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# Regulation of a Nonholonomic Autonomous Underwater Vehicle with Parametric Modeling Uncertainty using Lyapunov Functions

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**Abstract**

This paper addresses the problem of regulating the dynamic model of a nonholonomic underactuated autonomous underwater vehicle (AUV) to a point with a desired orientation. A time-invariant discontinuous controller is proposed that yields convergence of the trajectories of the closed-loop system in the presence of parametric modeling uncertainty. Controller design relies on a non smooth coordinate transformation in the original state space followed by the derivation of a Lyapunov-based, adaptive, smooth control law in the new coordinates. Convergence of the regulation system is analyzed and simulation results are presented.

## 1 Introduction

### 1.1 Practical Motivation

Recently, there has been renewed interest in the development of stationary benthic stations to carry out experiments on the biology, geochemistry, and physics of deep sea sediments and hydrothermal vents in situ, over long periods of time. However, current methods of deploying and servicing benthic laboratories are costly and require permanent support from specialized crews resident on board manned submersibles or surface ships. As a contribution to overcoming some of the abovementioned problems, a European team led by IFREMER, France developed a prototype autonomous underwater shuttle vehicle named SIRENE to automatically transport and position a large range of stationary benthic laboratories on the seabed, at a desired target point, down to depths of 4000 meters. The reader is referred to [4] for a general description of the project carried out by the partners IFREMER (FR), IST (PT), THETIS (GER), and VWS (GER).

The Sirene autonomous underwater vehicle (AUV) - depicted in Figure 1 - has an open-frame structure and is 4.0 m long, 1.6 m wide, and 1.96 m high. It has a dry weight of 4000 Kg and a maximum operating depth of 4000 m. The vehicle is equipped with two back thrusters for surge and yaw motion control in the horizontal plane, and one vertical thruster for heave control. In the figure, the vehicle carries a representative benthic lab which is cubic-shaped with a volume of approximately $2.3m^3$. The dynamic model of the Sirene can be found in [2].

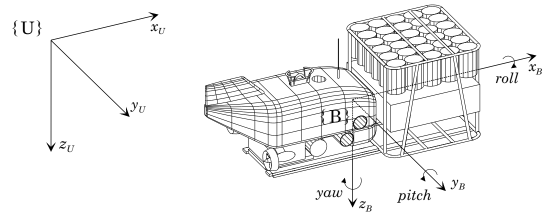


**Figure 1:** The vehicle SIRENE coupled to a benthic laboratory. Body-fixed $\{B\}$ and earth-fixed $\{U\}$ reference frames

### 1.2 Underactuated AUVs

The problem of steering an underactuated AUV to a point with a desired orientation has only recently received special attention. This task raises some challenging questions in control system theory, because the vehicle is underactuacted and, in general, falls in the category of so-called nonholonomic systems. Furthermore, as will be shown, its dynamics are complicated due to the presence of complex hydrodynamic terms. This rules out any attempt to design a steering system for the AUV that would rely on its kinematic equations only. Pioneering work in this field is reported in [8], where open loop small-amplitude periodic time-varying control laws are used to re-position and re-orient underactuated AUVs. A feedback control law that gives exponential convergence of a nonholonomic AUV to a constant desired configuration is introduced in [5]. The design of a continuous, periodic feedback control law that asymptotically stabilizes an underactuated AUV and yields exponential convergence to the origin is described in [9]. In [10], a time-varying feedback control law is proposed that yields global practical stabilization and tracking for an underactuated ship using a combined integrator backstepping and averaging approach.

It is important to point out that some of the control laws developed so far for underactuated underwater vehicles do not take explicitly into account their dynamics and are therefore unrealistic. Furthermore, even when the dynamics are taken into account the resulting closed loop system trajectories are often not "natural".

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### 1.3 Outline

Motivated by the above considerations, this paper addresses the problem of regulating a nonholonomic underactuated AUV in the *horizontal plane* to a point with a desired orientation. *A discontinuous, adaptive state feedback controller is derived that yields convergence of the trajectories of the closed loop system in the presence of parametric modeling uncertainty.* This is achieved by resorting to a polar representation of the kinematic model of the AUV that relies on a non smooth transformation in the original state space, followed by the derivation of a smooth, time-invariant control law in the new coordinates. The design of the new control algorithm proposed, together with the analysis of its convergence properties, build on Lyapunov stability theory, LaSalle's invariance principle, and backstepping design techniques [7]. The reader will find in [1] an introduction to the circle of ideas explored in this paper, as well as their application to the regulation of a nonholonomic wheeled robot with parametric modeling uncertainty.

