Synchronization in multi-agent systems with switching topologies and non-homogeneous communication delays

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***Abstract*— We study the synchronization problem for n single state agents with linear continuous time dynamics. The agent states are required to synchronize and travel at a desired common speed. This problem arises naturally in the design of coordinated path-following algorithms problem [9] and in studies on the synchronization of Kuramoto oscillator networks [17]. When the desired speed is zero or there are no time delays, it has been shown in the literature that a so-called neighboring control rule makes the states synchronize asymptotically under some connectivity conditions on the union of the underlying communication graphs. We will show that when both the desired speed and the communication delay are non-zero, the behavior of the synchronization system changes significantly. We start by considering asymmetric networks and switching topologies with homogeneous time delays. We then address some issues related to the behavior of the synchronization system in the presence of heterogeneous time delays. We provide connectivity conditions under which the synchronization problem is solved and introduce synchronization laws that compensates for the effect of non-zero speed and time delays. Simulations illustrate the synchronization of three agents.**

I. Introduction

Increasingly challenging mission scenarios and the advent of powerful embedded systems and communication networks have spawned widespread interest in the problem of coordinated motion control of multiple autonomous vehicles. The types of applications envisioned are numerous and include aircraft and spacecraft formation flying control [1], [10], [25], coordinated control of land robots [9], [23], control of multiple surface and underwater vehicles [6], [19], [27], and networked robots [4].

To meet the requirements imposed by these applications, a new control paradigm is needed that departs considerably from classical centralized control strategies. Centralized controllers deal with systems in which a single controller possesses all the information needed to achieve the desired control objectives (including stability and performance requirements). However, in many of the applications envisioned, because of the highly distributed nature of vehicles' sensing and actuation modules and due to the nature of the inter-vehicle communications network, it is impossible to tackle the problems in the framework of centralized control theory. For these reasons, there has been over the past few years a flurry of activity in the area of multi-agent networks with application to engineering and science problems. Namely, in such topics as parallel computing [28], synchronization of oscillators [24], [26], collective behavior and flocking [16], consensus [21], multi-vehicle formation control [5], asynchronous protocols [7], graph theory and graph connectivity [18], and Voronoi tessellation techniques [3].

In spite of significant progress in these challenging areas, much work remains to be done to develop strategies capable of yielding robust performance of a fleet of vehicles in the presence switching communication networks, information transmission time delays, and severe communication constraints. These difficulties are specially challenging in the field of marine robotics, since underwater communications and positioning rely heavily on acoustic systems, which are plagued with intermittent failures, latency, and multipath effects. This paper addresses some of the problems that arise in these kinds of applications. We review existing results and highlight their most important properties and develop synchronization laws to make a set of agents asymptotically synchronize and travel at a desired speed. We show that a simple neighboring rule fails to synchronize the agents in the case of latency in the exchange of information.

The key contributions of the paper are clearly reflected in its organization. Section II introduces the motivation for the proposed problem, that is, decentralized synchronization control of a network of single state agents with continuous dynamics under switching communication topologies and time delays and formulates the problem under consideration. Section III provides some preliminaries of Graph theory and reviews existing results in the literature on agreement problems. Section IV presents the main results of the paper and shows that the behavior of the coordination dynamics changes significantly in the presence of switching communication networks and time delays if the states are required to travel at a non-zero speed. We further establish the conditions under which synchronization is recovered, and provide some synchronization rules that maintain the common connectivity conditions. Several examples in Section V compare the behavior of the synchronization closed-loop systems in different situations. Section VI contains the main conclusions.

II. Motivation

Consider a group of n vehicles numbered $1, .., n$ and the problem of steering the vehicles along given paths while holding a desired formation pattern. The solution to this problem, henceforth referred to as the Coordinated Path-Following problem, unfolds in two basic steps. First, a path-following control law is used to drive each vehicle to its assigned path regardless of the temporal speed profile adopted.

