PATH-FOLLOWING OR REFERENCE-TRACKING?

An answer relaxing the limits to performance

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Abstract: In *path-following* the control objective is to force the output to follow a geometric *path* without a timing law assigned to it. We highlight a fundamental difference between the path-following and the standard *reference-tracking* by demonstrating that performance limitation due to unstable zero-dynamics can be removed in the path-following problem.

Keywords: Path-following, reference-tracking, zero-dynamics.

1. INTRODUCTION

Path-following problems are primarily concerned with the design of control laws that drive an object (robot arm, mobile robot, ship, aircraft, etc.) to reach and follow a geometric *path*. A secondary goal is to force the object moving along the path to satisfy some additional dynamic specification (dynamic assignment task). An approach to the path-following problem is to introduce a path variable θ as an "extra" control input (Hauser and Hindman, 1995; Encarnação and Pascoal, 2000; Al-Hiddabi and McClamroch, 2002; Skjetne, Fossen and Kokotović, 2004; Aguiar and Hespanha, 2004). For a system

$$\dot{x} = f(x,u), \qquad y = h(x,u), \qquad t \geq 0, \qquad (1)$$

with state $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^k$, output $y \in \mathbb{R}^m$, and a desired geometric path

$$\{\rho(\theta) \in \mathbb{R}^m : \theta \in [0,\infty)\},$$

defined as a function of the parameter θ, the objective is to drive to zero the *path-following error*

$$e_P(t) := y(t) - \rho\big(\theta(t)\big), \quad t \geq 0, \qquad (2)$$

where $\theta : [0,\infty) \to [0,\infty)$ is a *timing law* to be specified. There may be constraints on θ, e.g., on average of $\dot{\theta}$ or on its instantaneous values.

Path-following is more flexible than *reference-tracking*, where the objective is to force the output to follow a reference signal, a given function of time $r : [0,\infty) \to [0,\infty)$, and to drive to zero the *tracking error*

$$e_T(t) := y(t) - r(t), \quad \forall t \geq 0. \qquad (3)$$

It is possible to restrict the path-following problem to reference-tracking by pre-specifying a timing law, such as

$$\theta(t) := \omega t, \quad \forall t \geq 0,$$

where $\omega > 0$ is a given constant. In this paper we argue against this approach because it can introduce limits on the achievable performance that may not be inherent to path-following. We show this to be the case in systems with unstable zero-dynamics. We present examples for which reference-tracking necessarily results in errors whose $\mathcal{L}_2$-norm cannot be made arbitrarily

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small. In contrast, we construct controllers that result in path-following errors with arbitrarily small $\mathcal{L}_2$-norms.

Section 2 reviews fundamental limitations of reference-tracking. To keep the exposition brief we limit our discussion to the linear case for which detailed results are available. In Section 3, we present a framework for path-following and show how it can avoid some limitations of reference-tracking. In Sections 4 and 5, we use two examples to illustrate the ideas presented. The first of them is very simple and is used mostly to build intuition, whereas the second example more realistically illustrates the potential of the proposed approach. Section 6 contains concluding remarks and directions for future research.

2. FUNDAMENTAL LIMITATIONS TO REFERENCE-TRACKING

Consider a linear time-invariant system

$$\dot{x} = Ax + Bu, \qquad y = Cx + Du, \tag{4}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ the input, and $y \in \mathbb{R}^q$ the output. The classical tracking problem is to design a feedback controller for (4) such that the closed-loop system is internally stable, and for any reference signal $r(t)$ in a prescribed family $\mathcal{F}$, the system's output asymptotically approaches $r(t)$. Depending on $\mathcal{F}$, we can distinguish several types of problems. If $\mathcal{F}$ is described by an exosystem,

$$\dot{\omega} = S\omega, \quad r = G\omega, \tag{5}$$

it is called *the regulator or servomechanism problem*, investigated by Davison (1976), Francis and Wonham (1976), Francis (1977), Isidori and Byrnes (1990), among others. The *perfect tracking problem*, requires that

$$J := \int_0^\infty \|y(t) - r(t)\|^2\, dt = \int_0^\infty \|e_T(t)\|^2\, dt < \delta,$$

where the word "perfect" stresses that $\delta > 0$ can be specified to be arbitrarily small.

