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Global stabilization of an underactuated autonomous underwater vehicle via logic-based switching[1]

António Pedro Aguiar   António M. Pascoal

ISR/IST - Institute for Systems and Robotics and Dept. Electrical Engineering,
Instituto Superior Técnico, Torre Norte 8, Av. Rovisco Pais, 1049-001 Lisboa, Portugal
E-mail:{antonio.aguiar,antonio}@isr.ist.utl.pt

Abstract

This paper considers the problem of global stabilization of an underactuated autonomous underwater vehicle (AUV) to a point, with a desired orientation. Controllability and stabilizability properties of the vehicle model are discussed and a logic-based hybrid controller is proposed that yields global convergence of the AUV to an arbitrarily small neighborhood of the target point. Convergence and stability of the closed loop system are analyzed. To illustrate the control law developed, simulation results are presented using the model of the *Sirene* AUV.

1 Introduction

The problem of steering an underactuated autonomous underwater vehicle (AUV) to a point with a desired orientation has only recently received special attention in the literature. This task raises some challenging questions in control system theory, because the vehicle is underactuacted. Furthermore, as will be shown, its dynamics are complicated due to the presence of complex hydrodynamic terms. This rules out any attempt to design a steering system for the AUV that would rely on its kinematic equations only. Pioneering work in this field is reported in [14], where open loop small-amplitude periodic time-varying control laws were proposed to re-position and re-orient underactuated AUVs. A feedback control law that gives exponential convergence of a nonholonomic AUV to a constant desired configuration is introduced in [9]. The design of a continuous, periodic feedback control law that asymptotically stabilizes an underactuated AUV and yields exponential convergence to the origin is described in [16]. See also [17] for an extension of these results to address robustness issues. In [19], a time-varying feedback control law is proposed that yields global practical stabilization and tracking for an underactuated ship using a combined integrator backstepping and averaging approach. Practical applications of these results can be found in [18]. More recent work is described in [6], where the problem of regulating a dynamic model of a nonholonomic and underactuated AUV to a desired point with a given orientation is addressed and solved. This is done by using a discontinuous, nonlinear adaptive state feedback controller that yields convergence of the trajectories of the closed loop system in the presence of parametric modeling uncertainty.

This paper addresses the problem of stabilizing an underactuated AUV in the horizontal plane using a new technique that builds on hybrid control theory. To the best of the authors knowledge, this work is the first application of hybrid control to the stabilization of underactuated marine vehicles. *A feedback logic-based hybrid control law is derived that yields global stabilization of an underactuated AUV to an arbitrarily small neighborhood of a target position with a desired orientation.* Control systems design is done by transforming the AUV dynamic model into extended nonholonomic double integrator (ENDI) form plus a drift vector field, followed by the derivation of a controller for that system that explores previous work on feedback hybrid control of the ENDI [2, 5]. It is worth pointing out that the technique proposed in this paper is not a simply extension of the methodology developed in [2, 5]. In fact, point stabilization of an underactuated AUV poses considerable challenges to control system designers, since the models of those vehicles typically include a drift vector field that is not in the span of the input vector fields, thus precluding the use of input transformations to bring them to driftless form.

The paper is organized as follows: Section 2 describes the dynamical model of a prototype AUV named *Sirene* that is the focal point of this work. A coordinate transformation is introduced and some important results on stabilizability and controllability of the underactuated AUV model are discussed. Section 3 proposes a piecewise smooth controller for AUV stabilization based on hybrid systems theory, and discusses the stability of the resulting closed loop. It is shown that closed loop system is stable and that for any initial condition the AUV converges to a small neighborhood of the desired final position with a desired orientation. The radius of the neighborhood can be chosen arbitrarily close to zero (depending only on the controller parameters). Section 4 contains simulation results that illustrate the performance of the proposed control strategy. The paper concludes with a summary of results and recommendations for further research.

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2 The Autonomous Underwater Vehicle

This section describes the simplified equations of motion of the underactuated AUV *Sirene* and addresses the problem of controlling it to a point with a desired orientation. The controllability and stabilizability properties of the vehicle are discussed. The *Sirene*

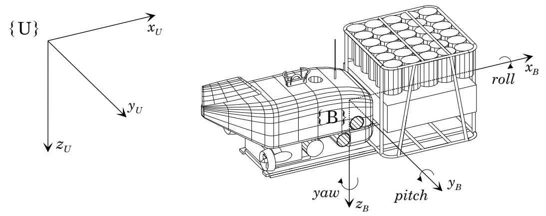

Figure 1: The vehicle *Sirene* coupled to a benthic laboratory. Body-fixed $\{B\}$ and earth-fixed $\{U\}$ reference frames

vehicle - depicted in Fig. 1 - has an open-frame structure and is 4.0 m long, 1.6 m wide, and 1.96 m high. It has a dry weight of 4000 Kg and a maximum operating depth of 4000 m. The vehicle is equipped with two back thrusters for surge and yaw motion control in the horizontal plane and one vertical thruster for heave control. Roll and pitch motion are left uncontrolled, since the metacentric height[1] is sufficiently large (36 cm) to provide adequate static stability. The AUV has no side thruster. In the figure, the vehicle carries a representative benthic lab that is cubic-shaped and has a volume of $2.3m^3$. See [1, 3] for complete details on the AUV dynamic model.

