varphi_i|| \tag{8}$$

where β_{1i}, β_{2i} and β_{3i} are some positive values that depend on $\mathbf{M}_i$ and on the first and second derivatives of $\mathbf{p}_{d_i}(\gamma_i)$ and $v_{r_i}(t)$ with respect to γ_i and t, respectively.

In sequel, we show that the PF subsystems are input-to-state stable (ISS) from inputs η_i. See [16] and [22] for the definition of ISS.

Lemma 2: The path-following subsystem of vehicle i with inputs η_i and δ_i and states $x_{p_i} = [e_i, \varphi_i, z_{2_i}]$ is ISS.

Proof: We use (7) and (8) and Young's inequality, to compute

$$\begin{aligned}\dot{V}_i \leq \ & -(k_{e_i}m_i - \tfrac{1}{4\lambda_1} - \tfrac{\beta_{1i}}{4\lambda_2})||e_i||^2 \\ & -(k_{\varphi_i} - \tfrac{\beta_{2i}k_{e_i}+\beta_{3i}}{4\lambda_3})||\varphi_i||^2 - k_{z_2i}||z_{2_i}||^2 \\ & +\lambda_1||\delta_i||^2 + ((\beta_{2i}k_{e_i} + \beta_{3i})\lambda_3 + \beta_{1i}\lambda_2)|\eta_i|^2\end{aligned}$$

for some $\lambda_1, \lambda_2, \lambda_3 > 0$, where $m_i = ||\mathbf{M}_i^{-1}||$, $k_{\varphi_i} = ||K_{\varphi_i}||$ and $k_{z_2i} = ||K_{z_2i}||$. Choose

$$\lambda_1 = \tfrac{\gamma_1}{2\beta_{1i}^2}, \quad \lambda_2 = \tfrac{\gamma_1}{2\beta_{1i}}, \quad \lambda_3 = \tfrac{\gamma_1}{2(\beta_{2i}k_{e_i}+\beta_{3i})},$$

$$k_{e_i} = \tfrac{2\beta_{1i}^2}{\gamma_1 m_i}, \qquad k_{\varphi_i} = \tfrac{(\beta_{2i}k_{e_i}+\beta_{3i})^2}{\gamma_1}$$

for some $\gamma_1 > 0$ to get

$$\begin{aligned}\dot{V}_i \leq & -\frac{1}{2} m_i k_{e_i} ||e_i||^2 - \frac{1}{2} k_{\varphi_i} ||\varphi_i||^2 - k_{z_2 i} ||z_{2_i}||^2 \\ & + \lambda_1 ||\delta_i||^2 + \gamma_1 |\eta_i|^2 .\end{aligned}$$

Therefore there exists $\lambda_p > 0$ such that

$$\dot{V}_i \leq -\lambda_p V_i + \lambda_1 ||\delta_i||^2 + \gamma_1 |\eta_i|^2 \tag{9}$$

which completes the proof. Notice that one can make the ISS-gains small at will by making γ_1 small enough. This makes the control gains k_{e_i} and k_{φ_i} increase. ■

IV. Coordinated path-following with a switching communication topology

Consider the multiple vehicle coordination problem with a switching communication topology parameterized by p as defined in Section II. Define the "graph-induced coordination error" as $\theta := \bar{M}^T \gamma \in \mathbb{R}^{n-1}$, where $\gamma := [\gamma_i]_{n\times 1}$ is the vector of coordination states and $\bar{M}$ defined in Lemma 1. Because of the properties of $\bar{M}$, $\gamma_i = \gamma_j, \forall i, j$ is equivalent to $\theta = \mathbf{0}$. Consequently, if θ is driven to zero asymptotically, so are the coordination errors $\gamma_i - \gamma_j$ and the problem of coordinated path-following (defined in Section II) is solved.