## 2 The AUV. Control Problem Formulation

This section describes the kinematic and dynamic equations of motion of the AUV of Figure 1 in the horizontal plane, and formulates the problem of controlling it to a point with a desired orientation. The control inputs are the thruster surge force $\tau_u$ and the thruster yaw torque $\tau_r$. The AUV has no side thruster. Using the results of [11] and the fact that the elements of the gravitational field corresponding to the unactuated dynamics are zero, it follows that the AUV is a second-order nonholonomic systems and therefore it cannot be $C^1$ asymptotically stabilizable to a single equilibrium point.

### 2.1 Vehicle Modeling

Following standard practice, the general kinematic and dynamic equations of motion of the vehicle can be developed using a global coordinate frame $\{U\}$ and a body-fixed coordinate frame $\{B\}$, as depicted in Figure 1. In the horizontal plane, the kinematic equations of motion of the vehicle can be written as

$$\dot{x} = u\cos\psi - v\sin\psi, \tag{2.1a}$$
$$\dot{y} = u\sin\psi + v\cos\psi, \tag{2.1b}$$
$$\dot{\psi} = r, \tag{2.1c}$$

where, following standard notation, $u$ (surge speed) and $v$ (sway speed) are the body fixed frame components of the vehicle's velocity, $x$ and $y$ are the cartesian coordinates of its center of mass, $\psi$ defines its orientation, and $r$ is the vehicle's angular speed. Furthermore, neglecting the motions in heave, roll, and pitch the simplified equations of motion for surge, sway and heading yield [6]

$$m_u\dot{u} - m_v vr + d_u u = \tau_u, \tag{2.2a}$$
$$m_v\dot{v} + m_u ur + d_v v = 0, \tag{2.2b}$$
$$m_r\dot{r} - m_{uv} uv + d_r r = \tau_r, \tag{2.2c}$$

where $m_u = m - X_{\dot{u}}$, $m_v = m - Y_{\dot{v}}$, $m_r = I_z - N_{\dot{r}}$, and $m_{uv} = m_u - m_v$ capture the effect of mass and hydrodynamic added mass terms, and $d_u = -X_u - X_{|u|u}\,|u|$, $d_v = -Y_v - Y_{|v|v}\,|v|$, and $d_r = -N_r - N_{|r|r}\,|r|$ capture the hydrodynamic damping effects. The symbols $\tau_u$ and $\tau_r$ denote the external force in surge and the external torque about the $z$ axis of the vehicle, respectively. In the equations, and for clarity of presentation, it was assumed that the AUV is neutrally buoyant and that the centre of buoyancy coincides with the centre of gravity.

### 2.2 Problem formulation

Let $\{G\}$ be a goal reference frame and assume for simplicity of presentation that $\{G\} = \{U\}$. The problem considered in this paper can be formulated as follows.

*Given the nonholonomic underactuated AUV with kinematics and dynamics equations* (2.1) *and* (2.2)*, derive a feedback control for* $\tau_u$ *and* $\tau_r$ *to regulate* $\{B\}$ *to* $\{G\}$ *in the presence of parametric model uncertainty.*

The type of parametric uncertainty considered includes the general case where all the hydrodynamic coefficients of the vehicle's dynamic model are all to deviate from their nominal values. The presentation that follows borrows from and extends the results described in [1].

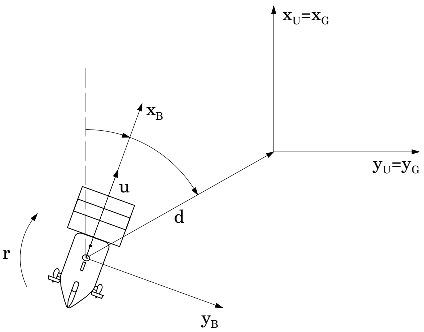


**Figure 2:** Coordinate Transformation.

### 2.3 Coordinate Transformation

Consider the coordinate transformation (see Figure 2)

$$e = \sqrt{x^2 + y^2} \tag{2.3a}$$
$$\psi + \beta = \tan^{-1}\left(\frac{-y}{-x}\right) \tag{2.3b}$$

where $d$ is the vector from $O_B$ to $O_U$, $e$ is the length of $d$, and $\beta$ denotes the angle measured from $x_B$ to $d$. Notice that in equation (2.3b) care must be taken to select the proper quadrant for $\beta$. Differentiating (2.3) with respect to time, the kinematics equations of motion of the AUV in the new coordinate system for $e \neq 0$ can be written as

$$\dot{e} = -u\cos\beta - v\sin\beta \tag{2.4a}$$
$$\dot{\beta} = \frac{\sin\beta}{e}u - \frac{\cos\beta}{e}v - r \tag{2.4b}$$
$$\dot{\psi} = r \tag{2.4c}$$

*Remark 1* Notice that the coordinate transformation (2.3) is only valid for non zero values of the distance error $e$, since for $e = 0$ the angle $\beta$ is undefined. This will introduce a discontinuity in the control law that will be derived later, which will obviate the basic limitations imposed by the result of Brockett.