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This is done by making each vehicle approach a conveniently defined virtual target that moves along the path. In the second step, the speeds of the virtual targets are adjusted so as to synchronize their positions (also called coordination states) along the paths, thus achieving coordination, while making the formation as a whole travel at a given speed $v_L(t)$. Since the vehicles converge to their corresponding virtual targets that will synchronize asymptotically, the closed-loop error signals tend to zero and the vehicles asymptotically reach a desired formation[1]. Following the nomenclature in [8], each path to be followed is parameterized by $\gamma_i \in \mathbb{R}$ and the vehicles reach the desired formation pattern iff $\gamma_i = \gamma_j; \forall i, j \in \mathcal{N} := \{1, ..., n\}$. In a simplified version, the dynamics of each state are governed by

$$\dot{\gamma}_i(t) = v_{r_i}(t); \;\; i \in \mathcal{N}, \tag{1}$$

where the speed-profiles v_{r_i} are taken as control signals that must be assigned to yield coordination of the states γ_i. To achieve this objective, information is exchanged over an inter-vehicle communications network. Typically, all-to-all communications are impossible to achieve. In general, v_{r_i} will be a function of γ_i and of the coordination states of the so-called neighboring vehicles defined by the set N_i that represents the set of vehicles that vehicle i communicates with. We will consider asymmetric communications networks where the communication links may be directional, that is, $i \in N_j$ does not necessarily imply $j \in N_i$. Another important issue in coordination control systems is delayed information. In a number of applications involving underwater cooperative control systems that rely on acoustic communications, time delays in the communication channels become significant. In this paper, we start by addressing homogenous time delays, that is, we assume that all the transmission links introduce the same time delay $\tau \geq 0$. Some of the issues studied will be further analyzed in the context of synchronization systems with heterogeneous time delays.

Definition 1: Coordination Consider a set of agents $i \in \mathcal{N}$ with dynamics (1), and a formation speed assignment $v_L(t)$. Assume γ_i and γ_j; $j \in N_i$ are available to vehicle $i \in \mathcal{N}$. Derive a control law for v_{r_i} such that the coordination errors $\gamma_i - \gamma_j$ and the formation speed tracking errors $\dot{\gamma}_i - v_L$; $\forall i, j \in \mathcal{N}$ converge to zero as $t \to \infty$.

A similar problem arises in the synchronization of networks of Kuramoto oscillators. See [24], [26] for further details.

With the set-up adopted, Graph Theory becomes the tool par excellence to model the constraints imposed by the communication topology among the vehicles, as embodied in the definition of sets N_i; $i \in \mathcal{N}$. We now recall some key concepts from algebraic graph theory [11] and agreement algorithms and derive some basic tools that will be used in the sequel.

[1]See [9], [8] and [15] where coordinated path-following is addressed.

III. Preliminaries and basic results

A. Graph theory

Let $\mathcal{G}(\mathcal{V}, \mathcal{E})$ (abbv. $\mathcal{G}$) be a *directed graph* or *digraph* induced by the inter-vehicle communication network, with $\mathcal{V}$ denoting the set of n *nodes* (each corresponding to a vehicle) and set $\mathcal{E}$ of ordered pairs $(V_i, V_j) \in \mathcal{E}$, henceforth referred to as *arcs* (each standing for a data link). Given an arc $(V_i, V_j) \in \mathcal{E}$, its first and second elements are called the *tail* and *head* of the arc, respectively. It is assumed that the flow of information in an arc is directed from its head to its tail. The *in-degree* (*out-degree*) of a node V_i is the number of arcs with V_i as its head (tail). If $(V_i, V_j) \in \mathcal{E}$, then we say that V_i and V_j are *adjacent*.

A *path* of length r from V_i to V_j in a digraph is a sequence of $r+1$ distinct nodes starting with V_i and ending with V_j such that consecutive nodes are adjacent. If there is a path in $\mathcal{G}$ from node V_i to node V_j, then V_j is said to be *reachable* from V_i. In this case, there is a path of consecutive communication links directed from vehicle j (transmitter) to vehicle i (receiver). A node V_i is *globally reachable* if it is reachable from every other node. Graph $\mathcal{G}$ is *quasi strongly connected* (QSC) if it has a globally reachable node[2].

The *adjacency matrix* of a digraph $\mathcal{G}$, denoted A, is a square matrix with rows and columns indexed by the nodes, such that the i, j-entry of A is 1 if $(V_i, V_j) \in \mathcal{E}$ and zero otherwise. The *degree matrix* D of a digraph $\mathcal{G}$ is a diagonal matrix where the i, i-entry equals the out-degree of node V_i. Notice that the out-degree of node V_i equals $|N_i|$, the cardinality of N_i. The *Laplacian* of a digraph is defined as $L = D - A$. Every row sum of L equals zero, that is, $L\mathbf{1} = \mathbf{0}$, where $\mathbf{1} := [1]_{n \times 1}$ and $\mathbf{0} := [0]_{n \times 1}$.