As shown in (Kwakernaak and Sivan, 1972; Davison, 1976; Francis, 1977; Davison and Scherzinger, 1987), the regulator problem is solvable if and only if (A, B) is stabilizable, (C, A) is detectable, the number of inputs is at least as large as the number of outputs ($m \geq q$) and the transmission zeros of (C, A, B, D) exclude the poles of (5). The solvability of the perfect tracking problem requires in addition that (4) be minimum phase, that is, the transmission zeros of (4) must be all contained in the open left complex half-plane. If (4) is a non-minimum phase system, a fundamental limitation exists in the achievable transient performance J, revealed as the limit

$$J_0 := \lim_{\epsilon \to 0} J_\epsilon$$

of the "cheap" quadratic cost functional

$$J_\epsilon = \min_{\tilde{u}} \int_0^\infty (e_T' e_T + \epsilon^2 \tilde{u}'\tilde{u})\, dt,$$

where $\tilde{u}$ is the transient of u. Qiu and Davison (1993) showed that for a constant reference $r(t) = \eta$, the optimal performance J_ϵ is bounded from below by

$$J_0 = \eta' H \eta, \quad \operatorname{trace} H = 2\sum_{i=1}^{p} \frac{1}{\lambda_i}$$

and for $r(t) = \eta_1 \sin\omega t + \eta_2 \cos\omega t$, $\eta = [\eta_1, \eta_2]'$ by

$$J_0 = \eta' M \eta, \;\; \operatorname{trace} M = \sum_{i=1}^{p} \left(\frac{1}{z_i - j\omega} + \frac{1}{z_i + j\omega} \right)$$

where $z_1, z_2, \ldots, z_p$ denote the zeros of (A, B, C, D) contained in the open right half of $\mathbb{C}$. Thus, the locations of the zeros in $\mathbb{C}^+$ determine the best attainable performance. For

$$r(t) = \sum_{k=-r_n}^{r_n} v_k e^{j\omega_k t}, \tag{6}$$

where $\omega_0 = 0$, ω_k, $k = \pm 1, \ldots, \pm r_n$, are distinct frequencies ($\omega_{-k} = -\omega_k$) and v_k ($v_{-k} = \bar{v}_k$) are complex vectors, Su, Qiu and Chen (2003) give explicit formulas for the best tracking performance $J_{opt} := \inf_{K \in \mathcal{K}} J$ achievable with a linear stabilizing controller K. For plants with no zeros at $j\omega_k$, $k = 0, \pm 1, \ldots, \pm r_n$, the best tracking performance is

$$J_{opt} = \sum_{i=1}^{p} 2Re(z_i) \left| \sum_{k=-r_n}^{r_n} \frac{< \eta_{\omega_k i}, v_k >}{z_i - j\omega_k} \right|^2,$$

where $z_1, z_2, \ldots, z_p$ are the nonminimum phase zeros of the plant with corresponding Blaschke vectors $\eta_{\omega_k 1}, \ldots, \eta_{\omega_k p}$. For an overall performance measure in tracking all references of the type (6), one measure is to average J over all possible $v := [v_{-r_n}, \cdots, v_{r_n}]'$ whose entries have zero mean, are mutually uncorrelated, and have unit variance, i.e., $E = \mathrm{E}\{J(v) : \mathrm{E}(v) = 0, \mathrm{E}(vv^\star) = I\}$, where E is the expectation operator. The average performance limit $E_{opt} = \inf_{K \in \mathcal{K}} E$, given by the formula

$$E_{opt} = 2\sum_{i=1}^{p} \sum_{k=-r_n}^{r_n} \frac{1}{z_i - j\omega_k}. \tag{7}$$

sums the contributions of all nonminimum phase zeros at all reference signal frequencies.