## 3 Nonlinear Controller Design

This section proposes a nonlinear control law to regulate the motion of the AUV described by equations (2.1) and (2.2) to a target point with a desired orientation. The Lyapunov based control law builds on previous work described in [1]. Only the rationale for the control law proposed is introduced, a formal proof of convergence of solutions being omitted. See [3]. For details, the structure of the control law can best be derived by introducing a candidate Lyapunov functions recursively, in a sequence of logical steps that are directly related to vehicle heading regulation and target distance regulation. This methodology borrows heavily from the techniques of backstepping [7]. A switching term is introduced in the control law at the last stage in order to solve the indeterminacy at $e = 0$ caused by the polar representation adopted.

*Step 1. (Heading regulation)* Define the variables

$$\rho = \frac{u}{e}, \quad \xi = \frac{v}{e}, \quad \delta = \beta + \frac{1}{\gamma}\psi,$$
$$\sigma = \beta + \psi + \int_{t_0}^{t} \xi \cos\beta d\tau + \frac{1}{\gamma}\int_{t_0}^{t} \rho \frac{\sin\beta}{\beta}\psi\, d\tau,$$

where $\gamma$ is a positive even integer constant. Rewrite the equations of motion (2.2) and (2.4) as

$$\dot{\delta} = \rho\sin\beta - \xi\cos\beta - \frac{\gamma-1}{\gamma} r \qquad (3.1a)$$
$$\dot{\sigma} = \rho\sin\beta + \frac{1}{\gamma}\rho\frac{\sin\beta}{\beta}\psi \qquad (3.1b)$$
$$\dot{\xi} = -\frac{m_u}{m_v}\rho\, r - \frac{d_v}{m_v}\xi + \xi\rho\cos\beta + \xi^2\sin\beta \qquad (3.1c)$$
$$\dot{r} = \frac{1}{m_r}\left[\tau_r + m_{uv}uv - d_r r\right] \qquad (3.1d)$$

and

$$\dot{e} = -\rho\cos\beta e - \xi\sin\beta e \qquad (3.2a)$$
$$\dot{\rho} = \frac{1}{m_u e}\left[\tau_u + m_v vr - d_u u\right] + \rho^2\cos\beta + \rho\xi\sin\beta \qquad (3.2b)$$

where the final system has been divided in two subsystems that will henceforth be referred to as the heading and distance subsystems, respectively. Consider first the heading subsystem (3.1), where the control objective is to regulate the variables $\delta$, $\sigma$, $\xi$, and $r$ to zero, and $\psi$ to a point in $\mathcal{O}_\psi = \{\psi = 2\pi n;\ n \in \mathbb{Z}\}$. In order to do this, observe from equations (3.1a), (3.1b), and (3.1d) that $r$ can be viewed as a virtual control input. Define the positive definite function

$$V_1 = \frac{1}{2}\delta^2 + \frac{1}{2}k_\sigma\sigma^2,$$

and compute its time derivative along the trajectories of (3.1) to obtain

$$\dot{V}_1 = \delta\left[k_\sigma\sigma\rho\frac{\sin\beta}{\beta} + \rho\sin\beta - \xi\cos\beta - \frac{\gamma-1}{\gamma}r\right].$$

Following the methodology in [7], let $r$ be a virtual control input and

$$\alpha = k_\sigma\sigma\rho\frac{\sin\beta}{\beta} + \rho\sin\beta - \xi\cos\beta + k_2\delta, \quad k_2 > 0, \qquad (3.3)$$

a virtual control law. Introduce the error variable

$$z_1 = \frac{\gamma-1}{\gamma}r - \alpha, \qquad (3.4)$$

and compute $\dot{V}_1$ to obtain $\dot{V}_1 = -k_2\delta^2 - \delta z_1$.