We will be dealing with situations where the communication links are time-varying in the sense that links can appear and disappear (switch) due to intermittent failures and/or communication links scheduling. The mathematical set-up required is described next.

A complete graph is a graph with all possible arcs (an arc between each pair of nodes). Let $\mathcal{G}$ be a complete graph with arcs numbered $1, ..., \bar{n}$. Consider a communication network among n vehicles. To model the underlying switching communication graph, let $p = [p_i]_{\bar{n} \times 1}$, where each $p_i(t) : [0, \infty) \to \{0, 1\}$ is a piecewise-continuous time-varying binary function which indicates the existence of arc i in the graph $\mathcal{G}$ at time t. Therefore, given a switching signal $p(t)$, the dynamic communication graph $\mathcal{G}_{p(t)}$ is the pair $(\mathcal{V}, \mathcal{E}_{p(t)})$, where, if $i \in \mathcal{E}_{p(t)}$ then $p_i(t) = 1$, otherwise $p_i(t) = 0$. For example, $p(t) = [1, 0, ..., 0]^T$ means that at time t only link number 1 is active. Denote by L_p the explicit dependence of the graph Laplacian on p and likewise for the degree matrix D_p and adjacency matrix A_p. Further let $N_{i,p(t)}$ denote the set of the neighbors of vehicle i at time t.

We discard infinitely fast switchings. Formally, let S_{dwell} denote the class of piecewise constant switching signals such that any consecutive discontinuities are separated by no less than some fixed positive constant time τ_D, the dwell time. We assume that $p(t) \in S_{dwell}$. See [13] for more details.

[2]In [20] a digraph is defined as QSC if its opposite digraph has a globally reachable node.

B. Switching topologies and connectedness

We will consider switching topologies where the communication graph may fail to be connected at any instant of time; however, we assume there is a finite time $T > 0$ such that over any interval of length T the union of the different graphs is somehow connected. This statement will be made precise in the sequel. We now present some key results for time-varying communication graph and agreement problems that borrow from [20], [21], [22].

Let $\mathcal{G}_i$; $i = 1, ..., q$ be q graphs defined on n nodes, and denote by L_i their corresponding graph Laplacians. Define the *union graph* $\mathcal{G} = \cup_i \mathcal{G}_i$ as the graph whose arcs are obtained from the union of the arcs $\mathcal{E}_i$ of $\mathcal{G}_i$; $i = 1, ..., q$. Since $p \in S_{dwell}$ (only a finite number of switchings are allowed over any bounded time interval), the union graph is defined over time intervals in the obvious manner. Formally, given two real numbers $0 \leq t_1 \leq t_2$, the union graph $\mathcal{G}([t_1, t_2))$ is the graph whose arcs are obtained from the union of the arcs $\mathcal{E}_{p(t)}$ of graph $\mathcal{G}_{p(t)}$ for $t \in [t_1, t_2)$.

Definition 2: Uniformly Quasi Strongly Connected A switching communication graph $\mathcal{G}_{p(t)}$ is *uniformly quasi strongly connected* (UQSC), if there exists $T > 0$ such that for every $t \geq 0$ the union graph $\mathcal{G}([t, t + T))$ is QSC.

C. Agreement

Consider n agents with dynamics (1) with a supporting communication network whose underlying communication graph is modeled by $\mathcal{G}_p$ subjected to the switching signal $p(t)$. Assume that the communication network causes no delays in delivering the information and let the control signals v_{r_i} be computed as

$$v_{r_i} = -k_i \sum_{j \in N_{i,p(t)}} \gamma_i(t) - \gamma_j(t) \tag{2}$$

(the so-called neighboring rule), where $k_i > 0$. Then the closed-loop system takes the form of linear switching dynamics

$$\dot{\gamma} = -KL_p\gamma \tag{3}$$

where $\gamma = [\gamma_i]_{n\times 1}$, K is a positive definite diagonal matrix and L_p is the Laplacian matrix of dynamic graph $\mathcal{G}_p$. It is known, see for example [21], that

Theorem 1: Agreement Consider n variables γ_i with dynamics (3) and assume the switching communication graph $\mathcal{G}_{p(t)}$ is UQSC. Then the coordination errors $\gamma_i - \gamma_j; \forall i, j \in \mathcal{N}$ converge to zero and $\dot{\gamma}_i \to 0; \forall i \in \mathcal{N}$ as $t \to \infty$; (exponentially fast). We say the agreement problem is solved.

