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Dynamic positioning and way-point tracking of underactuated AUVs in the presence of ocean currents[1]

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Abstract

This paper addresses the problem of dynamic positioning and way-point tracking of underactuated autonomous underwater vehicles (AUVs) in the presence of constant unknown ocean currents and parametric model uncertainty. A nonlinear adaptive controller is proposed that steers an AUV so as to track a sequence of points consisting of desired positions (x, y) in a inertial reference frame, followed by vehicle positioning at the final target point. The controller is first derived at the kinematic level assuming that the ocean current disturbance is known. An exponential observer is then designed and convergence of the resulting closed loop system trajectories is analyzed. Finally, integrator backstepping and Lyapunov based techniques are used to extend the kinematic controller to the dynamic case and to deal with model parameter uncertainty. Simulation results are presented and discussed.

1 Introduction

In an underactuated dynamical system, the dimension of the space spanned by the control vector is less than the dimension of the configuration space. Consequently, systems of this kind necessarily exhibit constraints on accelerations. See [11] for a survey of these concepts. The motivation for the study of controllers for underactuated systems, namely mobile robots is manifold and includes the following:

i) *Practical applications.* There is an increasing number of real-life underactuated mechanical systems. Mobile robots, walking robots, spacecraft, aircraft, helicopters, missiles, surface vessels, and underwater vehicles are representative examples.

ii) *Cost reduction.* For example, for underwater vehicles that work at large depths, the inclusion of a lateral thruster is very expensive and represents large capital costs.

iii) *Weight reduction.* This issue is of critical importance for aerial vehicles.

iv) *Thruster efficiency.* Often, an otherwise fully actuated vehicle may become underactuated when its speed changes. This happens in the case of AUVs that are designed to maneuver at low speeds using thruster control only. As the forward speed increases, the efficiency of the side thrusters decreases sharply, thus making it impossible to impart pure lateral motions on the vehicle.

v) *Reliability considerations.* Even for fully actuated vehicles, if one or more actuator failures occur, the system should be capable of detecting them and engaging a new control algorithm specially designed to accommodate the respective fault, and complete its mission if at all possible.

vi) *Complexity and increased challenge that this class of systems bring to the control area.* In fact, most underactuated systems are not fully feedback linearizable and exhibit nonholonomic constraints.

Necessary and sufficient conditions for an underactuated manipulator to exhibit second-order nonholonomic, first-order nonholonomic, or holonomic constraints are given in [10]. See also [12] for an extension of these results to underactuated vehicles (e.g. surface vessels, underwater vehicles, aeroplanes, and spacecraft). The work in [12] shows that if so-called unactuated dynamics of a vehicle model contain no gravitational field component, no continuously differentiable, constant state-feedback control law will asymptotically stabilize it to an equilibrium condition. This result brings out the importance of studying advanced control laws for underactuated systems.

The underactuated vehicle under consideration in this paper is the *Sirene* autonomous underwater vehicle (AUV). The *Sirene* vehicle - depicted in Fig. 1 - has an open-frame structure and is 4.0 m long, 1.6 m wide, and 1.96 m high. It has a dry weight of 4000 Kg and a maximum operating depth of 4000 m. The vehicle is equipped with two back thrusters for surge and yaw motion control in the horizontal plane, and one vertical thruster for heave control. Roll and pitch motion are left uncontrolled, since the metacentric height[1] is sufficiently large (36 cm) to provide adequate static stability. The AUV has no side thruster. In the figure, the vehicle carries a representative benthic lab which is cubic-shaped with a volume of approximately $2.3m^3$.

The problem of steering an underactuated AUV to a point with a desired orientation has only recently received special attention in the literature. This task

[1]This work was supported in part by the EC under the FREESUB network and by the PDCTM programme of the FCT of Portugal under projects DREAM and MAROV.

[1]distance between the center of buoyancy and the center of mass.

raises some challenging questions in control system theory because in addition to being underactuated the vehicle exhibit complex hydrodynamic effects that must necessarily be taken into account during the controller design phase. Namely, the vehicle exhibits sway and heave velocities that generate non-zero angles of sideslip and attack, respectively. This rules out any attempt to design a steering system for the AUV that would rely on its kinematic equations only.