Define the coordination control law with an auxiliary state z as

$$\begin{aligned}\dot{\gamma} &= v_L \mathbf{1} + z - A_1^{-1} L_p \gamma \\ \dot{z} &= -(A_1 + A_2) z + L_p \gamma\end{aligned} \tag{10}$$

where $A_1 = \operatorname{diag}[a_{1_i}]$ and $A_2 = \operatorname{diag}[a_{2_i}]$ are positive definite matrices. In decentralized form, (10) yields

$$\begin{aligned}\dot{\gamma}_i &= v_L + z_i - \frac{1}{a_{1_i}} \textstyle\sum_{j \in N_{i,p}} (\gamma_i - \gamma_j) \\ \dot{z}_i &= -(a_{1_i} + a_{2_i}) z_i + \textstyle\sum_{j \in N_{i,p}} (\gamma_i - \gamma_j).\end{aligned}$$

Let $x_c := (\theta, z)$ be the state of the coordination control (CC) subsystem and define

$$\begin{aligned}A_c(p) &:= \begin{pmatrix} -\bar{M}^T A_1^{-1} \bar{M} U_p^T U_p & \bar{M}^T \\ \bar{M} U_p^T U_p & -A_1 - A_2 \end{pmatrix}, \\ C_c(p) &:= \begin{pmatrix} -A_1^{-1} \bar{M} U_p^T U_p & I \end{pmatrix}.\end{aligned} \tag{11}$$

With this notation, the dynamics of x_c are governed by the the Linear Parametrically Varying (LPV) system

$$\begin{aligned}\dot{x}_c &= A_c(p) x_c \\ \eta &= C_c(p) x_c.\end{aligned} \tag{12}$$

where $\eta := [\eta_i]_{n\times 1}$ is the vector of speed error variables. We now present the main result of the paper.

Theorem 1: For any brief connectivity losses satisfying $\alpha < 1$ and bounded T_0, there exist control gains such that the interconnected system consisting of the n PF subsystems and the CC subsystem with input $\delta = [\delta_i]_{n\times 1}$ and states $x_p = [x_{pi}]_{n\times 1}$ and x_c is ISS.

To prove the theorem, we need the following lemmas.

Lemma 3: Consider the LPV system (12) with $A_1 = a_1 I, A_2 = a_2 I$. Then there exist $X > 0$, $\lambda_c > 0$ and $\lambda_d > 0$ such that

$$\begin{aligned}A_c(p) X + X A_c(p)^T &\leq -\lambda_c X, \quad \forall p \in P_c \\ A_c(p) X + X A_c(p)^T &\leq +\lambda_d X \quad \forall p \in P_{dc}.\end{aligned} \tag{13}$$

and λ_d / λ_c can be made arbitrarily small by proper choice of gains a_1 and a_2.

Proof: Let $\lambda_i \in \sigma(U_p^T U_p)$ and define $\bar{\lambda}_c := \max_{p \in P_c} \lambda_i$, and $\underline{\lambda}_c := \min_{p \in P_c} \lambda_i$.

Now, choose $X = \left(\begin{smallmatrix} I & 0 \\ 0 & xI \end{smallmatrix}\right)$ for some $x > 0$. By substituting X in (13) and using Schur's decomposition, it is straightforward to check that the inequalities in (13) are satisfied for

$$\begin{aligned}\lambda_d &= \sqrt{(a_1 + a_2)^2 + x} - (a_1 + a_2) \\ \lambda_c &= \lambda_p (a_1 + 2a_2) / (2 a_1 a_2) \\ x &= a_1^2 a_2 (\bar{\lambda}_c + \underline{\lambda}_c) / [(a_1 + 2a_2)(2 a_1 a_2 - \lambda_p)]\end{aligned} \tag{14}$$

where $\lambda_p := \bar{\lambda}_c \underline{\lambda}_c / (\bar{\lambda}_c + \underline{\lambda}_c)$. It is clear when $a_2 \to \infty$, then $\lambda_d \to 0$ and $\lambda_c \to \lambda_p / a_1$. ■

Remark 1: λ_c and λ_d computed by (14) are generally conservative. For specific communication graphs, better bounds can be obtained numerically by finding feasible solutions to the LMIs in (13).