To make the states γ_i travel with a desired speed profile $v_L(t)$ while synchronized, modify (2) to

$$v_{r_i} = v_L(t) - k_i \sum_{j \in N_{i,p(t)}} \gamma_i(t) - \gamma_j(t) \tag{4}$$

which in vector form yields

$$\dot{\gamma} = v_L\mathbf{1} - KL_p\gamma. \tag{5}$$

To show that the states synchronize make the change of variables

$$\tilde{\gamma} = \gamma - \mathbf{1}\int_0^t v_L(s)ds.$$

Using (5), the dynamics of $\tilde{\gamma}$ can be written as

$$\dot{\tilde{\gamma}} = -KL_p\tilde{\gamma}.$$

If the dynamic graph $\mathcal{G}_p$ is UQSC, according to Theorem 1, $\tilde{\gamma}_i - \tilde{\gamma}_j$ and $\dot{\tilde{\gamma}}_i$ tend to zero as $t \to \infty$, for all $i, j \in \mathcal{N}$. Therefore $\gamma_i - \gamma_j$ and $\dot{\gamma}_i - v_L$ converge to zero, exponentially fast.

We now consider the delayed version of (3). Let the coordination states γ_i evolve according to

$$\dot{\gamma}(t) = -KD_{p(t)}\gamma(t) + KA_{p(t)}\gamma(t - \tau) \tag{6}$$

where $D_{p(t)}$ and $A_{p(t)}$ are the degree matrix and the adjacency matrix of $\mathcal{G}_{p(t)}$, respectively. The following statement can be derived from [22].

Theorem 2: Agreement-delayed information Consider n variables γ_i with dynamics (6) and $\tau > 0$, and assume the switching communication graph $\mathcal{G}_{p(t)}$ is UQSC. Then, the results of Theorem 1 are valid.

The main results of the paper are stated in the next section where both $v_L(t)$ and τ are nonzero. It will be shown that the behavior of the closed-loop dynamics change significantly. We will also provide extra conditions needed to guarantee similar results, that is, coordination of the states γ_i while traveling at speed v_L.

IV. Synchronization: Delayed Information and Switching Topologies

Consider the coordination problem (Definition 1) for the case where the communication channels have homogenous time delays $\tau > 0$. In this case, the neighboring control law for the reference speed v_{r_i} becomes a function of delayed information, that is

$$v_{r_i} = v_L - k_i \sum_{j \in N_{i,p(t)}} \gamma_i(t) - \gamma_j(t - \tau). \tag{7}$$

Using (1) and (7), the closed-loop coordination system can be written as

$$\dot{\gamma}(t) = v_L\mathbf{1} - KD_{p(t)}\gamma(t) + KA_{p(t)}\gamma(t - \tau). \tag{8}$$

We are now interested in determining conditions under which coordination is achieved, that is, in the existence of a continuous time signal $\gamma_0(t) \in \mathbb{R}$ such that $\gamma = \gamma_0(t)\mathbf{1}$ is a solution of (8). If this is the case, then by substituting this "solution" in (8) and using the fact that $A_p = D_p - L_p$, we obtain

$$\dot{\gamma}_0\mathbf{1} = v_L\mathbf{1} - KD_p\gamma_0(t)\mathbf{1} + K(D_p - L_p)\gamma_0(t - \tau)\mathbf{1}$$

which simplifies to

$$(\dot{\gamma}_0 - v_L)\mathbf{1} = -(\gamma_0(t) - \gamma_0(t - \tau))KD_p\mathbf{1}. \tag{9}$$

Here, we used the fact that $L_p\mathbf{1} = \mathbf{0}$. Equality in (9) is valid iff all the rows of the right-hand side are equal for all time. Two cases are possible.

p1 $\gamma_0(t)$ is either a constant or a periodic signal with period τ. In this case $\gamma_0(t) - \gamma_0(t - \tau) = 0$ and the right-hand side of (9) equals zero. Thus (9) holds with $\dot{\gamma}_0 = v_L$ where the formation speed v_L

must be set to either zero or a periodic signal with period τ. These are not of interest from a practical standpoint.

p2 $\forall t, KD_{p(t)} = kI$ for some $k > 0$. This requires that the out-degrees of the nodes of the switching communication graph $\mathcal{G}_p$ never vanish, that is, $|N_{i,p}| \neq 0$, $\forall t$, so that the degree matrix is always nonsingular and we can set the control gains to $K = kD_p^{-1}$. Therefore, the control gains become piecewise constant as functions of p.