In practice, an AUV must often operate in the presence of unknown ocean currents. Interestingly enough, even for the case where the current is constant, the problem of regulating an AUV to a desired point with an arbitrary desired orientation does not have a solution. In fact, if the desired orientation does not coincide with the direction of the current, normal control laws will yield one of two possible behaviors: *i)* the vehicle will diverge from the desired target position, or *ii)* the controller will keep the vehicle moving around a neighborhood of the desired position, trying insistently to steer it to the given point, and consequently inducing an oscillatory behavior.
Motivated by this consideration, [4] addresses the problem of dynamic positioning of an AUV in the horizontal plane in the presence of unknown, constant ocean currents. To tackle this problem, the approach considered was to drop the specification on the final desired orientation and use this extra degree of freedom to force the vehicle to converge to the desired point. Naturally, the orientation of the vehicle at the end will be aligned with the direction of the current.

Another problem that extends the previous one is that of designing a guidance scheme to achieve way-point tracking before the AUV stops at the final goal position. The AUV can then be made to track a predefined reference path that is specified by a sequence of way points. Way-point tracking can in principle be done in a number of ways. Most of them have a practical flavor and lack a solid theoretical background. Perhaps the most widely known is so-called line-of-sight scheme [7]. In this case, vehicle guidance is simply done by issuing heading reference commands to the vehicle's steering system so as to approach the line of sight between the present position of the vehicle and the way-point to be reached. Tracking of the reference command is done via a properly designed autopilot. Notice, however, that the separation of guidance and autopilot functions may not yield stability.

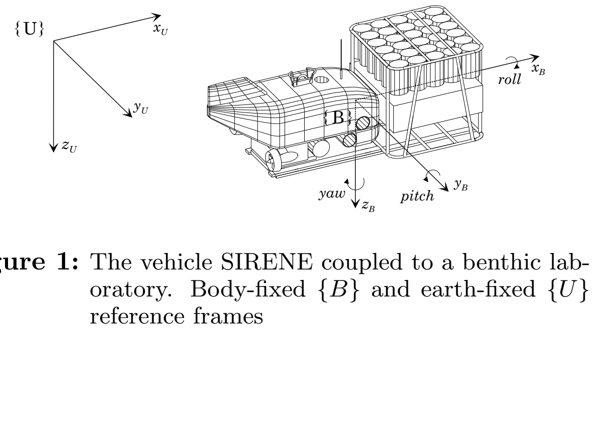

Figure 1: The vehicle SIRENE coupled to a benthic laboratory. Body-fixed $\{B\}$ and earth-fixed $\{U\}$ reference frames

Motivated by the above considerations, this paper extends the strategy proposed in [4] to position the AUV *Sirene* at the origin to actually force the AUV to track a sequence of points consisting of desired positions (x, y) in a inertial reference frame before it converges to the finally desired point. See [5] for related work in the area of wheeled robots. A nonlinear adaptive controller is proposed that yields convergence of the trajectories of the closed loop system in the presence of a constant unknown ocean current disturbance and parametric model uncertainty. Controller design relies on a non smooth coordinate transformation in the original state space followed by the derivation of a Lyapunov-based, adaptive, control law in the new coordinates and an exponential observer for the ocean current disturbance. For the sake of clarity of presentation, the controller is first derived at the kinematic level, assuming that the ocean current disturbance is known. Then, an observer is designed and convergence of the resulting closed loop system is analyzed. Finally, resorting to integrator backstepping and Lyapunov techniques [9], a nonlinear adaptive controller is developed that extends the kinematic controller to the dynamic case and deals with model parameter uncertainties. See [2] for full details.

2 The AUV. Control Problem Formulation

This section describes the kinematic and dynamic equations of motion of the AUV of Fig. 1 in the horizontal plane and formulates the problem of dynamic positioning and way-point tracking. The control inputs are the thruster surge force and the thruster yaw torque. The AUV has no side thruster. See [1, 3] for model details.

2.1 Vehicle Modeling

Following standard practice, the general kinematic and dynamic equations of motion of the vehicle can be developed using a global coordinate frame $\{U\}$ and a body-fixed coordinate frame $\{B\}$ that are depicted in Fig. 1. In the horizontal plane, the kinematic equations of motion of the vehicle, can be written as

$$\dot{x} = u\cos\psi - v\sin\psi, \tag{1a}$$
$$\dot{y} = u\sin\psi + v\cos\psi, \tag{1b}$$
$$\dot{\psi} = r, \tag{1c}$$

where u (surge speed) and v (sway speed) are the body fixed frame components of the vehicle's velocity, x and y are the cartesian coordinates of its center of mass, ψ defines its orientation, and r is the vehicle's angular speed. In the presence of a constant and irrotational ocean current, $(u_c, v_c)' \neq 0$, u and v are given by $u = u_r + u_c$ and $v = v_r + v_c$, where $(u_r, v_r)'$ is the relative body-current linear velocity vector.
Neglecting the motions in heave, roll, and pitch the simplified equations of motion for surge, sway and heading yield [6]