*Step 2. (Backstepping)* The function $V_1$ is now augmented with a quadratic term in $z_1$ to obtain the new candidate Lyapunov function

$$V_2 = V_1 + \frac{1}{2}z_1^2.$$

The time derivative of $V_2$ can be written as

$$\dot{V}_2 = -k_2\delta^2 + z_1\Big[\frac{1}{m_r}\frac{\gamma-1}{\gamma}\big(\tau_r + m_{uv}uv - d_r r\big) - \dot{\alpha} - \delta\Big].$$

Let the control law for $\tau_r$ be chosen as

$$\tau_r = -m_{uv}uv + d_r r + m_r\frac{\gamma}{\gamma-1}\left(\dot{\alpha} + \delta - k_3 z_1\right), \qquad (3.5)$$

$k_3 > 0$. Then $\dot{V}_2 = -k_2\delta^2 - k_3 z_1^2 \leq 0$, that is, $\dot{V}_2$ is negative semidefinite.

*Step 3. (Free dynamics analysis)* In this step, the dynamic motion of $\beta$ and $\xi$ in the manifold $E = \{(\delta, \sigma, z_1) \in \mathbb{R}^3 : \dot{V}_2 = 0\}$ is analyzed. We assume $\rho = k_1 > 0$, where $k_1$ is a positive scalar (see also Step 4). First, observe that since $V_2$ is positive definite, radially unbounded, and has negative semidefinite derivative, then it follows that $\delta$, $\sigma$, and $z_1$ are globally bounded. Furthermore, LaSalle's theorem guarantees convergence of these variables to the largest invariant set $M$ contained in $E$. Thus, $\delta(t) \to 0$ and $z_1(t) \to 0$ as $t \to \infty$. Observe also that in the set $E$ the variables $\delta$ and $\dot{\delta}$ are zero and therefore (3.1a), (3.3), and (3.4), imply that

$$k_1 k_\sigma \sigma\frac{\sin\beta}{\beta} = 0.$$

The above expression is verified if *i)* $\sin\beta = 0$, with $\beta \neq 0$ or *ii)* $\sigma = 0$. In the first case, one obtains that

$$\dot{\xi} = -\Big[\frac{d_v}{m_v} - \Big(\frac{m_u}{m_v}\frac{\gamma}{\gamma-1} + 1\Big)k_1\cos\beta\Big]\xi.$$

in $E$. Consequently, $\lim_{t\to\infty} \xi(t) = 0$ if the controller parameter $k_1$ is chosen such that

$$\frac{d_v}{m_v} > \Big(\frac{m_u}{m_v}\frac{\gamma}{\gamma-1} + 1\Big)k_1.$$

In the second case, important conclusions about $\{\beta, \xi\}$ restricted to manifold $E$ can be derived by resorting to the candidate Lyapunov function

$$V = \gamma\frac{m_u}{m_v}k_1^2(1-\cos\beta) + \frac{1}{2}\xi^2. \tag{3.6}$$

Computing its time derivative, yields

$$\dot V = -\begin{bmatrix} k_1\sqrt{\gamma\frac{m_u}{m_v}}\sin\beta \\ \xi \end{bmatrix}^T Q \begin{bmatrix} k_1\sqrt{\gamma\frac{m_u}{m_v}}\sin\beta \\ \xi \end{bmatrix},$$

where

$$Q = \begin{bmatrix} \frac{1}{\gamma-1}k_1 & \frac{k_1}{2(\gamma-1)}\sqrt{\gamma\frac{m_u}{m_v}}(1-\cos\beta) \\ \frac{k_1}{2(\gamma-1)}\sqrt{\gamma\frac{m_u}{m_v}}(1-\cos\beta) & \frac{d_v}{m_v} - \Big(\frac{m_u}{m_v}\frac{\gamma}{\gamma-1}+1\Big)k_1\cos\beta - \xi\sin\beta \end{bmatrix}.$$

Under the assumption that $\xi$ is bounded, i.e., assuming there exists a positive number $r_\xi$ such that

$$|\xi(t)| \le r_\xi, \quad t \ge t_0, \tag{3.7}$$

it can be checked that $Q$ is positive definite if the inequalities $\frac{1}{\gamma-1}k_1 > 0$ and $\frac{d_v}{m_v} > \Big[2\frac{\gamma}{\gamma-1}\frac{m_u}{m_v} + 1\Big]k_1 + r_\xi$ hold. In this case,

$$\dot V \le -\lambda_{min}(Q)(1+\cos\beta)V \le 0,$$

where $\lambda_{min}(Q)$ denotes the minimum eigenvalue of the positive matrix $Q$. Hence, it can be concluded that $\lim_{t\to\infty}\dot V(t) = 0$ which implies that $\{\sin\beta, \xi\}$ converges to zero as $t \to \infty$. From the definition of $\delta$, and since $\gamma$ takes an even integer value, it follows that $\psi \to \mathcal{O}_\psi$.