3. CONCEPTUAL FRAMEWORK

To gain further insight, we express the system dynamics (1) in the Isidori's normal form:

$$y^{(n_r)} = u, \qquad \dot{\zeta} = g(\zeta, y, \dot{y}, \ldots, y^{(n_r - 1)}), \tag{8}$$

were n_r denotes the system's relative degree, $y^{(k)}$ the kth derivative of its output, u its input, and ζ the zero-dynamics state. Equation (8) is obtained from (1) by a coordinate and input transformation. For simplicity, we consider a SISO system affine in the control with well-defined relative degree.

The zero-dynamics of (8) can be written in terms of the tracking error (3) as follows

$$\dot{\zeta} = g(\zeta, r + e_T, \dot{r} + \dot{e}_T, \ldots, r^{(n_r-1)} + e_T^{(n_r-1)}). \tag{9}$$

This expression highlights the difficulty in stabilizing systems with "unfriendly" zero-dynamics: Regarding e_T and its derivatives as inputs to (9), one sees that some tracking error must be "allocated" to the task of keeping ζ bounded which is harder when the (time-varying) system

$$\dot{\zeta} = g(\zeta, r, \dot{r}, \ldots, r^{(n_r-1)}), \tag{10}$$

is unstable. Measuring e_T with its $\mathcal{L}_2$ norm, this leads to the following optimal control problem: Find v to stabilize

$$e_T^{(n_r)} = v, \quad \dot{\zeta} = g(\zeta, r + e_T, \dot{r} + \dot{e}_T, \ldots), \tag{11}$$

while minimizing

$$J_{\mathrm{e_T}} := \int_0^\infty \|e_T(\tau)\|^2 d\tau.$$

When $n_r = 1$, $r(t)$ is a step reference signal, and the zero-dynamics (10) are anti-stable, the minimum $J_{\mathrm{e_T}}$ is strictly positive and turns out to be exactly the smallest achievable $\mathcal{L}_2$ norm of the tracking error (Seron, Braslavsky, Kokotović and Mayne, 1999).

The above discussion assumed that our only goal was to keep the zero-dynamics stable. In practice, this is not the case and one may want, e.g., to keep ζ below some given bounds. In this case, even with stable zero-dynamics one cannot achieve arbitrarily small tracking error e_T, especially if the zero-dynamics have lightly-damped modes, as in underactuated vehicles (Aguiar and Hespanha, 2004).

The above discussion of the performance limitations is the starting point to examine to what extent the path-following problem may suffer from the same limitations. We now express the zero-dynamics in terms of the path-following error (2) to obtain

$$\begin{aligned}\dot{\zeta} = g\big(\zeta, \rho(\theta) + e_P, \rho'(\theta)\dot{\theta} + \dot{e}_P,\\ \rho''(\theta)\dot{\theta}^2 + \rho'(\theta)\ddot{\theta} + \ddot{e}_P, \ldots,\\ \rho^{(n_r-1)}(\theta)\dot{\theta}^{(n_r-1)} + \ldots\\ \cdots + \rho'(\theta)\theta^{(n_r-1)} + e_P^{(n_r-1)}\big), \end{aligned} \tag{12}$$

where the derivative of ρ are with respect to θ, whereas the derivatives of θ are with respect to time. Contrasting (9) with (12) we see that we now have *two* control variables to stabilize ζ: the path error derivative $e_P^{(n_r)}$ and the timing law derivative $\theta^{(n_r)}$. So even when (10) is unstable it may still be possible to stabilize (12) by a choice of the timing law θ, while keeping the path-following error arbitrarily small.

A fundamental question that arises in path-following is then whether or not for the (time-invariant) system

$$\theta^{(n_r)} = \omega, \tag{13}$$

$$\dot{\zeta} = g\big(\zeta, \rho(\theta) + e_P, \rho'(\theta)\dot{\theta} + \dot{e}_P, \ldots\big), \tag{14}$$

ω can be used to render (14) input-to-state stable (Sontag, 1989) with respect to e_P, $\dot{e}_P, \ldots$, as the inputs, and whether or not this task can be accomplished with arbitrarily small $\mathcal{L}_2$ norm of e_P. Since (14) depends on $\rho(\theta)$, the ability to accomplish this depends on the geometry of the trajectory to be followed.