2.1 Vehicle Modeling

General equations of motion

Following standard practice, the kinematic and dynamic equations of motion of the AUV can be developed using a global coordinate frame $\{U\}$ and a body-fixed coordinate frame $\{B\}$, as depicted in Fig. 1. In the horizontal plane, the kinematic equations of motion of the vehicle can be written as

$$\dot{x} = u\cos\psi - v\sin\psi, \quad (1a)$$
$$\dot{y} = u\sin\psi + v\cos\psi, \quad (1b)$$
$$\dot{\psi} = r, \quad (1c)$$

where, following standard notation, u (surge speed) and v (sway speed) are the body fixed frame components of the vehicle's velocity, x and y are the cartesian coordinates of its center of mass, ψ defines its orientation, and r is the vehicle's angular speed.

Neglecting the motions in heave, roll, and pitch, the simplified equations of motion for surge, sway and heading yield [10]

$$m_u\dot{u} - m_v vr + d_u u = \tau_u, \quad (2a)$$
$$m_v\dot{v} + m_u ur + d_v v = 0, \quad (2b)$$
$$m_r\dot{r} - (m_u - m_v)uv + d_r r = \tau_r, \quad (2c)$$

[1]distance between the center of buoyancy and the center of mass.

where $m_u = m - X_{\dot{u}}$, $m_v = m - Y_{\dot{v}}$, $m_r = I_z - N_{\dot{r}}$, and $m_{uv} = m_u - m_v$ are mass and hydrodynamic added mass terms and $d_{u_r} = -X_u - X_{|u|u}|u_r|$, $d_{v_r} = -Y_v - Y_{|v|v}|v_r|$, and $d_r = -N_r - N_{|r|r}|r|$ capture hydrodynamic damping effects. The symbols τ_u and τ_r denote the external force in surge and the external torque about the z axis of the vehicle, respectively. In the equations, and for clarity of presentation, it is assumed that the AUV is neutrally buoyant and that the center of buoyancy coincides with the center of gravity.

Coordinate Transformation

Consider the global diffeomorphism given by the state and control coordinate transformation

$$\begin{aligned} z_1 &= \psi \\ z_2 &= x\cos\psi + y\sin\psi \\ z_3 &= -2(x\sin\psi - y\cos\psi) + \psi(x\cos\psi + y\sin\psi) \\ u_1 &= \frac{1}{m_r}\tau_r + \frac{m_u - m_v}{m_r}uv - \frac{d_r}{m_r}r \\ u_2 &= \frac{m_v}{m_u}vr - \frac{d_u}{m_u}u + \frac{1}{m_u}\tau_u - u_1\frac{z_1z_2 - z_3}{2} + vr \\ &\quad - r^2 z_2 \end{aligned}$$

that yields

$$\ddot{z}_1 = u_1 \quad (3a)$$
$$\ddot{z}_2 = u_2 \quad (3b)$$
$$\dot{z}_3 = z_1\dot{z}_2 - z_2\dot{z}_1 + 2v \quad (3c)$$

and transforms the second order constraint (2b) for the sway velocity into

$$m_v\dot{v} + m_u\left(\dot{z}_2 + \dot{z}_1\frac{z_1z_2 - z_3}{2}\right)\dot{z}_1 + d_v v = 0. \quad (4)$$

Throughout the paper, $q = (z_1, z_2, z_3, \dot{z}_1, \dot{z}_2, v)'$, and $u = (u_1, u_2)'$ denote the state vector and the input vector of the system described by equations (3) and (4), respectively.

2.2 Controllability and Stabilizability Results

The AUV falls into the class of control affine nonlinear systems with drift described by

$$\dot{q} = f(q) + \sum_{i=1}^{m} g_i(q)u_i$$

where $q \in M$, M is a smooth n-dimensional manifold, $u \in \mathbb{R}^m$, and the mappings $f, g_1, \ldots, g_m$ are smooth vector fields on M. The theorem that follows establishes basic results on nonlinear accessibility and controllability of (1)-(2). See [12, 15, 20] for relevant background.

Theorem 1 *Consider the underactuated AUV model described by (1)-(2). Let M_e be the set of equilibrium solutions corresponding to $\tau_u = \tau_r = 0$, that is, $M_e = \{(x, y, \psi, u, v, r)' \in \mathbb{R}^6 : u = v = r = 0\}$. Then, the AUV model satisfies the following properties:*

1. *There is no time-invariant continuously differentiable feedback law that asymptotically stabilizes the closed loop system to $(x_e, y_e, \psi_e, 0, 0, 0)' \in M_e$.*

2. *The AUV system is locally strongly accessible for any $(x, y, \psi, u, v, r)' \in \mathbb{R}^6$.*
3. *The AUV system is small time locally controllable (STLC) at any equilibrium $(x_e, y_e, \psi_e, 0, 0, 0)' \in M_e$.*

Proof. See [2]. □

3 Hybrid Controller Design

This section proposes a piecewise smooth controller to stabilize the AUV that builds on hybrid system theory. The objective is to design a feedback law for system (3)-(4) that will make the state $q = (z_1, z_2, z_3, \dot{z}_1, \dot{z}_2, v)'$ converges to an arbitrarily small neighborhood of the origin despite the drift vector field introduced by the sway velocity v that is not in the span of the input vector fields. Consequently, it is also necessary to guarantee that the sway velocity v subject to the constraint (4) goes to zero.