To estimate a region of attraction for $\xi$ in the manifold $E$ (in order to validate assumption (3.7)) observe that

$$\frac{1}{2}\xi^2 \le V \le V_0 \le 2\gamma\frac{m_u}{m_v}k_1^2 + \frac{1}{2}\xi_0^2.$$

Thus, for any $\xi(t_0) = \xi_0$ such that

$$|\xi_0| \le \sqrt{r_\xi^2 - 4\gamma\frac{m_u}{m_v}k_1^2}, \tag{3.8}$$

and $(\delta, \sigma, z_1) \in E$, $\lim_{t\to\infty}\xi(t) = 0$.

*Step 4. (Distance regulation)* Consider now the distance subsystem (3.2) and in particularly examine equation (3.2a). Since $\{\sin\beta, \xi\} \to 0$, then, intuitively, a possible strategy to force $e$ to converge to zero is the following: make the variable $\rho$ converge to a positive value if $cos\beta \to 1$ as $t \to \infty$; for the particular case where $cos\beta \to -1$ make $\rho$ converge to a negative value. In order to apply this strategy, a new error variable

$$z_2 = \begin{cases} \rho + k_1 & \text{if } (q, \beta, \xi)' \in \mathcal{R}_\delta, \\ \rho - k_1 & \text{otherwise,} \end{cases}$$

is defined. The region $\mathcal{R}_\delta$ is a subset of

$$\mathcal{R}_\epsilon = \Big\{ q = \big(\delta, \sigma\frac{\sin\beta}{\beta}, z_1, z_2\big)', \beta, \xi : \; \|q\| \le \epsilon_q, \\ |\sin\beta| \le \epsilon_\beta < 1, \, \cos\beta < 0, \, |\xi| \le \epsilon_\xi \Big\},$$

where $\epsilon_q$, $\epsilon_\beta$, $\epsilon_\xi$ are positive constants such that $\mathcal{R}_\epsilon$ is an invariant set. Consider now a third candidate Lyapunov function given by

$$V_3 = V_2 + \frac{1}{2}z_2^2.$$

Computing its time derivative gives

$$\dot V_3 = -k_2\delta^2 - k_3 z_1^2 + z_2\Big[\frac{1}{m_u e}(\tau_u + m_v v r - d_u u) \\ + \rho^2\cos\beta + \rho\xi\sin\beta\Big].$$

Now, by choosing the control input

$$\tau_u = -m_v v r + d_u u - m_u e\big[\rho^2\cos\beta + \rho\xi\sin\beta + k_4 z_2\big], \tag{3.9}$$

the time derivative of $V_3$ becomes

$$\dot V_3 = -k_2\delta^2 - k_3 z_1^2 - k_4 z_2^2 \le 0, \tag{3.10}$$

that is, $\dot V_3$ is negative semidefinite.

*Step 5. (Switching control law)* So far, it has been assumed that the AUV will never start at or reach the position $x = y = 0$ in finite time, because the polar representation (2.3) and consequently the control law described above are not defined at $e = 0$. To deal with this situation, a switching control law must be introduced at this stage. A possible solution is to make

$$\tau_u = 0, \tag{3.11a}$$
$$\tau_r = -k_r r - k_\psi \psi, \tag{3.11b}$$

where $k_r$ and $k_\psi$ are positive constants, when $e = 0$.

The complete control law is thus given by

$$\tau = \begin{bmatrix} \tau_u \\ \tau_r \end{bmatrix} = \begin{cases} (3.5), (3.9) & e \ne 0 \\ (3.11) & e = 0 \end{cases} \tag{3.12}$$

The following theorem can now be proved [3].

**Theorem 1** *Consider the closed loop nonlinear invariant system $\Sigma$ described by* (2.1), (2.2), *and* (3.12). *Consider also the set $\mathcal{R}(\delta_u, \delta_v, k_1)$,*

$$\mathcal{R} = \Big\{(u, v, e) \in \mathbb{R}^3 : e > 0, \Big|\frac{u}{e} - k_1\Big| \le \delta_u, \; \Big|\frac{v}{e}\Big| \le \delta_v\Big\}.$$

*Let $\mathcal{X}(t) = (x, y, \psi, u, v, r)' = \{\mathcal{X} : [t_0, \infty) \to \mathbb{R}^6\}$, $t_0 \ge 0$, be a solution of $\Sigma$. Then given any compact neighborhood $S \subset \mathbb{R}^4$ of $(x, y, \psi, r) = (0, 0, 2\pi n, 0)$, $n \in \mathbb{Z}$, one can find sufficiently small $k_1 > 0$, $\delta_u > 0$, and $\delta_v > 0$ such that, for any initial conditions $\mathcal{X}(t_0) = \mathcal{X}_0 \in S \cup \mathcal{R}$*