$$m_u\dot{u}_r - m_v v_r r + d_{u_r} u_r = \tau_u, \tag{2a}$$
$$m_v\dot{v}_r + m_u u_r r + d_{v_r} v_r = 0, \tag{2b}$$
$$m_r\dot{r} - m_{uv} u_r v_r + d_r r = \tau_r, \tag{2c}$$

where $m_u = m - X_{\dot{u}}$, $m_v = m - Y_{\dot{v}}$, $m_r = I_z - N_{\dot{r}}$, and $m_{uv} = m_u - m_v$ are mass and hydrodynamic added mass terms and $d_{u_r} = -X_u - X_{|u|u}|u_r|$, $d_{v_r} = -Y_v - Y_{|v|v}|v_r|$, and $d_r = -N_r - N_{|r|r}|r|$ capture hydrodynamic damping effects. The symbols τ_u and τ_r denote the external force in surge and the external torque about the z axis of the vehicle, respectively. In the equations, and for clarity of presentation, it is assumed that the AUV is neutrally buoyant and that the centre of buoyancy coincides with the centre of gravity.

2.2 Problem Formulation

Observe Fig. 2. The problem considered in this paper can be formulated as follows:

Consider the underactuated AUV with the kinematic and dynamic equations given by (1) *and* (2). *Let* $p = \{p_1, p_2, \ldots, p_n\}$*;* $p_i = (x_i, y_i)$, $i = 1, 2, \cdots, n$ *be a given sequence of points in* $\{U\}$. *Associated with each* p_i*;* $i = 1, 2, \cdots, (n-1)$ *consider the closed ball* $N_{\epsilon_i}(p_i)$ *with center* p_i *and radius* $\epsilon_i > 0$. *Derive a feedback control law for* τ_u *and* τ_r *so that the vehicle's center of mass* (x, y) *converges to* p_n *after visiting (that is, reaching) the ordered sequence of neighborhoods* $N_{\epsilon_i}(p_i)$*;* $i = 1, 2, \cdots, (n-1)$ *in the presence of a constant unknown ocean current disturbance and parametric model uncertainty.*

Notice how the requirement that the neighborhoods be visited only applies to $i = 1, 2, \cdots, (n-1)$. In fact, for the last way-point the vehicle will be steered using the controller developed in [4] (see Section 4). Details are omitted.

3 Nonlinear Controller Design

This section proposes a nonlinear adaptive control law to steer the underactuated AUV through a sequence of neighborhoods $N_{\epsilon_i}(p_i)$; $i = 1, 2, \cdots, (n-1)$, in the presence of a constant unknown ocean current disturbance and parametric model uncertainty.

3.1 Coordinate Transformation

Let $(x_d, y_d)'$ denote a generic way-point p_i. Let d be the vector from the origin of frame $\{B\}$ to $(x_d, y_d)'$, and e its length. Denote by β the angle measured from x_B to d. Consider the coordinate transformation (see Fig. 2)

$$e = \sqrt{(x - x_d)^2 + (y - y_d)^2}, \tag{3a}$$

$$x - x_d = -e\cos(\psi + \beta), \tag{3b}$$

$$y - y_d = -e\sin(\psi + \beta), \tag{3c}$$

$$\psi + \beta = \tan^{-1}\left(\frac{-(y - y_d)}{-(x - x_d)}\right). \tag{3d}$$

In equation (3d), care must be taken to select the proper quadrant for β. The kinematics equations of motion of the AUV can be rewritten in the new coordinate system to yield

$$\dot{e} = -u_r\cos\beta - v_r\sin\beta - V_c\cos(\beta + \psi - \phi_c), \tag{4a}$$

$$\dot{\beta} = \frac{\sin\beta}{e}u_r - \frac{\cos\beta}{e}v_r - r + \frac{V_c}{e}\sin(\beta + \psi - \phi_c), \tag{4b}$$

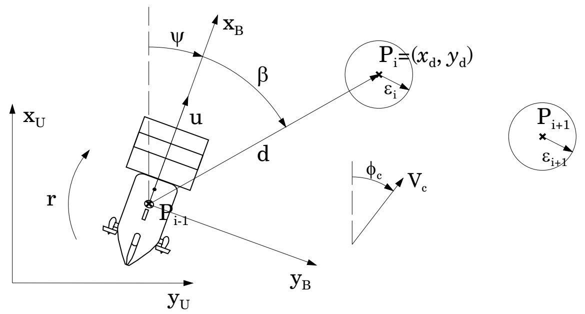

Figure 2: Coordinate Transformation.

$$\dot{\psi} = r. \tag{4c}$$

Notice that the coordinate transformation (3) is only valid for non zero values of the variable e, since for $e = 0$ the angle β is undefined.