Hybrid systems are specially suited to deal with the combination of continuous dynamics and discrete events. The literature on hybrid systems is extensive and discusses different modeling techniques [7, 21]. In this paper, a continuous-time hybrid system Σ is defined as [11]

$$\dot{x}(t) = f_{\sigma(t)}\big(x(t), t\big), \qquad t \geq t_0 \tag{5a}$$

$$\sigma(t) = \phi\left(x(t), \sigma(t^-)\right) \tag{5b}$$

where $\sigma(t) \in \mathcal{I} \triangleq \{1, \ldots, N\}$ and $x(t) \in \mathcal{X} \triangleq \cup_{\sigma=1}^{N} \mathcal{X}_\sigma \subset \mathbb{R}^n$. Here, the differential equation (5a) models the continuous dynamics, where the vector fields $f_\sigma : \mathcal{X}_\sigma \times \mathbb{R}^+ \to \mathcal{X}$, $\sigma \in \mathcal{I}$ are each locally Lipschitz continuous maps from $\mathcal{X}_\sigma$ to $\mathcal{X}$. The algebraic equation (5b), where $\phi : \mathcal{X} \times \mathcal{I} \to \mathcal{I}$, models the state of the decision-making logic. The discrete state $\sigma(t)$ is piecewise constant. The notation t^- indicates that the discrete state is piecewise continuous from the right. The dynamics of the system Σ can now be described as follows: starting at (x_0, i) with $x_0 \in \mathcal{R}_i \subset \mathcal{X}_i$, the continuous state trajectory $x(t)$ evolves according to $\dot{x} = f_i(x, t)$. When $\phi\big(x(\cdot), i\big)$ becomes equal to $j \neq i$, (and this could only happen when $x(\cdot)$ hits the set $\mathcal{X} \backslash \mathcal{R}_i$), the continuous dynamics switches to $\dot{x} = f_j(x, t)$, from which the process continues. As in [11], the "logical dynamics" will be determined recursively by equation (5b) with $\sigma^-(t_0) = \sigma_0 \in \mathcal{I}$, where $\sigma^-(t)$ denotes the limit of $\sigma(\tau)$ from below as $\tau \to t$ and the transition function ϕ is defined by

$$\phi(x, \sigma) = \begin{cases} \sigma & \text{if } x \in \mathcal{R}_\sigma, \\ \max_{\mathcal{I}}\{k : x \in \mathcal{R}_k\} & \text{otherwise.} \end{cases} \tag{6}$$

Consider now the AUV model described by equations (3)-(4) with state vector $q = (z_1, z_2, z_3, \dot{z}_1, \dot{z}_2, v)' \in \mathbb{R}^6$. Define the function $W(q) : \mathbb{R}^6 \to \Omega \subset \mathbb{R}^2$ as

$$\omega \triangleq (\omega_1, \omega_2)' = W(\cdot) = \left[s^2, \lambda_1(\lambda - \lambda_1)(z_1)^2 + (\dot{z}_1)^2\right]',$$

where $s = \dot{z}_3 + \lambda z_3$ and λ and λ_1 are positive constants that satisfy $\lambda_1 < \lambda$. The image of W is the two-dimensional closed positive quadrant space $\Omega = \{(\omega_1, \omega_2) \in \mathbb{R}^2 : \omega_1 \geq 0, \omega_2 \geq 0\}$. The mapping W has the following property: if ω_1 is bounded (say $\omega_1 \leq \epsilon$) then z_3 is also bounded by

$$|z_3(t)| \leq e^{-\lambda(t-t_0)}|z_3(t_0)| + \frac{\sqrt{\epsilon}}{\lambda}. \tag{7}$$

In particularly, if $\omega_1(t)$ converges to zero then $z_3(t)$ also converges to zero.

Consider the following three overlapping regions in Ω (see Fig. 2) that play a key role in the definition of $\phi(q, \sigma)$ in (6)

$$\mathcal{R}_1 = \big\{(\omega_1, \omega_2) \in \Omega : \omega_1 > \epsilon_1 \wedge \omega_2 \leq \gamma_1\big\} \tag{8a}$$

$$\mathcal{R}_2 = \big\{(\omega_1, \omega_2) \in \Omega : \omega_1 > \epsilon_2 \wedge \omega_2 > 0\big\} \cup \big\{(\omega_1, \omega_2) \in \Omega : \epsilon_1 < \omega_1 \leq \epsilon_2\big\} \tag{8b}$$

$$\mathcal{R}_3 = \big\{(\omega_1, \omega_2) \in \Omega : \omega_1 \leq \varepsilon_2\big\}, \tag{8c}$$

where

$$\varepsilon_2 = \begin{cases} +\infty & \sigma = 3 \wedge z_1\dot{z}_1 > 0 \wedge |z_1| < \frac{|\dot{z}_1|}{\lambda}, \\ \epsilon_2 & \text{otherwise,} \end{cases}$$

$$\gamma_1 = \begin{cases} +\infty & |z_1(t_0)| < \frac{|\dot{z}_1(t_0)|}{\lambda}, \\ \gamma & \text{otherwise,} \end{cases}$$

and $\epsilon_2 > \epsilon_1$, ϵ_1, and γ are positive constants.