1. *$\mathcal{X}(t)$ exists, is unique and defined for all $t \ge t_0$;*
2. *$\mathcal{X}(t)$ is bounded;*
3. *The solution $\mathcal{X}(t)$ converges to an equilibrium point in $\mathcal{O} := \big\{(x, y, \psi, u, v, r)' = (0, 0, 2\pi n, 0, 0, 0)', \; n \in \mathbb{Z}\big\}$ as $t \to \infty$.*

## 4 Adaptive Nonlinear Controller Design

This section describes an extension of the previous control law to deal with parameter uncertainty. Again, only the rational for the resulting adaptive control law is described through the introduction of a general Lyapunov function that captures parameter deviations from the nominal values. See [3] for formal convergence proofs.

### 4.1 Control law

Consider the set of all parameters of the AUV model (2.2) concatenated in the vector

$$\Theta = \Big[m_u, m_v, m_{uv}, m_r, X_u, X_{|u|u}, N_r, N_{|r|r}, \frac{m_u}{m_v}, \frac{Y_v}{m_v}, \frac{Y_{|v|v}}{m_v}\Big]'$$

and define the parameter estimation error $\tilde{\Theta}$ as $\tilde{\Theta} = \Theta - \hat{\Theta}$, where $\hat{\Theta}$ denotes a nominal value of $\Theta$. Consider the augmented candidate Lyapunov function

$$V_4 = \frac{1}{2}\delta^2 + \frac{1}{2}k_\sigma\sigma^2 + \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2}\tilde{\rho}^2 + \frac{1}{2}\tilde{\xi}^2 + \frac{1}{2}\tilde{\Theta}^T P \Gamma^{-1}\tilde{\Theta},$$

where $\tilde{\rho} = \rho - \hat{\rho}$, $\tilde{\xi} = \xi - \hat{\xi}$, $\Gamma = \text{diag}\,\{\gamma_1, \gamma_2, ..., \gamma_{11}\}$, and $P = \text{diag}\Big\{\frac{1}{m_u}, \frac{1}{m_u}, \frac{1}{m_r}, \frac{1}{m_r}, \frac{1}{m_u}, \frac{1}{m_u}, \frac{1}{m_r}, \frac{1}{m_r}, 1, 1, 1\Big\}$, where $\gamma_i > 0$, $i = 1, 2, ...11$ are adaptation gains. Let the variable $z_1$ be now slightly modified and redefined as $z_1 = \frac{\gamma - 1}{\gamma} r - \hat{\alpha}$, where

$$\hat{\alpha}(\delta, \sigma, \beta, \hat{\rho}, \hat{\xi}) = k_\sigma \sigma \hat{\rho} \frac{\sin\beta}{\beta} + \hat{\rho}\sin\beta - \hat{\xi}\cos\beta + k_2\delta.$$

Notice the inclusion of the variables $\tilde{\rho}$ and $\tilde{\xi}$ in the Lyapunov function $V_4$. This was done because in the process of computing $\dot{\hat{\alpha}}$ the variables $\dot{\rho}$ and $\dot{\xi}$ must also be computed and these in turn are functions of the model parameters (see equations (3.1c) and (3.2b)). Motivated by the choices in steps 2 and 3, choose the control laws

$$\begin{aligned}\tau_r = &-\hat{\theta}_3 uv - \hat{\theta}_7 r - \hat{\theta}_8 |r|r + \hat{\theta}_4 \frac{\gamma}{\gamma - 1}\big[\dot{\hat{\alpha}} + \delta\big] \\ &- k_3 \frac{\gamma}{\gamma - 1} z_1,\end{aligned} \quad (4.1a)$$

$$\begin{aligned}\tau_u = &-\hat{\theta}_2 vr - \hat{\theta}_5 u - \hat{\theta}_6 |u|u \\ &- \hat{\theta}_1 e\big[\rho^2\cos\beta + \rho\xi\sin\beta\big] - k_4\, e\, z_2,\end{aligned} \quad (4.1b)$$

and the updating laws

$$\dot{\hat{\rho}} = \delta k_\sigma \sigma \frac{\sin\beta}{\beta} + \delta\sin\beta + k_\rho\tilde{\rho}, \quad k_\rho > 0 \quad (4.2a)$$

$$\begin{aligned}\dot{\hat{\xi}} = &-\hat{\theta}_9\, \rho r + \hat{\theta}_{10}\xi + \hat{\theta}_{11}|v|\xi + \rho\xi\cos\beta \\ &+ \xi^2\sin\beta - \delta\cos\beta + k_\xi\tilde{\xi}, \quad k_\xi > 0\end{aligned} \quad (4.2b)$$