In what follows it is important to introduce the following notation. Let $\chi = (x, y)'$ and $\chi_d = (x_d, y_d)'$. Clearly, $e = \|\chi - \chi_d\|_2$. Notice that $e = e(i)$; $i = 1, 2, \cdots, (n-1)$, that is, the error depends on what current way-point $\chi_d = p_i$ is selected. Let $\mathbb{Z}_n$ be the set $\mathbb{Z}_n = \{1, 2, \ldots, n\}$. Consider the piecewise constant signal $\sigma : [t_0, \infty) \to \mathbb{Z}_n$ that is continuous from the right at every point and is defined recursively by

$$\sigma = \eta(\chi, \sigma^-), \quad t \geq t_0 \tag{5}$$

where $\sigma^-(\tau)$ is equal to the limit from the left of $\sigma(\tau)$ as $\tau \to t$. The operator $\eta : \mathbb{R}^3 \times \mathbb{Z}_n \to \mathbb{Z}_n$ is the transition function defined by

$$\eta(\chi, i) = \begin{cases} i, & e = e(i) > \epsilon_i \\ i + 1, & e = e(i) \leq \epsilon_i;\ i \neq n \\ n, & i = n. \end{cases} \tag{6}$$

In order to single out the last way-point as a desired target towards which the AUV should converge, and inspired by the work of [4], $(x_d, y_d)'$ is formally defined as

$$(x_d, y_d) = \begin{cases} p_\sigma & \text{if } \sigma < n, \\ p_\sigma - \gamma(\cos\phi_c, \sin\phi_c) & \text{if } \sigma = n. \end{cases} \tag{7}$$

3.2 Kinematic Controller

At the kinematic level it will be assumed that u_r and r are the control inputs. At this stage, the relevant equations of motion of the AUV are simply (4) and (2b). It is important to stress out that the dynamics of the sway velocity v must be explicitly taken into account, since the presence of this term in the kinematics equations (1) is not negligible (as is usually the case for wheeled mobile robots).

Returning now to the control problem, observe equations (4). The strategy for controller design consists basically of *i)* for $i = 1, 2, \cdots (n-1)$, fixing the surge velocity to a constant positive value U_d, *ii)* manipulating r to regulate β to zero (this will align x_B with vector d), and *iii)* for $i = n$ (the final target), actuating on u_r to force the vehicle to converge to position p_n. At this stage, it is assumed that the intensity V_c and the direction ϕ_c of the ocean current disturbance are known. The following result applies for the case where $i < n$.

Theorem 1 *Consider the sequence of points* $\{p_1, p_2, \ldots, p_n\}$ *and the associated neighborhoods* $\{N_{\epsilon_1}(p_1), N_{\epsilon_2}(p_2), \ldots, N_{\epsilon_{n-1}}(p_{n-1})\}$. *Let* $\underline{\epsilon} = \min_{1 \le i < n} \epsilon_i$ *and* U_d, k_2, *and* $\underline{k}_2 > 0$ *be positive constants. Consider the nonlinear system* Σ_{kin} *described by the AUV nonlinear model* (1) *and* (2b) *and assume that*

$$k_2 \ge \frac{U_d + V_c}{\underline{\epsilon}} + \underline{k}_2, \quad U_d > V_c, \quad \frac{d_{v_r}}{m_u} > \frac{U_d}{\underline{\epsilon}}. \tag{8}$$

Let the control law $u_r = \alpha_1$ *and* $r = \alpha_2$ *be given by*

$$\alpha_1 = U_d, \tag{9a}$$

$$\alpha_2 = k_2 \beta + \frac{V_c}{e} \sin(\psi - \phi_c) \cos\beta - \frac{v_r}{e} \cos\beta \tag{9b}$$

with β *and* e *as given in* (3) *where* $(x_d, y_d)'$ *is computed using* (5)-(7).
Let $\mathcal{X}_{kin}(t) = (x, y, \psi, v_r)' = \{\mathcal{X}_{kin} : [t_0, \infty) \to \mathbb{R}^4\}$, $t_0 \ge 0$, *be a solution to* Σ_{kin}. *Then, for any initial conditions* $\mathcal{X}_{kin}(t_0) \in \mathbb{R}^4$ *the control signals and the solution* $\mathcal{X}_{kin}(t)$ *are bounded. Furthermore, there are finite instants of time* $t_1^m \le t_1^M \le t_2^m \le t_2^M, \ldots, \le t_{n-1}^m \le t_{n-1}^M$ *such that* $\big(x(t), y(t)\big)'$ *stays in* $N_{\epsilon_i}(p_i)$ *for* $t_i^m \le t \le t_i^M$, $i = 1, 2, \ldots, n-1$.