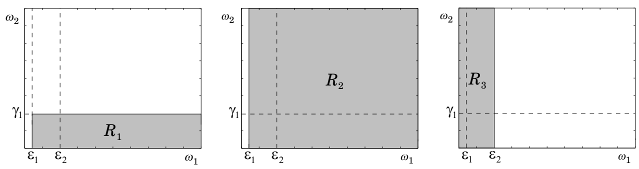

Figure 2: Definition of regions $\mathcal{R}_1$, $\mathcal{R}_2$, and $\mathcal{R}_3$.

The following control law is proposed

$$u = (u_1, u_2)' = h_\sigma(q), \tag{9}$$

where the vector fields $h_\sigma : \mathbb{R}^6 \to \mathbb{R}^2$, $\sigma \in \mathcal{I} = \{1, 2, 3\}$ are given by

$$h_1(q) = \begin{bmatrix} -\lambda\dot{z}_1 + k_1 \\ -\lambda\dot{z}_2 \end{bmatrix}, \; h_2(q) = \begin{bmatrix} -\lambda\dot{z}_1 \\ -\lambda\dot{z}_2 + k_2 \operatorname{sat}(\frac{s}{\sqrt{\epsilon_1}}) \end{bmatrix}, \tag{10}$$

$$h_3(q) = \begin{bmatrix} -\lambda\dot{z}_1 - \lambda_1(\lambda - \lambda_1)z_1 \\ -\lambda\dot{z}_2 - \lambda_1(\lambda - \lambda_1)z_2 - k_3 z_1 s \end{bmatrix}, \tag{11}$$

with $k_1 = \operatorname{sgn}\big(z_2(t_0) + \frac{\dot{z}_2(t_0)}{\lambda}\big)\operatorname{sgn}(s)$, $k_2 = -\operatorname{sgn}\big(z_1(t_0) + \frac{\dot{z}_1(t_0)}{\lambda}\big)$, and k_3 a positive constant. The function $\operatorname{sgn}(\cdot)$ is defined by $\operatorname{sgn}(x) = 1$ if $x \geq 0$, and $\operatorname{sgn}(x) = -1$ if $x < 0$. The function $\operatorname{sat}(\cdot)$ is in turn defined by $\operatorname{sat}(x) = \operatorname{sgn}(x)$ if $|x| > 1$, and $\operatorname{sat}(x) = x$ if $|x| \leq 1$. The switching signal $\sigma(t)$ is piecewise constant, takes values in $\mathcal{I} = \{1, 2, 3\}$, and is determined recursively by

$$\sigma(t) = \phi\left(\omega(t), \sigma^-(t)\right), \quad \sigma^-(t_0) = \sigma_0 \in \mathcal{I} \tag{12}$$

where the transition function is defined according to (6).
The control laws for each region were designed according to the following simple rule: if $\sigma = 1$, the variable $\omega_2(t)$ must move away from zero; when $\sigma = 2$, $\omega_1(t)$ must decrease and reach a given bound in finite time and $\omega_2(t)$ must not increase; finally, when $\sigma = 3$, the variable $\omega_2(t)$ and therefore the state $z_2(t)$ must converge to zero while $\omega_1(t)$ should remain near ϵ_1 (in this case its behavior will be dictated by the drift state v). A sketch of a typical trajectory in the W-space is shown in Fig. 3.

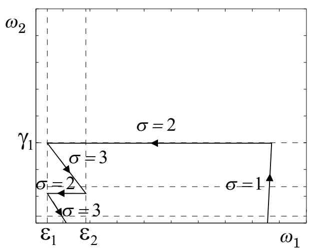

Figure 3: Image of a representative trajectory in the (ω_1, ω_2)-space.

The following is the key result of this paper.

Theorem 2 *Consider the hybrid system Σ_{AUV} described by* (3)-(4), (8)-(12), *and* (6). *Let $\{q(t), \sigma(t)\} = \{q : [t_0, \infty) \to \mathbb{R}^6, \sigma : [t_0, \infty) \to \mathcal{I}\}$ be a solution to Σ_{AUV}. Then, the following properties hold.*

1. *Given an arbitrary pair $\{x_0, \sigma_0\} \in \mathbb{R}^5 \times \mathcal{I}$ (initial condition), there exists a unique solution $\{x(t), \sigma(t)\}$ for all $t \geq t_0$ such that $\{x(t_0), \sigma^-(t_0)\} = \{x_0, \sigma_0\}$.*
2. *For any set of initial conditions $\{q(t_0), \sigma^-(t_0)\} = \{q_0, \sigma_0\} \in \mathbb{R}^6 \times \mathcal{I}$, there exists a finite time T such that for $t > T$ the state variables $z_1(t)$, $\dot{z}_1(t)$, $z_2(t)$, $\dot{z}_2(t)$, and $v(t)$ converge to zero, and $\omega_1(t) \leq \epsilon_2$, where $\epsilon_2 > 0$ is a controller parameter that can be chosen arbitrarily small.*
3. *The origin $q = 0$ is a Lyapunov uniformly stable equilibrium point of Σ_{AUV}.*