that yield

$$\begin{aligned}\dot{V}_4 = &-k_2\delta^2 - \frac{k_3}{m_r}z_1^2 - [z_2, \tilde{\rho}]\, Q_1\, [z_2, \tilde{\rho}]' \\ &- k_\xi\tilde{\xi}^2 + \tilde{\Theta}^T P\big[Q_2 - \Gamma^{-1}\dot{\hat{\Theta}}\big],\end{aligned} \quad (4.3)$$

where $Q_1 = \begin{pmatrix} \frac{k_4}{m_u} & \frac{k_4}{2m_u} \\ \frac{k_4}{2m_u} & k_\rho \end{pmatrix}$ is a positive definite matrix if $k_\rho > \frac{k_4}{4m_u}$ and $Q_2$ is a diagonal matrix given by

$$\begin{aligned}Q_2 = \text{diag}\Big\{&(\tilde{\rho} + z_2)(\rho^2\cos\beta + \rho\xi\sin\beta), (\tilde{\rho} + z_2)\xi r, \\ &z_1\frac{\gamma - 1}{\gamma}uv, -z_1(\dot{\hat{\alpha}} + \delta), \rho(z_2 + \tilde{\rho}), \rho(z_2 + \tilde{\rho})|u|, \\ &z_1\frac{\gamma - 1}{\gamma}r, z_1\frac{\gamma - 1}{\gamma}|r|r, -\tilde{\xi}\rho r, \tilde{\xi}\xi, \tilde{\xi}|v|\xi\Big\}.\end{aligned}$$

Notice in equation (4.3) how the terms containing $\tilde{\Theta}_i$ have been grouped together. To eliminate them, choose the parameter adaptation law as

$$\dot{\hat{\Theta}} = \Gamma Q_2, \quad (4.4)$$

to yield

$$\dot{V}_4 = -k_2\delta^2 - \frac{k_3}{m_r}z_1^2 - [z_2, \tilde{\rho}]\, Q_1\, [z_2, \tilde{\rho}]' - k_\xi\tilde{\xi}^2 \le 0.$$

Thus, the complete adaptive control law is given by

$$\tau = \begin{bmatrix} \tau_u \\ \tau_r \end{bmatrix} = \begin{cases} (4.1), (4.2), (4.4) & e \neq 0 \\ (3.11) & e = 0 \end{cases} \quad (4.5)$$

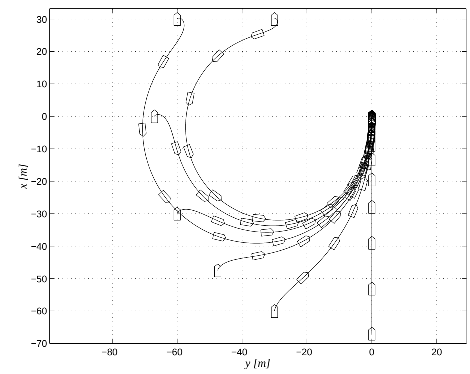


**Figure 3:** The Sirene AUV path for different initial conditions in $(x, y)$. The initial condition for $(\psi, u, v, r)'$ is zero.

## 5 Simulation Results

This section illustrates the performance of the proposed control scheme (in the presence of parametric uncertainty) using computer simulations. The objective is to regulate the position and attitude of the SIRENE AUV to zero. The control parameters were selected as follows: $k_1 = 0.03$, $k_2 = 0.5$, $k_3 = 100$, $k_4 = 20$, $k_\sigma = 1$, $\gamma = 2$, $k_\rho = 10$, $k_\xi = 10$, $k_\psi = 1.6$, $k_r = 1.9$, and $\Gamma = \text{diag}(10, 10, 10, 1, 1, 2, 2, 2, 20, 20, 20)$. The initial estimates for the vehicle parameters were disturbed by 50% from their true values.

Figures 3-5 show the simulation results for the nonlinear adaptive control law (4.5). Figure 3 illustrates the vehicle trajectory in the xy-plane for different initial conditions in $(x, y)$. Figures 4-5 display the time responses