In path-following, one wants to select a timing law (through the choice of ω) that satisfies some pre-specified constraints on $\dot{\theta}$. Clearly, one wants $\lim_{t\to\infty} \theta(t) = \infty$, but one may further impose a given average speed and/or its upper- and lower-bounds.

4. FOLLOWING A STRAIGHT PATH WITH UNSTABLE LOAD

The dynamics of a vehicle moving on a straight line, on top of which lies a mass, can be modeled by

$$\ddot{y} = 2f(\dot{z} - \dot{y}) + u, \quad \ddot{z} = 2f(\dot{y} - \dot{z}) + k(z - y). \tag{15}$$

with $f, k > 0$. The variable y denotes the position of the vehicle along the line, z the (inertial-frame) position of the mass, and u an applied force. The mass is "carried" with the vehicle by a viscous friction force given by $2f(\dot{y} - \dot{z})$. However, the top of the vehicle is not flat and gravity drives the mass away from the equilibrium position $z = y$ with a force $k(z - y)$. The transfer function of (15) from the force input u to the vehicle position y is given by

$$H(s) = \frac{s^2 + 2fs - k}{s^2(s^2 + 4fs - k)}, \tag{16}$$

exhibiting an unstable zero at $\alpha := \sqrt{f^2 + 4k} - f$. When the objective is to control the position of the vehicle y, the state z essentially plays the role of the zero-dynamics. The instability of the right equation in (15) (when y is seen as the input) directly translates into a non-minimum phase behavior.

We consider here the simple path-following task of moving the vehicle along a straight line. One approach to solving this problem is to recast it as a trajectory tracking problem by creating a reference signal that effectively accomplishes the path-planning task. E.g.,

$$r(t) = r_0 + v_0 t, \qquad \forall t \geq 0,$$

where $v_0 > 0$ denotes some desired velocity and r_0 an arbitrary constant. Defining $\zeta := z - r$ we can expresses the zero-dynamics as being driven by the tracking error as it was done in (11). For this example, we obtain

$$\ddot{e}_T = v, \quad \ddot{\zeta} + 2f\dot{\zeta} - k\zeta = 2f\dot{e}_T - ke_T. \quad (17)$$

It is straightforward to show that there exists a nonzero initial condition $\zeta(0)$, for which it is not possible to achieve an arbitrarily small $\mathcal{L}_2$-norm for e_T and simultaneously make (17) closed-loop stable by appropriate choice of v. This is a consequence of the instability of the right equation in (17) [or equivalently the nonminimum phaseness of (15)], which is perfectly consistent with the results in Section 2. The above argument shows that, when we attempted to solve the path-following problem by recasting it as a trajectory tracking one, we encountered a fundamental limitation in terms of the smallest $\mathcal{L}_2$-norm achievable for the error.

Suppose now that we approach our original path-following problem by constructing a path-following error as in (2), where we simply define

$$\rho(\theta) = \theta, \qquad \forall \theta \geq 0.$$

We can now express the zero-dynamics in terms of the path following-error, as it was done in (14). Defining $\zeta := z - \theta$, we obtain

$$\ddot{\theta} = \omega, \quad \ddot{\zeta} + 2f\dot{\zeta} - k\zeta = 2f\dot{e}_P - ke_P - \omega \quad (18)$$