Next, we will address case **p2** and state the first result of this section. To lift the constraint $|N_{i,p}| \neq 0$ and have a coordination algorithm applicable to more general types of switching topologies, we will later modify control law (7).

Proposition 1: Consider n agents with dynamics (1) and neighboring feedback control law (7). Assume that $N_{i,p(t)} \neq \emptyset$ (the empty set) for all t, and let the control gains be $k_i(t) = k/|N_{i,p(t)}|$. Then, if the underlying communication graph $\mathcal{G}_p$ is UQSC, $|\gamma_i - \gamma_j| \to 0$ and $\dot\gamma_i \to \dot\gamma_0$ as $t \to \infty$, where γ_0 is a solution of the delay differential equation

$$\dot\gamma_0 = -k(\gamma_0(t) - \gamma_0(t-\tau)) + v_L. \tag{10}$$

Moreover, if v_L is constant $\dot\gamma_i \to \frac{v_L}{1+k\tau}$; $\forall i \in \mathcal{N}$.

Proof: As explained above, with the control law (7), the coordination system takes the form (8). Let

$$\tilde\gamma(t) = \gamma(t) - \gamma_0(t)\mathbf{1} \tag{11}$$

and substitute γ from (11) in (8) to get

$$\begin{aligned}&\dot\gamma_0(t)\mathbf{1} + \dot{\tilde\gamma} = \\ &\quad v_L\mathbf{1} - K(t)D_{p(t)}\tilde\gamma(t) + K(t)A_{p(t)}\tilde\gamma(t-\tau)+ \\ &\quad -\gamma_0(t)K(t)D_{p(t)}\mathbf{1} + \gamma_0(t-\tau)K(t)A_{p(t)}\mathbf{1}\end{aligned} \tag{12}$$

which simplifies to

$$\dot{\tilde\gamma} = -k\tilde\gamma(t) + kD_p^{-1}A_p\tilde\gamma(t-\tau) \tag{13}$$

if $\gamma_0(t)$ is the solution of (10) and $K(t) = kD_p^{-1}$. From Theorem 2, the error states $\tilde\gamma_i - \tilde\gamma_j$ and $\dot{\tilde\gamma}_i$ vanish exponentially. Consequently, $\gamma_i - \gamma_j = \tilde\gamma_i - \tilde\gamma_j \to 0$ and $\dot\gamma_i \to \dot\gamma_0$ as $t \to \infty$.

For the particular case of constant v_L, one solution to (10) is $\gamma_0(t) = v_L^* t$ where $v_L^* = \frac{v_L}{1+k\tau}$, and the result follows. ■

Notice that due to the transmission delay τ, there is a finite error in the speed tracking, that is, $\dot\gamma_i$ converges to v_L^* and not to v_L.

Consider now the case where there are instants of time t for which $N_{i,p(t)} = \emptyset$ for some $i \in \mathcal{N}$. We will present this part only for the case where v_L is constant, that is, $\gamma_0(t) = v_L^* t$ and $v_L^* = \frac{v_L}{1+k\tau}$. In this case, (8) can be rewritten in terms of $\tilde\gamma$ defined in (11) as

$$\dot{\tilde\gamma} = -K(t)D_{p(t)}\tilde\gamma(t) + K(t)A_{p(t)}\tilde\gamma(t-\tau) + v_L^*\tau(kI - K(t)D_p)\mathbf{1}. \tag{14}$$

Clearly, when $\tau = 0$ or $v_L = 0$ agreement is achieved for any choice of positive definite K, due to Theorem 2. However, this is not the case when $\tau \neq 0$ and $v_L \neq 0$. For example, assume that the agreement dynamics (14) are at rest, that is, $\dot{\tilde\gamma}_i = 0$, and $\tilde\gamma_i = \tilde\gamma_j$ $\forall i, j \in \mathcal{N}$. Then, at the time that $|N_{i,p(t)}| = 0$ for some i, the dynamics (14) are disturbed by a signal of amplitude $v_L^*\tau k = v_L - v_L^*$ through the channel (row) i.