Proof.
Existence and Uniqueness
Since for each $i \in \mathcal{I}$ the vector field $h_i(q)$ is continuously differentiable with respect to q, then $h_i(q)$ is locally Lipschitz in q. Moreover, since for $i = 1, 2$ the Jacobian matrix $[\partial h_i / \partial q]$ is uniformly bounded in $\mathbb{R}^6$ it follows that $h_i(q)$ is globally Lipschitz. For $i = 3$, $h_3(q)$ is not globally Lipschitz. However, it can be shown that every solution of Σ_{AUV} with $q_0 \in \mathcal{R}_3$ and $\sigma_0 = 3$ lies entirely in a compact set $S \subset \mathbb{R}^6$. A proof of this statement arises naturally in the discussion below. Notice also that the distance between two points in the (ω_1, ω_2)-space where consecutive switching may occur is always nonzero. It now follows from classical arguments [13] that the hybrid system Σ_{AUV} has exactly one solution over $[t_0, \infty)$ for each initial condition $\{q_0, \sigma_0\} \in \mathbb{R}^6 \times \mathcal{I}$.

Convergence
An outline of the convergence proof is given based on the following five claims. See [2] for complete details.

Claim 1 *Given an initial time $t_0 \geq 0$, there exists a finite time $T \geq t_0$ such that $\sigma(t) \in \mathcal{I} \backslash \{1\}$ for all $t \geq T$.*
Proof. Consider first that $\sigma_0 = 1$ at $t = t_0$, and suppose that by contradiction $\sigma(t) = 1$ (and consequently, $\omega(t)$ remains in $\mathcal{R}_1$) for all $t \geq t_0$. In this case, the dynamics of z_1 satisfy $\ddot{z}_1 = -\lambda \dot{z}_1 + k_1$. Therefore, $\omega_2(t) = \lambda_1(\lambda - \lambda_1) z_1^2 + \dot{z}_1^2$ is unbounded and $\omega(t)$ leaves region $\mathcal{R}_1$ which is a contradiction. The remainder of the proof consist of showing that if $\sigma_0 \in \mathcal{I} \backslash \{1\}$, then σ will never switch to 1. From the definition of regions $\mathcal{R}_i$, $i = 1, 2, 3$(expression (8)) and according to the switching logic implemented for $\sigma(t)$ (see equation (6)) it can be easily checked that σ can only switch to 1 if there exists a finite time $\bar{t} > t_0$, such that $\omega(\bar{t}) \in \mathcal{R} = \{(\omega_1, \omega_2) \in \Omega : \omega_1 > \epsilon_2(t) \wedge \omega_2 = 0\}$. Assume (by contradiction) that this happens. Then there exists $\tau > 0$ such that for all $t \in [\bar{t} - \tau, \bar{t})$ one has $\sigma(t) = 2$ and the dynamics of $z_1(t)$ satisfy $\ddot{z}_1 = -\lambda \dot{z}_1$. This in turn shows that (since the initial condition $\omega_2(\bar{t} - \tau) \neq 0$), ω_2 will never be zero at a finite time $t = \bar{t}$.

Claim 2 *For any $T \geq t_{\sigma_2} \geq t_0$ such that $\sigma(t) = 2$ for $t \in [t_{\sigma_2}, T)$, there exist finite constants $\bar{\gamma}_0 > 0$, $\bar{\lambda} > 0$, and $\bar{\gamma} > 0$ such that the sway velocity $v(t)$ satisfies for $t \in [t_{\sigma_2}, T)$ the inequality*

$$|v(t)| \leq \bar{\gamma}_0 + \bar{\gamma}\, e^{-\bar{\lambda}(t - t_{\sigma_2})} + \int_{t_{\sigma_2}}^{t} e^{-\bar{\lambda}(t-\tau)} h(\tau) |s(\tau)| d\tau,$$

where $h(t) = \frac{m_u}{m_v} \frac{\dot{z}_1^2(t_{\sigma_2})}{2\lambda} e^{-2\lambda(t - t_{\sigma_2})}$. Moreover, if for $T = \infty$ one has $\lim_{t \to \infty} h(t)|s(t)| = 0$, then $\lim_{t \to \infty} v(t) = 0$.

Claim 3 *For any $t_{\sigma_2} \geq 0$ such that $\sigma(t_{\sigma_2}) = 2$, if $z_1(t_{\sigma_2}) \neq -\frac{\dot{z}_1(t_{\sigma_2})}{\lambda}$, then there exists a finite time $T \geq t_{\sigma_2}$ such that $\omega_1(T) = \epsilon_1$.*
Proof. To prove that $\omega_1(t)$ reaches the boundary $\omega_1 = \epsilon_1$ in finite time, observe that for $\sigma = 2$ the dynamics of ω_1 are given by

$$\dot{\omega}_1 = 2s\big[z_1 k_2 + 2(\dot{v} + \lambda v)\big], \quad \omega_1 \geq \epsilon_1. \tag{13}$$