and the dynamics of the path-following error are given by

$$\ddot{e}_P + 2f\dot{e}_P = 2f\dot{\zeta} - \omega + u, \quad (19)$$

where both u and ω are control variables. It turns out that now *there is no intrinsic limitation on the achievable $\mathcal{L}_2$-norm of the path-following error.* This is because, in setting-up the optimal control problem to determine the input u that makes the $\mathcal{L}_2$-norm of e_P arbitrarily small subject to the dynamics in (19), we are not constrained by the need to keep ζ bounded, as this can be achieved by suitable choice of ω in (18). The key observation is that the performance considerations that relate to (19) have been decoupled from the stability considerations that relate to (18). Moreover, since (18)–(19) is controllable from (u, ω), it is even straightforward to make $\lim_{t\to\infty} e_P(t) = 0$, and $\lim_{t\to\infty} \dot{\theta}(t) = v_0$, for any $v_0 > 0$. This means that we can follow the path with any assignable (asymptotic) velocity v_0.

Two key assumptions made the previous example especially simple: SISO systems and straight-line path. These assumptions kept the discussion within the universe of linear system theory, but the intuition gained from this example carries over to more complicated problems.

5. FOLLOWING A PLANAR PATH WITH UNSTABLE LOAD

We now consider the vehicle from Section 4 moving on the plane. Its dynamics are still given by (15), but now $y, z \in \mathbb{R}^2$, and f, k are substituted by diagonal matrices $F = \text{diag}(f_1, f_2) > 0$ and $K = \text{diag}(k_1, k_2) > 0$. We design a path-following controller which guarantees that the vehicle converges to and moves along a circular path $\rho(\theta)$ of radius R,

$$\rho(\theta) = \begin{bmatrix} R\sin\theta & R\cos\theta \end{bmatrix}^T, \qquad \forall \theta \geq 0, \quad (20)$$

$\lim_{t\to\infty} y(t) - \rho(\theta(t)) = 0$ and $\lim_{t\to\infty} \theta(t) = \infty$, while preventing the load from drifting off, i.e. keeping $z(t) - y(t)$ bounded. Moreover, we require $\dot{\theta}(t) \geq 0$ which disallows the backward movement of the vehicle along the path $\rho(\theta)$.

Applying a suitable state and feedback transformation, we obtain the normal form of (15)

$$\ddot{y} = \overline{u}, \quad (21)$$
$$\dot{\zeta}_s = A_s\zeta_s + B_s y, \quad (22)$$
$$\dot{\zeta} = A\zeta + By, \quad (23)$$

where $A_s = \text{diag}(\lambda_{1s}, \lambda_{2s})$, $B_s = \text{diag}(b_{s1}, b_{s2})$, $A = \text{diag}(\lambda_1, \lambda_2)$ and $B = \text{diag}(b_1, b_2)$. The subsystems (22), (23) correspond to the stable and unstable part of the zero-dynamics for (15), respectively. In fact, the eigenvalues of A_s, are equal to the stable transmission zeros, whereas the eigenvalues of A, to the unstable ones. Since A_s is stable, it suffices to prove that y is bounded to conclude boundedness of ζ_s. We will therefore ignore subsystem (22) from further analysis.

Introducing the error coordinates $e_P := y - \rho(\theta)$ and $\widetilde{u} := \overline{u} - \frac{\partial\rho}{\partial\theta}\ddot{\theta} - \frac{\partial^2\rho}{\partial\theta^2}\dot{\theta}^2$, we get

$$\ddot{e}_P = \widetilde{u}, \quad (24)$$
$$\dot{\zeta} = A\zeta + B(\rho(\theta) + e_P), \quad (25)$$
$$\ddot{\theta} = \omega. \quad (26)$$

Our control design consists of four steps. First, we construct a discontinuous feedback law for θ to ensure boundedness of (25) with $e_P(t) = 0$, and estimate the bound $d^\star$ such that any additive disturbance with norm smaller than $d^\star$ does not destroy this property. In the second step, such feedback law is approximated by a C^2 one, as required in the definition of $\widetilde{u}$, such that 'smoothing' does not exceed the effect of an additive disturbance with size $\frac{d^\star}{2}$. A C^2 $\theta(t)$ automatically defines the signal $\omega(t)$ in (26). Finally, a feedback law $\widetilde{u}$ is chosen such that the resulting $e_P(t)$ is also norm-bounded by $\frac{d^\star}{2}$.