This problem arises from the fact that if agreement is reached the formation traveling speed must be v_L^*. However, during the interval where $N_{i,p} = \emptyset$, the corresponding coordination state is governed by the dynamics $\dot\gamma_i = v_L$. This contradiction can be resolved by applying different reference speeds when a particular vehicle has no neighbors to communicate with. The solution is stated next.

Proposition 2: Consider n agents with dynamics (1) and feedback control law

$$v_{r_i} = \begin{cases} v_L - \frac{k}{|N_{i,p}|}\sum_{j\in N_{i,p}} \gamma_i(t) - \gamma_j(t-\tau), & N_{i,p} \neq \emptyset \\ v_L^*, & N_{i,p} = \emptyset \end{cases} \tag{15}$$

Then, if the underlying communication graph $\mathcal{G}_p$ is UQSC, $|\gamma_i - \gamma_j| \to 0$ and $\dot\gamma_i \to v_L^*$ as $t \to \infty$.

Proof: The closed-loop coordination dynamics in vector form is given by

$$\dot\gamma = -KD_p\gamma(t) + KA_p\gamma(t-\tau) + \frac{v_L - v_L^*}{k}KD_p\mathbf{1} + v_L^*\mathbf{1}.$$

where $K = \mathrm{diag}[k_i]$ and

$$k_i = \begin{cases} \frac{k}{|N_{i,p}|}, & N_{i,p} \neq \emptyset \\ 1, & N_{i,p} = \emptyset \end{cases}$$

Letting $\tilde\gamma(t) = \gamma(t) - v_L^* t\mathbf{1}$ simplifies the closed-loop dynamics to

$$\dot{\tilde\gamma} = -KD_p\tilde\gamma(t) + KA_p\tilde\gamma(t-\tau).$$

From Theorem 2, agreement is achieved and the results follow. ■

Notice that to implement the control law (15), the agents need to know the delay τ to compute v_L^*. This raises the question of estimating v_L^*. This issue will not be addressed in the paper.

We showed that an agreement protocol based on a simple neighboring rule over a switching network with delayed information leads to undesirable results. We then proved that this problem can be avoided if the time delay is known. We now study the case where the time-delays are nonhomogeneous, that is, each communication link may exhibit a different time-delay. As in the previous case, we assume the time delays can be measured. Let the communication links exhibit m distinct time delays $\{\tau_r \geq 0; r = 1, ..., m\}$ and let $N_{i,p}^r$ denote the neighbors of agent i whose information arrives with τ_r delay.

Let the control signal for agent $i \in \mathcal{N}$ be given by

$$\dot\gamma_i = v_L - k_i \sum_{r=1}^{m}\sum_{j\in N_{i,p}^r}[\gamma_i(t) - \gamma_j(t-\tau_r) - v_L\tau_r], \tag{16}$$

It is easily shown that (16) compensates for delays. Define a state transformation as $\tilde\gamma_i(t) = \gamma_i(t) - v_L t$ and substitute in (16) to obtain

$$\dot{\tilde\gamma}_i = -k_i \sum_{r=1}^{m}\sum_{j\in N_{i,p}^r}[\tilde\gamma_i(t) - \tilde\gamma_j(t-\tau_r)], \tag{17}$$

In general, proving that the states $\tilde\gamma_i$ governed by the above equations agree asymptotically is not an easy task. We conjecture that this is true if the dynamic graph is UQSC. This property has been shown for similar discrete dynamics in [2] and [29]. In what follows we address the case where the graphs are fixed.

To write the above equations in compact vector form, let the associated arcs with the time delay τ_r define a subgraph $\mathcal{G}_r$ with corresponding degree matrix D_r and adjacency matrix A_r. Since to each arc there is only one delay associated, the subgraphs are disjoint and $D = \sum_{r=1}^m D_r$ and $A = \sum_{r=1}^m A_r$, where D and A are the degree matrix and adjacency of the main (or union) graph, respectively. Then (17) yields

$$\dot{\tilde{\gamma}} = -KD\tilde{\gamma}(t) + K\sum_{r=1}^{m} A_r\tilde{\gamma}(t-\tau_r), \tag{18}$$

where $K = \mathrm{diag}[k_i]_{n\times n}$ is a positive definite matrix. We assume the underlying communication graph is symmetric, that is, $A_r = A_r^T; \forall r = 1, ..., m$. We are now ready to state the main result of the paper.

