Proc. IFAC - 15th World Congress, Barcelona, Spain, July 2002

DYNAMIC POSITIONING OF AN UNDERACTUATED AUV IN THE PRESENCE OF A CONSTANT UNKNOWN OCEAN CURRENT DISTURBANCE *

António Pedro Aguiar **António M. Pascoal**

Institute for Systems and Robotics, Instituto Superior Técnico, Torre Norte 8, Av. Rovisco Pais, 1049-001 Lisboa, Portugal
E-mail:{antonio.aguiar,antonio}@isr.ist.utl.pt

Abstract: This paper addresses the problem of dynamic positioning of an underactuated autonomous underwater vehicle (AUV) in the horizontal plane. A nonlinear adaptive controller is proposed that yields convergence of the trajectories of the closed loop system to a desired target point in the presence of a constant unknown ocean current disturbance and parametric model uncertainty. The controller is first derived at the kinematic level assuming that the ocean current disturbance is known. An exponential observer is then designed and convergence of the resulting closed loop system is analyzed. Finally, integrator backstepping and Lyapunov based techniques are used to extend the kinematic controller to the dynamic case and deal with model parameters uncertainties. Simulation results are presented and discussed.

Keywords: Underactuated Systems; Autonomous Underwater Vehicles; Nonlinear Adaptive Control; Regulation.

1. INTRODUCTION

The problem of steering an autonomous underwater vehicle (AUV) to a point with a desired orientation has only recently received special attention. This task raises some challenging questions in control system theory when the vehicle is underactuacted. Furthermore, as will be shown, its dynamics are complicated due to the presence of complex hydrodynamic terms. This rules out any attempt to design a steering system for the AUV that would rely on its kinematic equations only. Pioneering work in this field is reported in (Leonard, 1995), where open loop small-amplitude periodic time-varying control laws are used to reposition and re-orient underactuated AUVs. The design of a continuous, periodic feedback control law that asymptotically stabilizes an underactuated AUV and yields exponential convergence to the origin is described in (Pettersen and Egeland, 1996). In (Pettersen and Nijmeijer, 1998), a time-varying feedback control law is proposed that yields global practical stabilization and tracking for an underactuated ship using a combined integrator backstepping and averaging approach.

It is important to point out that some of the control laws developed so far for underactuated underwater vehicles do not take explicitly into account their dynamics and are therefore unrealistic. Furthermore, even when the dynamics are taken into account, the resulting closed loop system trajectories are often not "natural". This issue is discussed in (Aguiar and Pascoal, 2001*b*), where the problem of regulating a nonholonomic underactuated AUV in the horizontal plane to a point with a desired orientation and with parametric modeling uncertainty is posed and solved. The control algorithm proposed builds on a non smooth coordinate transformation, Lyapunov stability theory, and backstepping design techniques.

* This work was supported in part by the EC under the FREESUB network and by the PDCTM programme of the FCT of Portugal under projects DREAM and MAROV.

In practice, an AUV must often operate in the presence of unknown ocean currents. Interesting enough, even for the case where the current is constant, the problem of regulating an AUV to a desired point with an arbitrary desired orientation does not have a solution. In fact, if the desired orientation does not coincide with the direction of the current, normal control laws will yield one of two possible behaviors: *i)* the vehicle will diverge from the desired target position, or *ii)* the controller will keep the vehicle moving around a neighborhood of the desired position, trying insistently to steer it to the given point, and consequently inducing an oscillatory behavior.

Motivated by the above considerations, this paper addresses the problem of dynamic positioning of an AUV in the horizontal plane in the presence of unknown, constant ocean currents. To tackle this problem, the approach considered here is to drop the specification on the final desired orientation and use this extra degree of freedom to force the vehicle to converge to the desired point. Naturally, the orientation of the vehicle at the end will be aligned with the direction of the current.The vehicle under consideration (see Figure 1) only has two independent main back thrusters and is therefore underactuated. A nonlinear adaptive controller is proposed that gives convergence of the trajectories of the closed loop system in the presence of a constant unknown ocean current disturbance and parametric model uncertainty. Controller design relies on a non smooth coordinate transformation in the original state space followed by the derivation of a Lyapunov-based, adaptive, control law in the new coordinates and an exponential observer for the ocean current disturbance. For sake of clarity of presentation, the controller is first derived at the kinematic level, assuming that the ocean current disturbance is known. Then, an observer is designed and convergence of the resulting closed loop system is analyzed. Finally, resorting to integrator backstepping and Lyapunov techniques (Krstić *et al.*, 1995), a nonlinear adaptive controller is developed that extends the kinematic controller to the dynamic case and deals with model parameter uncertainties.

2. THE AUV. CONTROL PROBLEM FORMULATION

This section describes the kinematic and dynamic equations of motion of the AUV of Figure 1 in the horizontal plane and formulates the problem of controlling it to a point with a desired orientation. The control inputs are the thruster surge force τ_u and the thruster yaw torque τ_r. The AUV has no side thruster, see (Aguiar and Pascoal, 1997) for model details.