Step 1: Given a sampling time $T > 0$, we decompose $\theta(t)$ as $\theta(t) = \theta_k + \widetilde{\theta}_k(t)$ for $t \in [kT, (k+1)T)$, and define $v_k = [v_{1k}, v_{2k}]^T = [R\sin\theta_k, R\cos\theta_k]^T$. This leads to the following difference equation governing $\zeta(kT) := \zeta_k$

$$\zeta_{k+1} = A_d\zeta_k + B_d v_k + d_k \tag{27}$$

where $A_d = e^{AT}$, $B_d = \int_0^T e^{At}dtB$ and $d_k = d_{1k} + d_{2k}$,

$$\begin{aligned} d_{1k} &= \int_0^T e^{A(T-t)}Be_P(t+kT)dt, \\ d_{2k} &= \int_0^T e^{A(T-t)}B\left[\rho(\theta(t+kT)) - \rho(\theta_k)\right]dt. \end{aligned} \tag{28}$$

It can be shown that for a sufficiently small sampling time T, there exist constants $d^\star > 0$ and $c^\star > 0$ such that feedback law $v_k^\star$

$$v_k^\star := \arg\min_{\|v_k\|=R} \|B_d^{-1}A_d\zeta_k + v_k\|^2, \tag{29}$$

renders the set $\Omega(c^\star) = \{\zeta \in \mathbb{R}^2 : V \leq c^\star\}$ forward invariant for (27), (29) as long as $\|d(k)\| \leq d^\star$. From $v_k^\star$, we compute θ_k by solving

$$\theta_k = \min_{\tau > \theta_{k-1}} \{\tau \in \mathbb{R}^+ : R\sin\tau = v_{1k}^\star, R\cos\tau = v_{2k}^\star\} \tag{30}$$

Step 2: We define $\widetilde{\theta}_k(t+kT)$ by

$$\widetilde{\theta}_k(t+kT) = \begin{cases} \overline{\theta}_k(t), & t \in [0, T^\star] \\ 0, & t \in [T^\star, T] \end{cases}, \tag{31}$$

where $T^\star$ is determined by substituting (31) and $\|\rho(\theta_k) - \rho(\theta(t+kT))\| \leq 2R$ in (28), and requiring that $\|d_{2k}\| \leq \frac{d^\star}{2}$. The function $\overline{\theta}_k(t)$ is any non-decreasing map which provides smooth transition from θ_{k-1} to θ_k, namely

$$\begin{aligned} \left[\overline{\theta}_k(0), \dot{\overline{\theta}}_k(0), \ddot{\overline{\theta}}_k(0)\right] &= \left[\theta_{k-1} - \theta_k, 0, 0\right], \\ \left[\overline{\theta}_k(T^\star), \dot{\overline{\theta}}_k(T^\star), \ddot{\overline{\theta}}_k(T^\star)\right] &= \left[0, 0, 0\right]. \end{aligned}$$

The rational for keeping $\theta(t)$ "essentially" piecewise constant, is that while $\theta(t) = \theta_k$, (25) is a linear system. This enables us to compute its exact discretization (27), and based on it, a stabilizing feedback law (29).

Step 3: To generate the desired $\theta(t)$ we use

$$\omega(t) = \frac{d^2}{dt^2}\theta(t). \tag{32}$$

Since (32) involves two differentiations, it may appear that such choice is non-robust with respect to zero-dynamics uncertainties. This is not the case because $\theta(t)$ depends only on discrete samples of $\zeta(t)$, and for $t \in (kT, (k+1)T)$ we have an explicit formula which can be symbolically differentiated.

