Multiple-Model Adaptive Estimators: Open Problems and Future Directions

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Over the past decades there have been a considerable research effort on the subject of adaptive observers. In simple terms, an adaptive observer or estimator can be defined as a process that provides in real time the estimate of the state (or some function of it) of the plant from partial and possibly noise measurements of the inputs and outputs, inexact knowledge of the initial condition and *in the presence of unknown parameters in the description of the plant*[1]. In many cases, adaptive observers also identifies the unknown parameters.

Several adaptive continuous/discrete (or even hybrid) observers using deterministic/stochastic approaches for linear/nonlinear plants can be found in the literature. An attractive class of adaptive observers is the one that uses multiple-model architectures. Multiple-model adaptive estimators (MMAE) are composed by a bank of local observers (typically Kalman filters if the plant is linear) running in parallel and a centralized part responsible by the identification and the computation of the state estimate which usually is a weighted sum of the estimates produced by the bank of observers. Each local observer was designed using one element of a set of models that represent the possible plant behavior patterns (also called system modes).

In the 1960s and 1970s, motivated by the need for accurate stochastic state estimation for dynamic systems subject to significant parameter uncertainty, the use of MMAE began to develop gradually [1]–[5]. Consider the state estimation of a linear time-invariant plant. If there is no parameter uncertainty in the plant state-space description, then the Kalman filter is the optimal state estimation algorithm in a well defined sense [6]. If the plant has an uncertainty parameter vector, the estimation accuracy provided by the standard Kalman filter is not adequate. In this case, one can then design a bank of Kalman filters where each filter uses in its implementation one element of a finite discrete parameter set that contains different realizations of the unknown parameters. Each Kalman filter generates a local state estimate and a residual signal together with state covariance and residual covariance matrices. We can then compute in real-time the posterior probability that each model is indeed the unknown plant. The state estimate is obtained by weighting the individual Kalman filters state-estimates by the respective posterior probabilities. It turns out that, under non-trivial stationary and ergodicity assumptions, if indeed the true plant parameter is identical to one of its discrete values, then the correct model is identified "almost surely" and the MMAE will generate the true conditional mean of the state as well as its conditional covariance [6]–[11].

The stochastic discrete-time MMAE was first proposed by Magill [1] and numerous results on its properties have been obtained over the years; key properties can be found e.g. in [6], [9]–[11]. The stochastic continuous-time MMAE was introduced in [7], [8], but no further research has been carried out until recently in [12], to the best of our knowledge.

To extend and overcome the limitations of linear models, a number of approaches have been proposed in the literature for nonlinear estimation using multiple-models. These include the "sum of Gaussians" filters [6], [13], [14], which utilizes banks of extended Kalman filters, and other related Bayesian based algorithms such as multiple-hypothesis tracking [15], bootstrap filters [16], unscented Kalman filters [17], particle filters [18], [19], etc.

A related area of research deals with multiple-model estimation of stochastic hybrid systems [20], [21]. A stochastic hybrid system can be viewed as a set of models, each on describing a particular system behavior pattern (or operational mode), together with a random jump process which governs the sudden transition of its system behavior patterns.

Over the past years, MMAE have been applied with great success in several important domains. These include surveillance and fusion algorithms involving multiple sensors and multiple targets [15], [22]–[24], adaptive control [25]–[33], fault detection and isolation [34]–[36], biomedical engineering [37], etc.

Notwithstanding the fact that significant progress has been made in the study and design of MMAE, *the available theoretical results are very limited*. Some important research problems need to be addressed in order to obtain advanced design methodologies for adaptive observers *with guaranteed stability and convergence, robustness and performance* in the presence of plant disturbances, sensor noises, *plant model uncertainty, and unmodeled dynamics*. A set of typical questions that warrant further research is the following:

1) What do we gain by using a multiple-model approach, compared with nondaptive observers and other adaptive observer schemes? and how do we fairly compare them?
2) How do we design MMAE with guaranteed stability and convergence of the algorithm ?
3) Assuming that the physical plant belongs to a given "legal family" and given the performance requirements, level of noise and disturbances, and the class of unmodeled dynamics, how many local observers should be implemented? and what is the minimum number of local observers needed?

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[1]Typical examples of plant model uncertainty include unknown constant parameters in the plant state-space description and unknown statistical values of measurement noise and disturbances.

4) How to quantize the parameter space optimally?
5) What kind of local observers? What are the key properties that each local observer should have?
6) How do we compute optimally the state estimate? and what is the relevant data for that purpose from the local observers? Should we use a switching strategy or a continuous combination of the outputs of the local observers?
7) Should we explore more sophisticated algorithms with *a)* variable number of local observers (this will imply online design of the observers) and/or *b)* adaptive local observers? and what do we gain by this increasing of complexity?

These questions pose considerable challenges and require substantial theoretical research work. They can only be answered in a specific and clear context. The Kalman filter or its extended version for nonlinear plants is quite efficient, robust, and practical for most problems when there are no substantial nonlinearities and plant model uncertainty. The multiple-model approach becomes relevant only when either of these conditions is not satisfied.

In summary, MMAE integrate dynamic hypotheses testing concepts with linear or nonlinear observers that lead to a system identification algorithm. They are at the heart of modern surveillance systems involving multiple sensors and multiple targets. Other important motivations for the study of MMAE include adaptive control and fault detection and isolation. Future work is necessary to derive practical multiple-model estimation architectures *with rigorous theoretical foundations*.

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