Proof. Consider the candidate Lyapunov function

$$V_{kin} = \frac{1}{2}\beta^2. \tag{10}$$

Computing its time derivative along the trajectories of system Σ_{kin} gives

$$\dot{V}_{kin} = -\beta^2 \Big[k_2 - \frac{U_d}{e}\frac{\sin\beta}{\beta} - \frac{V_c}{e}\frac{\sin\beta}{\beta}\cos(\psi - \phi_c)\Big]$$

which is negative definite if k_2 satisfies condition (8). Thus, $\beta \to 0$ as $t \to \infty$. To prove that v_r is bounded consider its dynamic motion in closed loop given by

$$\begin{aligned}\dot{v}_r = &-\Big[\frac{d_{v_r}}{m_v} - \frac{m_u}{m_v}\frac{U_d}{e}\cos\beta\Big] v_r \\ &- \frac{m_u}{m_v} U_d \Big[k_2\beta + \frac{V_c}{e}\cos\beta \sin(\psi - \phi_c)\Big],\end{aligned} \tag{11}$$

Clearly, if condition (8) holds, then v_r is bounded since $\lim_{|v_r| \to \infty} v_r \dot{v}_r = -\infty$. The convergence of e is shown by observing that

$$\dot{e} = -U_d \cos\beta - v_r \sin\beta - V_c \cos(\beta + \psi - \phi_c).$$

Thus, since $\beta \to 0$, v_r is bounded and $U_d > V_c$ it follows that there exist a time $T \ge t_0$ and a finite positive constant α such that $\dot{e} < -\alpha$ for all $t > T$. Consequently, the vehicle position (x, y) reaches the neighborhood $N_{\epsilon_i}(p_i)$ of p_i in finite time. □

Notice that Theorem 1 only deals with the first $n-1$ way-points. Steering the vehicle to the last way-point can be done using the control structure proposed in [4].

3.3 Observer Design

Let v_{c_x} and v_{c_y} denote the components of the ocean current disturbance expressed in $\{U\}$. Then, the kinematic equation (1a) can be rewritten as

$$\dot{x} = u_r \cos\psi - v_r \sin\psi + v_{c_x}.$$

A simple observer for the component v_{c_x} of the current is

$$\begin{aligned}\dot{\hat{x}} &= u_r \cos\psi - v_r \sin\psi + \hat{v}_{c_x} + k_{x_1}\tilde{x}, \\ \dot{\hat{v}}_{c_x} &= k_{x_2}\tilde{x},\end{aligned}$$

where $\tilde{x} = x - \hat{x}$. Clearly, the estimate errors $\tilde{x}$ and $\tilde{v}_{c_x} = v_{c_x} - \hat{v}_{c_x}$ are asymptotically exponentially stable if all roots of the characteristic polynomial $p(s) = s^2 + k_{x_1}s + k_{x_2}$ associated with the system

$$\begin{bmatrix}\dot{\tilde{x}} \\ \dot{\tilde{v}}_{c_x}\end{bmatrix} = \begin{bmatrix}-k_{x_1} & 1 \\ -k_{x_2} & 0\end{bmatrix}\begin{bmatrix}\tilde{x} \\ \tilde{v}_{c_x}\end{bmatrix}$$

have strictly negative real parts.

The observer for the component v_{c_y} can be written in an analogous manner.

Define the variables $\hat{V}_c$ and $\hat{\phi}_c$ as the module and argument of the vector $[\hat{v}_{c_x}, \hat{v}_{c_y}]$, respectively. The next theorem shows convergence of the kinematic control loop when the observer is included.