Since $\ddot{z}_1 = -\lambda \dot{z}_1$, it can be checked that there exists a finite time $\bar{t}_1 > t_{\sigma_2}$ such $z_1(t) k_2 = -|z_1(t)| \operatorname{sgn}(s)$ for all $t \geq \bar{t}_1$. Thus, from equation (13) it follows that

$$\dot{\omega}_1 = -2s\big[|z_1| \operatorname{sgn}(s) - 2(\dot{v} + \lambda v)\big], \quad t \geq \bar{t}_1$$

Notice that $\lim_{t \to \infty} h(t)|s(t)| = 0$ (see [4]) which implies, according to Claim 2, that $\lim_{t \to \infty} v(t) = 0$. Notice also that $z_1(t)$ converges to a value different from zero ($z_1(t_{\sigma_2}) + \frac{\dot{z}_1(t_{\sigma_2})}{\lambda} \neq 0$). Thus, there will be a finite time $\bar{t}_2 > \bar{t}_1$ such that $\forall t \geq \bar{t}_2$, $\dot{\omega}_1(t) < 0$. Therefore $\omega_1(t)$ will reach the boundary $\omega_1 = \epsilon_1$ in finite time.

Claim 4 *For any t_{σ_2} and any positive interval $\tau > 0$ such that $\sigma(t) = 2$ for $t \in [t_{\sigma_2}, t_{\sigma_2} + \tau]$, if the initial conditions $\big(z_1(t_{\sigma_2}), \dot{z}_1(t_{\sigma_2})\big)'$ satisfy*

$$z_1(t_{\sigma_2}) \dot{z}_1(t_{\sigma_2}) \leq 0, \; |z_1(t_{\sigma_2})| > \frac{|\dot{z}_1(t_{\sigma_2})|}{\lambda}, \tag{14}$$

then for all $t \in [t_{\sigma_2}, t_{\sigma_2} + \tau]$

$$\omega_2(t) \le \omega_2(t_{\sigma_2}). \qquad (15)$$

Proof. For $\sigma = 2$, $\ddot{z}_1 = -\lambda \dot{z}_1$. In this case $z_1(t)$ converges to $z_1(t_{\sigma_2}) + \frac{\dot{z}_1(t_{\sigma_2})}{\lambda}$ as $t \to \infty$ and if conditions (14) hold, then $|z_1(t)| \le |z_1(t_{\sigma_2})|$. Thus, since $\omega_2(t) = \lambda_1(\lambda - \lambda_1)z_1^2(t) + \dot{z}_1^2(t)$, it follows (15).

Claim 5 *There exists a finite time* $T \ge t_0$ *such that* $\sigma(t) = 3$ *and* $\omega_1(t) \le \epsilon_2$ *for all* $t > T$*. Furthermore, the state variables* $z_1(t)$, $\dot{z}_1(t)$, $z_2(t)$, $\dot{z}_2(t)$, *and* $v(t)$ *converge to zero.*

Proof. Consider first that $\sigma(t) = 3$ for all $t \ge T$. Clearly, from (3), (9), and (11) and the fact that $\lambda_1 > 0$ and $\lambda - \lambda_1 > 0$, it follows that $(z_1, \dot{z}_1)'$ is exponentially stable. Also, one can conclude that $(z_2, \dot{z}_2)'$ is exponentially stable if $|s(t)|$ is bounded. From (4), it can be inferred that $v(t)$ converges to zero (notice that $d_v > 0$) if $|s(t)|$ is bounded. Due to space limitations, the proof of boundedness of $|s(t)|$ is omitted. See [2].

To conclude the proof, it remains to show that there exists a finite time T such that for all $t \ge T$, $\sigma(t) = 3$. First, observe that one can always find positive constants $\omega_2^\star$ and $v^\star$ such that for any initial conditions $\sigma(t_{\sigma_3}) = 3$, $z_1(t_{\sigma_3})$, $\dot{z}_1(t_{\sigma_3})$, and $v(t_{\sigma_3})$ that satisfy $\omega_2(t_{\sigma_3}) \le \omega_2^\star$ and $|v(t_{\sigma_3})| \le v^\star$, the bound of $\omega_1(t)$ is less or equal to ϵ_2 for all $t \ge t_{\sigma_3}$. Notice also that for $\sigma = 3$, the closed loop dynamics of $z_1(t)$ are given by $\ddot{z}_1 = -\lambda z_1 - \lambda_1(\lambda - \lambda_1)z_1$. Thus, it follows that for any t_{σ_3} such that $\sigma(t_{\sigma_3}) = 3$, there exist a positive time interval $\tau > 0$ and a finite time $T \in [t_{\sigma_3}, t_{\sigma_3} + \tau]$ such that for all $t \in [T, t_{\sigma_3} + \tau]$

$$z_1(t)\dot{z}_1(t) \le 0 \quad \text{and} \quad |z_1(t)| > \frac{|\dot{z}_1(t)|}{\lambda}. \qquad (16)$$

From the definition of ε_2, it can be concluded that σ can not switch from 3 while condition (16) is not true (since $\varepsilon_2 = +\infty$). Thus, if $\sigma(t)$ switches from 3 to 2, it means that conditions (16) hold. Consequently, from Claim 4 and 3 it follows that $w_2(t)$ will not increase and also that, after a finite time, the signal σ will switch again to 3. Hence, there will be finite jumps between 2 and 3 until ω_2 becomes less or equal to $\omega_2^\star$. See Fig. 3 for a better understanding of the switching logic. From the proofs of Claims 2 and 3, it follows that after a finite time (say $t = T$) one has $|v(T)| \le v^\star$, and $\omega_2(T) \le \omega^\star$. Consequently, it can be concluded that for all $t \ge T$, $\omega_2(t) \le \epsilon_2$, $\sigma(t) = 3$, and $v(t)$ converges to zero. This conclude the proof of Claim 5 and naturally item 2 of Theorem 2.