Lemma 4: Consider the coordination control subsystem (12) with brief connectivity losses in the communication network, as defined in (5). If the asymptotic connectivity loss rate $\alpha < \lambda_c / (\lambda_c + \lambda_d)$, the states x_p and output η remain bounded and tend exponentially to zero.

Proof: Consider the control parameters as defined in Lemma 3 and define the Lyapunov function $V := x_c^T X^{-1} x_c$. The derivative of V along the solutions of (12) yields

$$\begin{aligned}\dot{V} &\leq -\lambda_c V, \quad p \in P_c \\ \dot{V} &\leq +\lambda_d V, \quad p \in P_{dc}.\end{aligned}$$

Integrating the above differential inequalities and doing computations similar to the ones in [15], it is possible to show that

$$V(t) \leq V(\tau) e^{-\lambda_c (t - \tau - T_p(\tau, t)) + \lambda_d T_p(\tau, t)}, \quad \forall t \geq \tau \geq 0$$

which yields

$$V(t) \leq e^{-[(1-\alpha)\lambda_c - \alpha \lambda_d](t - t_0)} V(t_0) e^{(1-\alpha)(\lambda_c + \lambda_d) T_0}, \forall t \geq t_0 \geq 0$$

if the system has brief connectivity losses defined in (5). From the assumptions, $\lambda := [(1 - \alpha)\lambda_c - \alpha \lambda_d] > 0$. Therefore, $V(t)$ remains bounded and tends to zero, so does x_c. Moreover, by choosing $r = \min(x^{-1}, a_1^2 / (\bar{\lambda}_c^2 + x a_1^2))$, then $r C_c(p)^T C_c(p) \leq X^{-1}, \quad \forall p$ and $\eta^T(t) \eta(t) \leq \frac{1}{r} V(t)$, thus completing the proof. ■

We are now ready to prove Theorem 1.

Proof: [Theorem 1] From Lemma 2, it follows that each path following subsystem with inputs η_i and δ_i is ISS with the gains as defined in (9). In Lemma 3, we have showed that $\frac{\lambda_d}{\lambda_c}$ can be made arbitrarily small by increasing the gain a_2. As a consequence $\alpha < 1/(1 + \frac{\lambda_d}{\lambda_c})$ and therefore, from Lemma 4, it follows that the CC subsystem is exponentially stable. Close examination of (9) and (12) shows that the CC and PF subsystems form an interconnected cascade system. Since the cascade interconnection of two ISS system is ISS, it follows that the resulting cascade system with input δ and states x_p and x_c is ISS. See [16] for details about ISS systems. ■

V. An illustrative example

This section illustrates the application of the previous results to underwater vehicles moving in three-dimensional space.

A. Path-following and coordination of underwater vehicles in 3-D space

Consider an ellipsoidal shaped underactuated AUV not necessarily neutrally buoyant. Let $\{\mathcal{B}\}$ be a body-fixed coordinate frame whose origin is located at the center of mass of the vehicle and suppose that we have available a pure body-fixed control force τ_u in the $x_{\mathcal{B}}$ direction and two independent control torques τ_q and τ_r about the $y_{\mathcal{B}}$ and $z_{\mathcal{B}}$ axes of the vehicle, respectively. The kinematics and dynamics equations of motion of the vehicle can be written as (1)–(2), where