Theorem 2 *Consider the nonlinear time invariant system* $\Sigma_{kin+Obs}$ *consisting of the nonlinear AUV model* (1), (2b), *the current observer, and the control law* (5)-(7), *together with* $u_r = \alpha_1$ *and* $r = \alpha_2$, *where* α_1 *and* α_2 *are given by* (9) *with* V_c *and* ϕ_c *replaced by their estimates* $\hat{V}_c$ *and* $\hat{\phi}_c$, *respectively. Assume that* U_d *and* k_2 *are positive constants and satisfy conditions* (8). *Consider the sequence of points* $\{p_1, p_2, \ldots, p_n\}$ *and the associated neighborhoods* $\{N_{\epsilon_1}(p_1), N_{\epsilon_2}(p_2), \ldots, N_{\epsilon_{n-1}}(p_{n-1})\}$. *Let* $\mathcal{X}_{kin+Obs}(t) = (x, y, \psi, v_r, \tilde{v}_{c_x}, \tilde{v}_{c_y})' = \{\mathcal{X}_{kin+Obs} : [t_0, \infty) \to \mathbb{R}^6\}$, $t_0 \ge 0$, *be a solution of* $\Sigma_{kin+Obs}$. *Then, for any initial conditions* $\mathcal{X}_{kin+Obs}(t_0) \in \mathbb{R}^6$ *the control signals and the solution* $\mathcal{X}_{kin+Obs}(t)$ *are bounded. Furthermore, there are finite instants of time* $t_1^m \le t_1^M \le t_2^m \le t_2^M, \ldots, \le t_{n-1}^m \le t_{n-1}^M$ *such that* $\big(x(t), y(t)\big)'$ *stays in* $N_{\epsilon_i}(p_i)$ *for* $t_i^m \le t \le t_i^M$, $i = 1, 2, \ldots, n-1$.

Proof. Consider first the case where $\hat{V}_c = V_c$ and $\hat{\phi}_c = \phi_c$ for all $t \ge t_0$. Then, from Theorem 1, one can conclude that for any initial conditions $\mathcal{X}_{kin+Obs}(t_0)$ on manifold $\{\tilde{v}_{c_x} = 0, \tilde{v}_{c_y} = 0\}$ the control signals and the solution $\mathcal{X}_{kin+Obs}(t)$ are bounded and the position (x, y) reaches the sequence of neighborhoods of points $p_1, p_2, \ldots, p_{n-1}$. Observe also that, from Section 3.3, $(\tilde{v}_{c_x}, \tilde{v}_{c_y}) \to 0$ as $t \to \infty$. Thus, to conclude the proof it remains to show that all off-manifold solutions are bounded. Starting with β, one has

$$\begin{aligned}\dot{\beta} = &-\beta^2 \Big[k_2 - \frac{U_d}{e}\frac{\sin\beta}{\beta} - \frac{V_c}{e}\frac{\sin\beta}{\beta}\cos(\psi - \phi_c)\Big] \\ &- \Big[\frac{\hat{V}_c}{e}\sin(\psi - \hat{\phi}_c) - \frac{V_c}{e}\sin(\psi - \phi_c)\Big]\cos\beta.\end{aligned}$$

Clearly it can be seen that β is bounded. Notice also that v_r is bounded, since its dynamics are given by (11) replacing V_c and ϕ_c by $\hat{V}_c$ and $\hat{\phi}_c$, respectively.

Since all off-manifold solutions are bounded and $\{\tilde{v}_{c_x}, \tilde{v}_{c_y}\}$ converge to zero, then, resorting to LaSalle's invariance principle and the positive limit set lemma [8, Lemma 3.1], Theorem 2 follows. □

3.4 Nonlinear Dynamic Controller Design

This section indicates how the kinematic controller can be extended to the dynamic case (details are omitted). This is done by resorting to backstepping techniques [9]. Following this methodology, let u_r and r be virtual control inputs and α_1 and α_2 (see equations (9a) and (9b)) the corresponding virtual control laws. Introduce the error variables

$$z_1 = u_r - \alpha_1, \quad z_2 = r - \alpha_2,$$

and consider the Lyapunov function (10), augmented with the quadratic terms z_1 and z_2, that is,

$$V_{dyn} = V_{kin} + \frac{1}{2} m_u z_1^2 + \frac{1}{2} m_r z_2^2.$$

The time derivative of V_{dyn} can be written as

$$\dot{V}_{dyn} \leq -\underline{k}_2 \beta^2 + z_1 \Big[\tau_u + m_v v_r r - d_{u_r} u_r - m_u \dot{\alpha}_1 + \frac{\sin\beta}{e}\beta \Big] + z_2 \Big[\tau_r + m_{uv} u_r v_r - d_r r - m_r \dot{\alpha}_2 - \beta \Big].$$

Let the control law for τ_u and τ_r be chosen as

$$\begin{aligned} \tau_u &= -m_v v_r r + d_{u_r} u_r + m_u \dot{\alpha}_1 - \frac{\sin\beta}{e}\beta - k_3 z_1, \\ \tau_r &= -m_{uv} u_r v_r + d_r r + m_r \dot{\alpha}_2 + \beta - k_4 z_2, \end{aligned}$$

where k_3 and k_4 are positive constants. Then,

$$\dot{V}_{dyn} \leq -\underline{k}_2 \beta^2 - k_3 z_1^2 - k_4 z_2^2$$

that is, $\dot{V}_{dyn}$ is negative definite.