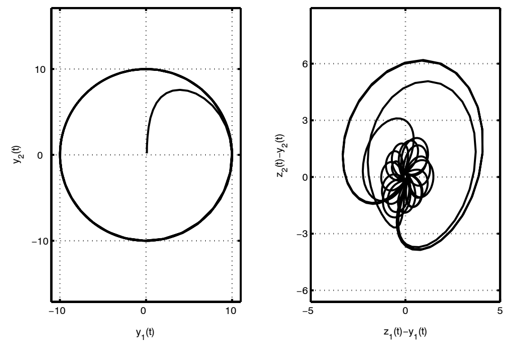

Fig. 1. Position of the vehicle (left), and difference in positions of the vehicle and the load (right)

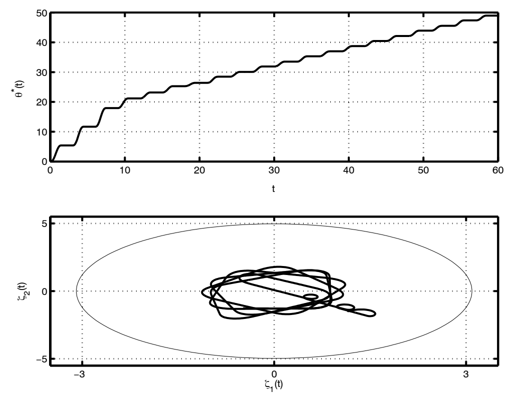

Fig. 2. Timing law $\theta(t)$ (top), and unstable modes of the zero-dynamics and $\Omega(c^\star)$ (bottom)

Step 4: We now consider the impact of the path-following error e_P. It can be shown that for all $e_P(0), \dot{e}_P(0) \in \mathbb{R}^2$, there exist gains $\kappa_1, \kappa_2 \in \mathbb{R}^2$, such that the feedback law

$$\widetilde{u}_1 = -\kappa_1^T e_P - \kappa_2^T \dot{e}_P \tag{33}$$

guarantees that $\|d_{1k}\| \leq \frac{d^\star}{2}$, $\forall k \geq 0$, see (28). With (32) and (33), we have $\|d_k\| \leq d^\star$, hence, $\zeta(t)$ is bounded $\forall t \geq 0$. Since $e_P(t) \to 0$, we get $y(t) \to \rho(\theta(t))$, which implies boundedness of $\zeta_s(t)$. This completes the design.

The path-following controller (32), (33) not only guarantees stability but it can also be used to achieve arbitrarily small $\mathcal{L}_2$-norm of $e_P(t)$,

$$J_{e_P} = \int_0^\infty \|e_P(t)\|^2 dt = \int_0^\infty \|y(t) - \rho(\theta(t))\|^2 dt.$$

which is the consequence of two facts. First, for any $\epsilon > 0$ and any initial condition $e_P(0), \dot{e}_P(0)$, there exist $\kappa_1^\epsilon, \kappa_2^\epsilon$ such that the corresponding solution of (24), (33) satisfies $J_{e_P} \leq \epsilon$, (Kwakernaak and Sivan, 1972). Moreover, such choice of (33) is beneficial for the stability of the zero-dynamics, because faster convergence of $e_P(t)$ to zero translates into a 'smaller' disturbance d_{1k} on (27).

We illustrate our design by simulating the closed-loop system (15), (32) and (33). The typical behavior is given on Figures 1 and 2, for the initial conditions

$$[y_1(0), y_2(0), \dot{y}_1(0), \dot{y}_2(0)]^T = [0.1, -0.2, 0, 0]^T,$$
$$[z_1(0), z_2(0), \dot{z}_1(0), \dot{z}_2(0)]^T = [0.06, -0.14, 0, 0]^T,$$

and the following parameter values: $F = \mathrm{diag}(8, 5)$, $K = \mathrm{diag}(3, 2)$, $\kappa_1 = \kappa_2 = [10, 10]^T$, $T = 3$, $R = 10$.

6. CONCLUSION

We highlighted a fundamental difference between the path-following problem and the standard reference-tracking problem by demonstrating on two examples that the performance limitation due to unstable zero-dynamics does not arise in path-following. Future research includes the generalization of the design presented in Section 5 to encompass more general linear and nonlinear systems, as well as providing a geometric characterization of the paths for which the proposed control design is applicable.

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