$$\begin{aligned}
&D_v(v) = \operatorname{diag}\{X_{v_1} + X_{|v_1|v_1}|v_1|, Y_{v_2} + Y_{|v_2|v_2}|v_2|,\\
&\qquad Z_{v_3} + Z_{|v_3|v_3}|v_3|\},\\
&D_\omega(\omega) = \operatorname{diag}\{K_{\omega_1} + K_{|\omega_1|\omega_1}|\omega_1|, M_{\omega_2} + M_{|\omega_2|\omega_2}|\omega_2|,\\
&\qquad N_{\omega_3} + N_{|\omega_3|\omega_3}|\omega_3|\},\\
&\mathbf{M} = \operatorname{diag}\{m_{11}, m_{22}, m_{33}\}, \mathbf{J} = \operatorname{diag}\{J_{11}, J_{22}, J_{33}\},\\
&u_1 = \tau_u, u_2 = (\tau_q, \tau_r)^T, g_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}, G_2 = \begin{pmatrix}0&0\\1&0\\0&1\end{pmatrix},\\
&\bar{g}_1(R) = R^T\begin{pmatrix}0\\0\\W-B\end{pmatrix}, \qquad \bar{g}_2(R) = S(r_{\mathcal{B}})R^T\begin{pmatrix}0\\0\\B\end{pmatrix},\\
&f_v = -D_v(v)v - \bar{g}_1(R), \quad f_\omega = -D_\omega(\omega)\omega - \bar{g}_2(R).
\end{aligned}$$

The gravitational and buoyant forces are given by $W = mg$ and $B = \rho g \nabla$, respectively, where m is the mass of the vehicle, ρ is the mass density of the water, and ∇ is the volume of displaced water. In the simulations presented here, the physical parameters match those of the *Sirene* AUV described in [3], [1].

B. Gain selection

Consider the problem of coordinated CPF of a group of 3 vehicles, that is $n = 3$. Vehicle 2 is allowed to communicate with vehicles 1 and 3, but the latter two do not communicate between themselves directly. We considered the loss of communications, in this case failure of both links, to be 75% of the time, with the failures occurring periodically with a period of 10[sec], that is, $\alpha = 0.75$ and $T_0 = 7.5$[sec]. The corresponding eigenvalues defined in the proof of Lemma 3 are given by $\bar{\lambda}_c = 3$, $\underline{\lambda}_c = 1$ and $\bar{\lambda}_d = 2$. For $a_1 = 1$ and $a_2 = 1.6$, we have $\lambda_c = 0.98$ and $\lambda_d = 0.12$. Increasing a_2 to 5 results in $\lambda_c = 0.85$ and $\lambda_d = 0.01$. However, for $a_1 = 1$ and $a_2 \leq 1.5$, there are no λ_c and λ_d that satisfy the conditions of Lemma 3.

C. Simulation results

This section contains the results of simulations that illustrate the performance obtained with the CPF control laws developed in the paper. In the simulations three underactuated AUVs are required to follow paths of the form

$$\mathbf{p}_{d_i}(\gamma_i) = \left[c_1\cos(\frac{2\pi}{T}\gamma_i + \phi_d), c_1\sin(\frac{2\pi}{T}\gamma_i + \phi_d), c_2\gamma_i + z_{0_i}\right],$$

with $c_1 = 20\,m$, $c_2 = 0.05\,m$, $T = 400$, $\phi_d = -\frac{3\pi}{4}$, and $z_{0_1} = -10\,m, z_{0_2} = -5\,m, z_{0_3} = 0\,m$. The initial conditions of the AUVs are $\mathbf{p}_1 = (x_1, y_1, z_1) = (5\,m, -10\,m, -5\,m)$, $\mathbf{p}_2 = (x_2, y_2, z_2) = (5\,m, -15\,m, 0\,m)$, $\mathbf{p}_3 = (x_3, y_3, z_3) = (5\,m, -20\,m, 5\,m)$, $R_1 = R_2 = R_3 = I$, and $v_1 = v_2 = v_3 = \omega_1 = \omega_2 = \omega_3 = \mathbf{0}$. The reference speed v_L was set to $v_L = 0.5\,s^{-1}$.

The vehicles are also required to keep a formation pattern whereby they are aligned along a common vertical line. Figure 1 shows the trajectories of the AUVs. Figure 2 illustrates the evolutions of the coordination errors and path-following errors while the communication links fail periodically. Clearly, the vehicles adjust their speeds to meet the formation requirements and the coordination errors $\gamma_{12} := \gamma_1 - \gamma_2$ and $\gamma_{13} := \gamma_1 - \gamma_3$ converge to zero.