3.5 Adaptive Nonlinear Controller Design

So far, it was assumed that the AUV model parameters are known precisely. This assumption is unrealistic. In this section the control law developed is extended to ensure robustness against uncertainties in the model parameters.

Consider the set of all parameters of the AUV model (2) concatenated in the vector $\Theta = \Big[m_u, m_v, m_{uv}, m_r, X_u, X_{|u|u}, N_r, N_{|r|r}, m_r \frac{m_u}{m_v}, m_r \frac{Y_v}{m_v}, m_r \frac{Y_{|v|v}}{m_v} \Big]'$, and define the parameter estimation error $\tilde{\Theta}$ as $\tilde{\Theta} = \Theta - \hat{\Theta}$, where $\hat{\Theta}$ denotes a nominal value of Θ. Consider the augmented candidate Lyapunov function

$$V_{adp} = V_{dyn} + \frac{1}{2} \tilde{\Theta}^T \Gamma^{-1} \tilde{\Theta}, \tag{14}$$

where $\Gamma = \operatorname{diag}\{\gamma_1, \gamma_2, ..., \gamma_{11}\}$, and $\gamma_i > 0$, $i = 1, 2, ...11$ are the adaptation gains.

Motivated by the choices in the previous sections, choose the control laws

$$\begin{aligned} \tau_u &= -\hat{\theta}_2 v_r r - \hat{\theta}_5 u_r - \hat{\theta}_6 |u_r| u_r + \hat{\theta}_1 \dot{\alpha}_1 - \frac{\sin\beta}{e}\beta - k_3 z_1, \\ \tau_r &= -\hat{\theta}_3 u_r v_r - \hat{\theta}_7 r - \hat{\theta}_8 |r| r + \hat{\theta}_4 \dot{\alpha}_{2_a} + \hat{\theta}_9 \frac{u_r}{e} r \cos\beta \\ &\quad + \hat{\theta}_{10} \frac{v_r}{e} \cos\beta + \hat{\theta}_{11} |v_r| \frac{v_r}{e} \cos\beta \\ &\quad + \hat{\theta}_4 \frac{v_r}{e} \Big(\frac{\dot{e}}{e} \cos\beta + \dot{\beta} \sin\beta \Big) + \beta - k_4 z_2, \end{aligned}$$

where $\hat{\theta}_i$ denotes the i-th element of vector $\hat{\Theta}$, $\alpha_{2_a} = k_2 \beta + \frac{V_c}{e} \sin(\psi - \phi_c) \cos\beta$, $\alpha_{2_b} = -\frac{v_r}{e} \cos\beta$. Then,

$$\dot{V}_{adp} \leq -\underline{k}_2 \beta^2 - k_3 z_1^2 - k_4 z_2^2 + \tilde{\Theta}^T \Big[Q - \Gamma^{-1} \dot{\hat{\Theta}} \Big],$$

where Q is a diagonal matrix given by $Q = \operatorname{diag}\Big\{ -\dot{\alpha}_1 z_1, z_1 v_r r, z_2 u_r v_r, -z_2 \dot{\alpha}_{2_a} - z_2 \frac{v_r}{e} \Big(\frac{\dot{e}}{e} \cos\beta + \dot{\beta} \sin\beta \Big), z_1 u_r, z_1 |u_r| u_r, z_2 r, z_2 |r| r, -u_r r \frac{z_2}{e} \cos\beta, \frac{v_r}{e} z_2 \cos\beta, \frac{v_r}{e} |v_r| z_2 \cos\beta \Big\}$. Notice in above equation how the terms containing $\tilde{\Theta}_i$ have been grouped together. To eliminate them, choose the parameter adaptation law as $\dot{\hat{\Theta}} = \Gamma Q$, to yield $\dot{V}_{adp} \leq -\underline{k}_2 \beta^2 - k_3 z_1^2 - k_4 z_2^2 \leq 0$.

4 Simulation results

In order to illustrate the performance of the way-point tracking control algorithm derived (in the presence of parametric uncertainty and constant ocean current disturbances), computer simulations were carried out with a model of the *Sirene* AUV. The vehicle dynamic model can be found in Section 2. See also [1, 3], for complete details.

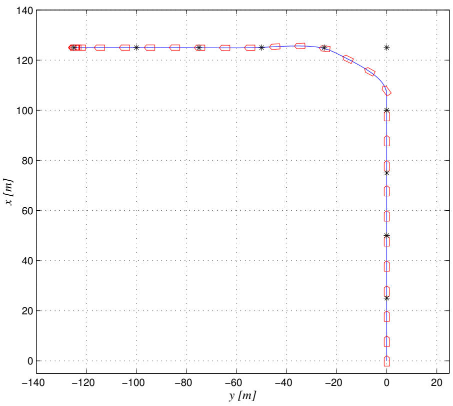

Figure 3: Way-point tracking with the *Sirene* AUV. $U_d = 0.5\, m/s$, $V_c = \phi_c = 0$.