Proposition 3: Consider n agents with a fixed symmetric communication network that exhibits m time delays $\tau_r; r = 1, ..., m$. Then, the control law given by (16) solves the coordination problem (Definition 1) if the underlying communication graph is QSC [3]. In particular, the coordination errors $\gamma_i - \gamma_j$ and the speed tracking errors $\dot{\gamma}_i - v_L; \forall i, j \in \mathcal{N}$ converge asymptotically to zero as $t \to \infty$.

Proof: Consider the Lyapunov-Krasovskii functional

$$V = \tilde{\gamma}^T K^{-1}\tilde{\gamma} + \sum_{r=1}^{m}\int_{-\tau_r}^{0}\tilde{\gamma}(\nu)^T D_r\tilde{\gamma}(\nu)d\nu$$

whose time derivative along the solutions of (18) yields

$$\begin{aligned}\dot{V} &= 2\tilde{\gamma}^T(-D\tilde{\gamma} + \textstyle\sum_r A_r\tilde{\gamma}_{\tau_r}) + \sum_r(\tilde{\gamma}^T D_r\tilde{\gamma} - \tilde{\gamma}_{\tau_r}^T D_r\tilde{\gamma}_{\tau_r})\\ &= -\textstyle\sum_r \tilde{\gamma}^T D_r\tilde{\gamma} - 2\tilde{\gamma}^T A_r\tilde{\gamma}_{\tau_r} + \tilde{\gamma}_{\tau_r}^T D_r\tilde{\gamma}_{\tau_r}\end{aligned}$$

where $\tilde{\gamma}_{\tau_r} = \tilde{\gamma}(t-\tau_r)$. Using Gersgorin theorem [14] and the fact that $L_r = D_r - A_r \le 0$, it is easily shown that

$$\mathcal{L}_r := \begin{pmatrix} D_r & -A_r \\ -A_r & D_r \end{pmatrix} \le 0; \forall r = 1, ..., m. \tag{19}$$

Therefore, $\dot{V} \le 0$. Let $\Omega := \{\mathrm{col}(\tilde{\gamma}, \tilde{\gamma}_{\tau_1}, ..., \tilde{\gamma}_{\tau_r}) : \dot{V} = 0\}$. Using (18) and the fact that $D_r\tilde{\gamma} = A_r\tilde{\gamma}_{\tau_r}$ on Ω, clearly $\dot{\tilde{\gamma}} = \mathbf{0}$ on Ω. That is, Ω is an invariant set. Since $\tilde{\gamma}$ is constant, $\tilde{\gamma}_{\tau_r} = \tilde{\gamma}$, then $D_r\tilde{\gamma} = A_r\tilde{\gamma}$, $L_r\tilde{\gamma} = \mathbf{0}$, and $L\tilde{\gamma} = \mathbf{0}$. Because the graph is QSC, $\tilde{\gamma} \in \mathrm{span}\{\mathbf{1}\}$ on Ω. The result follows by using LaSalle invariance principle. See [12] for details on stability in the sense of Lyapunov for delayed systems. ■

V. Examples

In this section, we consider a simple synchronization problem of three agents numbered $\{1, 2, 3\}$ in three situations:

1) with and without switching topologies
2) with and without time delays
3) zero and nonzero v_L

In all the simulations, the control gains were set to $K = 0.5I_3$ and the following communication switching topology were applied. A communication network periodically switch between neighboring sets $N_1 = \{1, 2\}, N_2 = \emptyset, N_3 = \emptyset$ and $N_1 = \emptyset, N_2 = \{1\}, N_3 = \{1\}$, with period 10[s] and duty cycle 50%. In Figures 1-4, the coordination errors $\gamma_1 - \gamma_2$ and $\gamma_1 - \gamma_3$ are shown.

Fig. 1 shows the coordination errors generated by control signals of the form (4) with $v_L = 1$[s^{-1}]. The switching topology reduces the convergence rate, but the states agree exponentially in both cases.

Fig. 2 illustrates the effect of a homogeneous time delay $\tau = 2$[s] with the control law of the form (7) and $v_L = 1$[s^{-1}]. Clearly, the states do not reach agreement.

Fig. 3 shows the evolution of the coordination errors for nonhomogeneous time delays. Links $(1, 2)$, $(2, 1)$, and links $(1, 3)$, $(3, 1)$ have time delays $\tau_1 = 2$[s] and $\tau_2 = 1$[s], respectively. The control law of the form (7) with $v_L = 1$[s^{-1}] is applied. Again, no agreement is obtained.