VI. Conclusions

The paper addressed the problem of steering a group of underactuated autonomous vehicles along given spatial paths, while holding a desired inter-vehicle formation pattern (coordinated path-following) in the presence of communication failures. A solution was derived that builds on recent results on path-following control [2], [4] and state-agreement (coordination) control [11], [12]. The effect of communication failures was addressed using the notion of brief instabilities and cascade systems. The solution proposed builds on Lyapunov based techniques and addresses explicitly the constraints imposed by the topology of the inter-vehicle communications network. Furthermore, it leads to a decentralized control law. Simulations illustrated the efficacy of the solution proposed. Further work is required to extend the methodology proposed to address the problems of robustness against communication delays.

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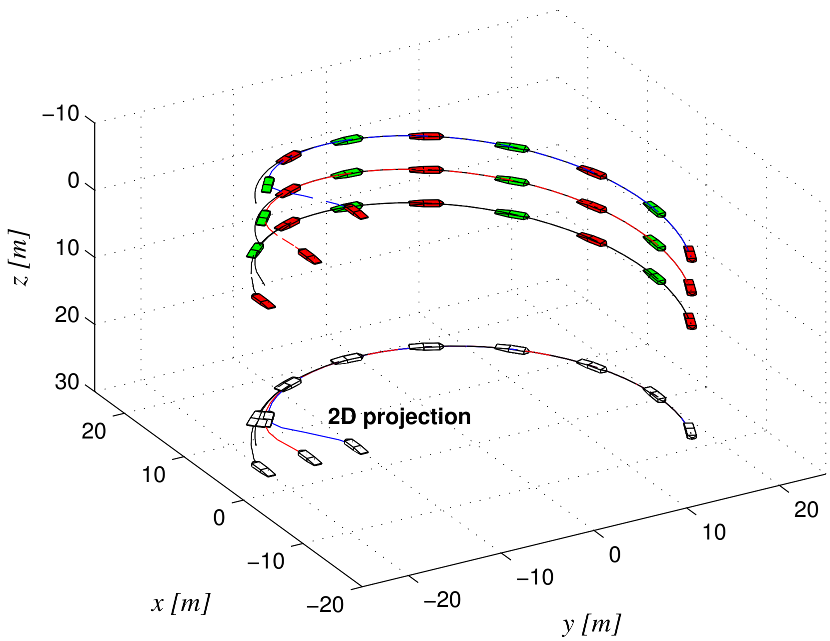

Fig. 1. Coordination of 3 AUVs, with communication failures

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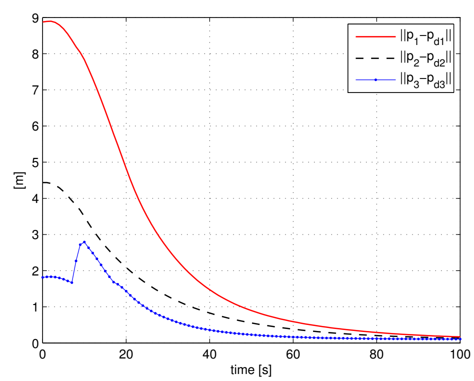
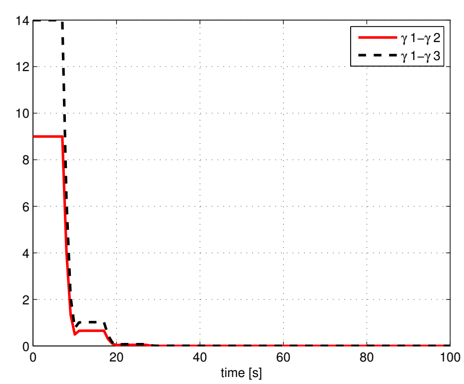


(a) path-following errors



(b) Vehicle coordination errors

Fig. 2. 75% of time communication failures