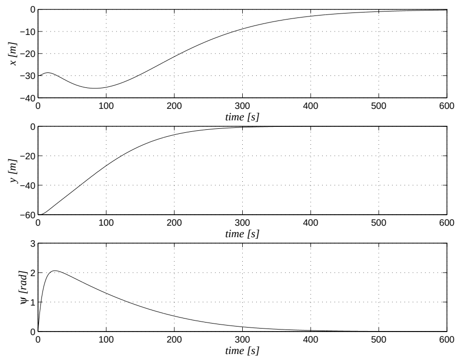


**Figure 4:** Time evolution of position variables $x(t)$ and $y(t)$ and orientation variable $\psi(t)$.

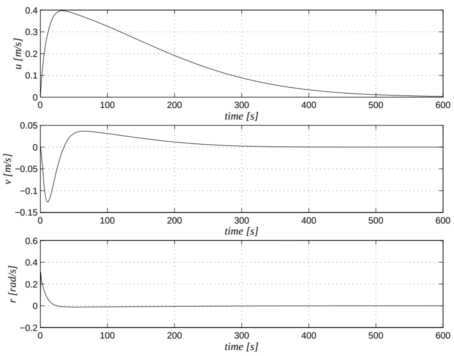


**Figure 5:** Time evolution of linear velocity in $x_B$-direction (surge) $u(t)$, linear velocity in $y_B$-direction (sway) $v(t)$, and angular velocity $r(t)$.

of the relevant state space variables for the initial condition $(x_0, y_0, \psi_0, u_0, v_0, r_0) = (-30m, -60m, 0, 0, 0, 0)$. Notice how, in spite of parameter uncertainty and the drift term (see the sway velocity activity in Figure 5), the vehicle converges asymptotically to the origin with a "natural", smooth trajectory.

## 6 Conclusions

This paper proposed a new solution to the problem of regulating the dynamic model of an underactuated, nonholonomic AUV to a point with a desired orientation. A discontinuous, bounded, time invariant, nonlinear adaptive control law that yields convergence of the trajectories of the closed loop system in the presence of parametric modeling uncertainty was derived. Simulation results show that the control objectives were achieved successfully. Future research will address the problem of control and analysis of mechanical nonholonomic systems in the presence of noise measurements, actuator saturation constraints, and observer dynamics. Notice that for the AUV case, robust control schemes are mandatory due to significant model uncertainty, measurement noise, and the strong influence of external disturbances such as underwater currents. Another open problem is the application of this theoretical developments to real world practical applications.

## References

[1] A. P. Aguiar, A. N. Atassi, and A. M. Pascoal, *Regulation of a nonholonomic dynamic wheeled mobile robot with parametric modeling uncertainty using Lyapunov functions*, Proc. 39th IEEE Conference on Decision and Control (Sydney, Australia), December 2000.

[2] A. P. Aguiar and A. M. Pascoal, *Modeling and control of an autonomous underwater shuttle for the transport of benthic laboratories*, Proc. of the Oceans 97 Conference (Halifax, Canada), October 1997.

[3] A. P. Aguiar and A. M. Pascoal, *Regulation of a nonholonomic and underactuated autonomous underwater vehicle with parametric modeling uncertainty using lyapunov functions*, Tech. Report 0001, ISR/IST Institute for Systems and Robotics and Instituto Superior Técnico, Lisbon, Portugal, March 2000.

[4] L. Brisset, M. Nokin, D. Semac, H. Amann, W. Shneider, and A. Pascoal, *New methods for deep sea intervention on future benthic laboratories: analysis, development, and testing*, Proc. Second Mast Days and Euromar Market (Sorrento, Italy), 1995, pp. 1025–1037.

[5] O. Egeland, M. Dalsmo, and O. Sørdalen, *Feedback control of a nonholonomic underwater vehicle with a constant desired configuration*, The Intern. Journal Of Robotics Research, MIT **15** (1996), no. 1, 24–35.

[6] T. I. Fossen, *Guidance and control of ocean vehicles*, John Wiley & Sons, England, 1994.

[7] M. Krstić, I. Kanellakopoulos, and P. Kokotović, *Nonlinear and adaptive control design*, John Wiley & Sons, Inc., New York, USA, 1995.

[8] N. E. Leonard, *Control synthesis and adaptation for an underactuated autonomous underwater vehicle*, IEEE J. of Oceanic Eng. **20** (1995), no. 3, 211–220.

[9] K. Y. Pettersen and O. Egeland, *Position and attitude control of an underactuated autonomous underwater vehicle*, Proc. of the 35th IEEE Conf. on Decision and Control (Kobe, Japan), 1996, pp. 987–991.

[10] K. Y. Pettersen and H. Nijmeijer, *Global practical stabilization and tracking for an underactuated ship - a combined averaging and backstepping approach*, Proc. IFAC Conference on Systems Structure and Control (Nantes, France), July 1998, pp. 59–64.

[11] K. Y. Wichlund, O. Sørdalen, and O. Egeland, *Control properties of underactuated vehicles*, Proc. of the 1995 IEEE International Conference on Robotics and Automation (Nagoya, Japan), IEEE Computer Society Press, May 1995, pp. 2009–2014.