Figures 3-5 display the resulting vehicle trajectory in the xy-plane for three different simulations scenarios using the nonlinear adaptive control law for $i < n$ and the controller described in [4] for $i = n$ (the last point). The control parameters (for $i < n$) were selected as following: $k_2 = 1.8$, $k_3 = 1 \times 10^3$, $k_4 = 500$, $k_{x_1} = 1.0$, $k_{x_2} = 0.25$, $k_{y_1} = 1.0$, $k_{y_2} = 0.25$, and $\Gamma = \operatorname{diag}(10, 10, 10, 1, 1, 2, 2, 2, 1, 0.1, .1) \times 10^3$. The parameters satisfy the constraints (8). The initial estimates for the vehicle parameters were disturbed by 50% from their true values. The sequence of points are $p = \{(25.0, 0.0), (50.0, 0.0), (75.0, 0.0), (100.0, 0.0), (125.0, 0.0), (125.0, -25.0), (125.0, -50.0), (125.0, -75.0), (125, -100.0), (125.0, -125.0), (125.0, -125.0)\}$. The maximum admissible deviations from p_i; $i = 1, 2, \cdots, 10$ were fixed to $\epsilon_i = 5\,m$, except for $i = 5$, where $\epsilon_5 = 20\,m$. In both simulations, the initial conditions for the vehicle were $(x, y, \psi, u, v, r) = 0$. In the first simulation (see Fig. 3) there is no ocean current. The other two simulations capture the situation where the ocean current (which is unknown from the point of view of the controller) has intensity $V_c = 0.2\,m/s$ and direction $\phi_c = \frac{\pi}{4}$ rad, but with different values on the controller parameter U_d. See Figures 4 and 5 for $U_d = 0.5$ and $U_d = 1.0$, respectively. The figures show the influence of the ocean current on the resulting xy-trajectory. Clearly, the influence is stronger for slow forward speeds u_r. In spite of that, notice that the vehicle always reaches the sequence of neighborhoods of the points $p_1, p_2, \ldots, p_{10}$ until it finally converges to the desired position $p_{11} = (125, 125)\,m$.

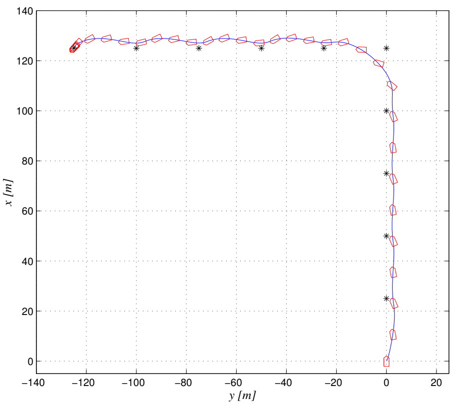

Figure 4: Way-point tracking with the *Sirene* AUV. $U_d = 0.5\,m/s$, $V_c = 0.2\,m/s$, $\phi_c = \frac{\pi}{4}\,rad$.

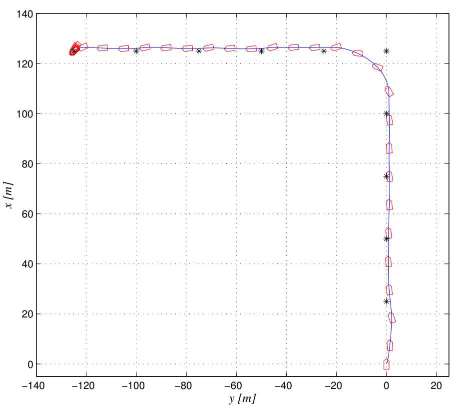

Figure 5: Way-point tracking with the *Sirene* AUV. $U_d = 1.0\,m/s$, $V_c = 0.2\,m/s$, $\phi_c = \frac{\pi}{4}\,rad$.

5 Conclusions

A solution to the problem of dynamic positioning and way-point tracking of an underactuated AUV (in the horizontal plane) in the presence of a constant unknown ocean current disturbance and parametric model uncertainty was proposed. Convergence of the resulting nonlinear system trajectories was analyzed and simulations were performed to illustrate the behavior of the proposed control scheme. Simulation results show that the control objectives were achieved successfully. Future research will address the application of the new control strategy developed to the operation of a prototype marine vehicle.

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