Fig. 4(a) depicts a similar situation but with $v_L = 0$. As shown before, *contrary* to the previous case the states reach agreement.

From the above simulations, one can conclude that the agreement fails in cases where both the desired speed v_L and the time delays are nonzero.

Fig. 4(b) shows how the control signal (16) can compensate for the time delays. The simulations were done for nonzero v_L, switching topologies, and nonhomogeneous time delays.

VI. Conclusion

We studied the synchronization problem of n agents with continuous time linear dynamics that are required to travel at a desired speed.

We considered asymmetric networks, switching topologies with homogenous time delays, and provided the connectivity conditions under which the synchronization problem is solved.

We showed that when both the desired speed and the communication delay are non-zero, the application of a simple neighboring control rule does not yield asymptotic synchronization.

Assuming that the time delays are known, we derived synchronization laws that compensate for the effect of nonzero speed and time delays. Several simulations illustrate the performance obtained for the synchronization of three agents in the presence of switching communication networks and delayed information.

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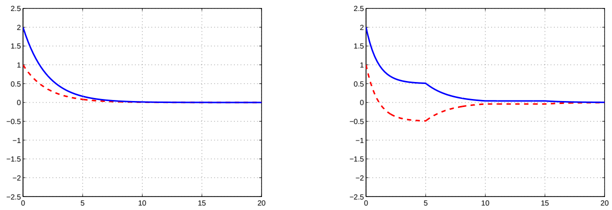
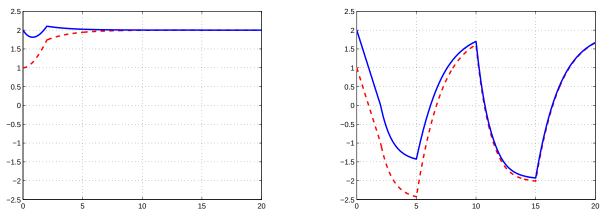
[3] When the graph is symmetric, being QSC is equivalent to being connected.



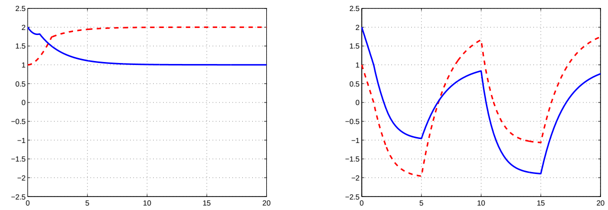
(a) No switchings (b) With switchings

Fig. 1. Synchronization with no time delays, $v_L = 1[\mathrm{s}^{-1}]$. Solid line - $\gamma_1 - \gamma_2$; Dash line - $\gamma_1 - \gamma_3$



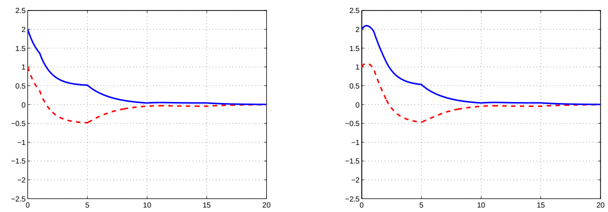
(a) No switchings (b) With switchings

Fig. 2. Synchronization with homogeneous delay $\tau = 2[\mathrm{s}]$, $v_L = 1[\mathrm{s}^{-1}]$. Solid line - $\gamma_1 - \gamma_2$; Dash line - $\gamma_1 - \gamma_3$



(a) No switchings (b) With switchings

Fig. 3. Synchronization with non-homogeneous delays $\tau_1 = 2[\mathrm{s}]$, $\tau_2 = 1[\mathrm{s}]$, $v_L = 1[\mathrm{s}^{-1}]$. Solid line - $\gamma_1 - \gamma_2$; Dash line - $\gamma_1 - \gamma_3$



(a) Without delay compensation, $v_L = 0\ \mathrm{s}^{-1}$ (b) With delay compensation, $v_L = 1\ \mathrm{s}^{-1}$

Fig. 4. Synchronization with non-homogeneous delays $\tau_1 = 2[\mathrm{s}]$, $\tau_2 = 1[\mathrm{s}]$. Solid line - $\gamma_1 - \gamma_2$; Dash line - $\gamma_1 - \gamma_3$

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