Stability

From the proof of Claim 5 it can be concluded that there exists a positive constant $\bar{r}$ such that for any $q(t_0) \in B_{\bar{r}}(0)$, $\sigma(t) = 3$ for all $t \ge t_0$. Moreover, given any $\epsilon > 0$, there exists a positive constant $r \le \bar{r}$ such that with $q(t_0) \le r$ it follows that $|z_3(t)| \le \epsilon$ for all $t \ge t_0$ (see (7)). Also, from the closed loop system equations for $\sigma = 3$, it can be easily proved that the other components of the state q, *i.e.*, z_1, $\dot{z}_1$, z_2, $\dot{z}_2$, and v are asymptotically stable. Therefore, the hybrid system Σ_{AUV} is Lyapunov uniformly stable by definition.

This concludes the outline of the proof of Theorem 2. □

4 Simulation Results

This section illustrates the performance of the proposed control scheme with a model of the *Sirene* AUV. The parameters of the complete model can be found in [1].

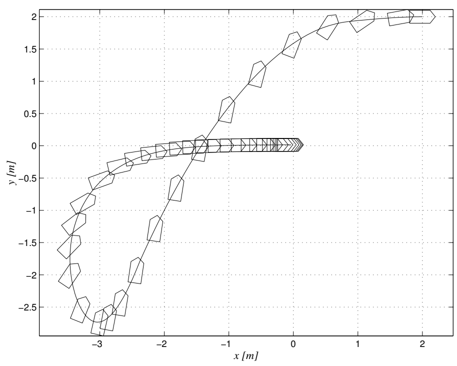

Figure 4: Path of the underactuated *Sirene* AUV.

The simulation results for a sample initial condition given by $(x, y, \theta, u, v, r) = (2\,m, 2\,m, 0, 0, 0, 0)$, or equivalently $q_0 = (z_1, z_2, z_3, \dot{z}_1, \dot{z}_2)' = (0, 2, 4, 0, 0, 0)'$, and $\sigma(0^-) = 1$ are shown in Figures 4-6. The control parameters were chosen to be $\epsilon_1 = 0.1$, $\epsilon_2 = 0.2$, $\gamma_1 = 0.6$, $\lambda = 1.0$, $\lambda_1 = 0.95$, and $k_3 = 100$.

Fig. 4 shows the AUV trajectory in the horizontal plane. The vehicle converges to a small neighborhood of the target position with a desired orientation. Fig. 5 is a plot of the vehicle linear and angular velocities. Notice that the most aggressive motions occur during the first 20 seconds. This is clearly mirrored in the sway velocity activity over that time period. During the rest of the maneuver, as expected, the sway velocity $v(t)$ converges to zero as the trajectory of the vehicle straightens out.

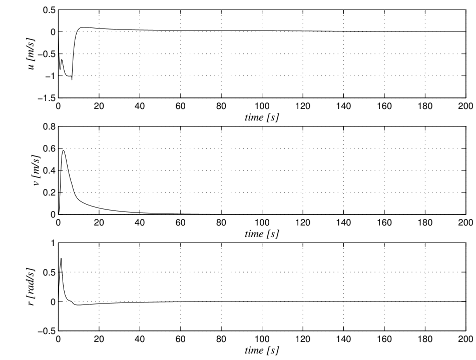

Figure 5: Time evolution of linear velocity in x-direction (surge) u, linear velocity in y-direction (sway) v, and angular velocity r.

To better understand the action of the hybrid control law proposed, examine Fig. 6 that shows the time evolution of the variables $\omega_1(t)$, $\omega_2(t)$, and $\sigma(t)$. It can be seen that $\omega(t)$ starts in region $\mathcal{R}_1$ and, consequently, ω_2 grows until reaches γ_1. At that moment, the signal σ switches to 2, ω_2 does not increase, and ω_1 decreases until it reaches the boundary $\omega_1 = \epsilon_1$. Then, σ switches to 3 and $z_1(t)$, $z_2(t)$, and $v(t)$ converge to zero. This

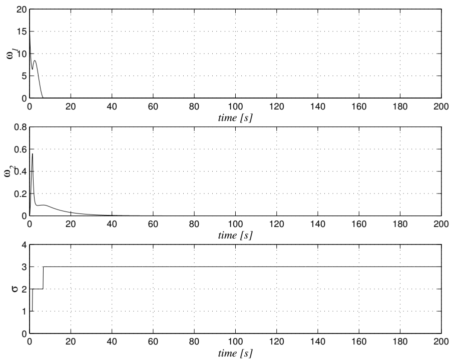

Figure 6: Time evolution of variables ω_1, ω_2, and $\sigma(t)$.

implies that the state $x(t)$ and $\theta(t)$ also converge to zero.

5 Conclusions

A feedback hybrid control law was proposed to globally stabilize an underactuated AUV in the horizontal plane to a point with a desired orientation. The control law derived builds on hybrid systems theory and exhibits a simple structure. Convergence of the AUV to an arbitrarily small neighborhood of the origin and stability of the resulting closed loop system were analyzed. To illustrated the control law developed, simulations were conducted using a dynamic model of the *Sirene* AUV. The simulation results show that the control objectives were achieved successfully. Notice, however that due to the existence in the control law of a coordinate transformation of the AUV model to extended nonholonomic double integrator with drift terms, the resulting path is hard to predict and may not be a "natural maneuver".

Future research will address this problem and that of robustness against parametric model uncertainty. Further work is also required to bridge the gap between theory and practice, effectively bringing the theoretical results derived to bear on the development of controllers for underactuated marine robots.

References

[1] A. P. Aguiar, *Modeling, control, and guidance of an autonomous underwater shuttle for the transport of benthic laboratories*, Master's thesis, Dept. Electrical Engineering, Instituto Superior Técnico, IST, Lisbon, Portugal, 1998.

[2] A. P. Aguiar, *Nonlinear motion control of nonholonomic and underactuated systems*, Ph.D. thesis, Dept. Electrical Engineering, Instituto Superior Técnico, IST, Lisbon, Portugal, 2002.

[3] A. P. Aguiar and A. M. Pascoal, *Modeling and control of an autonomous underwater shuttle for the transport of benthic laboratories*, Proceedings of the Oceans 97 Conference (Halifax, Nova Scotia, Canada), October 1997.

[4] A. P. Aguiar and A. M. Pascoal, *Stabilization of an underactuated autonomous underwater vehicle via a logic-based hybrid controller*, Tech. Report 9903, ISR/IST Institute for Systems and Robotics and Instituto Superior Técnico, Lisbon, Portugal, June 1999.

[5] A. P. Aguiar and A. M. Pascoal, *Stabilization of the extended nonholonomic double integrator via logic-based hybrid control*, SYROCO'00 - 6th International IFAC Symposium on Robot Control (Vienna, Austria), September 2000.

[6] A. P. Aguiar and A. M. Pascoal, *Regulation of a nonholonomic autonomous underwater vehicle with parametric modeling uncertainty using Lyapunov functions*, Proc. 40th IEEE Conference on Decision and Control (Orlando, Florida, USA), December 2001.

[7] M. S. Branicky, *Multiple Lyapunov functions and other analysis tools for switched and hybrid systems*, IEEE Transactions on Automatic Control **43** (1998), no. 4, 475–482.

[8] R. W. Brockett, *Pattern generation and feedback control of nonholonomic systems*, Proc. of the Workshop on Mechanics, Holonomy and Control, IEEE, 1993.

[9] O. Egeland, M. Dalsmo, and O. Sørdalen, *Feedback control of a nonholonomic underwater vehicle with a constant desired configuration*, The International Journal Of Robotics Research, MIT **15** (1996), no. 1, 24–35.

[10] T. I. Fossen, *Guidance and control of ocean vehicles*, John Wiley & Sons, England, 1994.

[11] J. P. Hespanha, *Stabilization of nonholonomic integrators via logic-based switching*, Proc. 13th World Congress of IFAC (S. Francisco, CA, USA), vol. E, June 1996, pp. 467–472.

[12] A. Isidori, *Nonlinear control systems*, 2^{nd} ed., Springer-Verlag, Berlin, Germany, 1989.

[13] H. K. Khalil, *Nonlinear systems*, 2^{nd} ed., Prentice-Hall, New Jersey, USA, 1996.

[14] N. E. Leonard, *Control synthesis and adaptation for an underactuated autonomous underwater vehicle*, IEEE Journal of Oceanic Engineering **20** (1995), no. 3, 211–220.

[15] H. Nijmeijer and A. J. van der Schaft, *Nonlinear dynamical control systems*, Springer-Verlag, New York, USA, 1990.

[16] K. Y. Pettersen and O. Egeland, *Position and attitude control of an underactuated autonomous underwater vehicle*, Proceedings of the 35th IEEE Conference on Decision and Control (Kobe, Japan), 1996, pp. 987–991.

[17] K. Y. Pettersen and O. Egeland, *Robust attitude stabilization of an underactuated AUV*, Proceedings of 1997 European Control Conference (Brussels, Belgium), July 1997.

[18] K. Y. Pettersen and T. I. Fossen, *Underactuated dynamic positioning of a ship - experimental results*, IEEE Transactions on Control Systems Technology **8** (2000), no. 5, 856–863.

[19] K. Y. Pettersen and H. Nijmeijer, *Global practical stabilization and tracking for an underactuated ship - a combined averaging and backstepping approach*, Proc. IFAC Conference on Systems Structure and Control (Nantes, France), July 1998, pp. 59–64.

[20] H. J. Sussmann, *A general theorem on local controllability*, SIAM Journal of Control and Optimization **25** (1987), no. 1, 158–194.

[21] H. Ye, A. N. Michel, and L. Hou, *Stability theory for hybrid dynamical systems*, IEEE Transactions on Automatic Control **43** (1998), no. 4, 461–474.