2.1 *Vehicle Modeling*

Following standard practice, the general kinematic and dynamic equations of motion of the vehicle can be developed using a global coordinate frame $\{U\}$ and a body-fixed coordinate frame $\{B\}$ that are depicted in Figure 1. In the horizontal plane, the kinematic equations of motion of the vehicle, can be written as

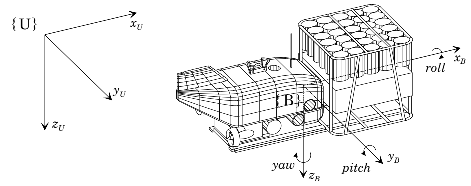

Fig. 1. The vehicle SIRENE coupled to a benthic laboratory. Body-fixed $\{B\}$ and earth-fixed $\{U\}$ reference frames

$$\dot{x} = u\cos\psi - v\sin\psi, \tag{1a}$$

$$\dot{y} = u\sin\psi + v\cos\psi, \tag{1b}$$

$$\dot{\psi} = r, \tag{1c}$$

where, following standard notation, u (surge speed) and v (sway speed) are the body fixed frame components of the vehicle's velocity, x and y are the cartesian coordinates of its center of mass, ψ defines its orientation, and r is the vehicle's angular speed. In the presence of a constant and irrotational ocean current, $(u_c, v_c)' \neq 0$, u and v are given by $u = u_r + u_c$ and $v = v_r + v_c$, where $(u_r, v_r)'$ is the relative body-current linear velocity vector.

Neglecting the motions in heave, roll, and pitch the simplified equations of motion for surge, sway and heading yield (Fossen, 1994)

$$m_u\dot{u}_r - m_v v_r r + d_{u_r} u_r = \tau_u, \tag{2a}$$

$$m_v\dot{v}_r + m_u u_r r + d_{v_r} v_r = 0, \tag{2b}$$

$$m_r\dot{r} - m_{uv} u_r v_r + d_r r = \tau_r, \tag{2c}$$

where $m_u = m - X_{\dot{u}}$, $m_v = m - Y_{\dot{v}}$, $m_r = I_z - N_{\dot{r}}$, and $m_{uv} = m_u - m_v$ are mass and hydrodynamic added mass terms and $d_{u_r} = -X_u - X_{|u|u}|u_r|$, $d_{v_r} = -Y_v - Y_{|v|v}|v_r|$, and $d_r = -N_r - N_{|r|r}|r|$ capture hydrodynamic damping effects. The symbols τ_u and τ_r denote the external force in surge and the external torque about the z axis of the vehicle, respectively. In the equations, and for clarity of presentation, it is assumed that the AUV is neutrally buoyant and that the centre of buoyancy coincides with the centre of gravity.

2.2 *Problem Formulation*

Let $\{G\}$ be a goal reference frame and assume for simplicity of presentation that $\{G\} = \{U\}$, see Figure 2. Then, the problem considered in this paper can be formulated as follows:

Consider the underactuated AUV with the kinematic and dynamic equations given by (1) *and* (2). *Derive a feedback control law for τ_u and τ_r so that (x, y) converges to the origin of $\{G\}$ as $t \to \infty$ in the presence of a constant unknown ocean current disturbance and parametric model uncertainty.*

3. NONLINEAR CONTROLLER DESIGN AND CONVERGENCE ANALYSIS

This section proposes a nonlinear adaptive control law to regulate the motion of the underactuated AUV to a given point in the presence of a constant unknown ocean current disturbance and parametric model uncertainty. The controller is first derived at the kinematic level, that is, by assuming that the control signals are the surge velocity u_r and the yaw angular velocity r. It is also assumed that the ocean current disturbance intensity V_c and its direction ϕ_c are known. This assumption will be lifted latter.

3.1 *Coordinate Transformation*

Let (x_d, y_d) be a vector in $\mathbb{R}^2$. Further let d be the vector from the origin of frame $\{B\}$ to $(x_d, y_d)'$ and e its length. Denote by β the angle measured from x_B to d. Vector $(x_d, y_d)'$ plays an important role in the development of the controller for the AUV, as explained later. Figure 2 illustrates the particular case where $(x_d, y_d)'$ is the origin of $\{G\}$. Consider the coordinate transformation (see Figure 2)

$$e = \sqrt{(x-x_d)^2 + (y-y_d)^2}, \tag{3a}$$

$$x - x_d = -e\cos(\psi+\beta), \tag{3b}$$

$$y - y_d = -e\sin(\psi+\beta), \tag{3c}$$

$$\psi + \beta = \tan^{-1}\left(\frac{-(y-y_d)}{-(x-x_d)}\right). \tag{3d}$$

In equation (3d), care must be taken to select

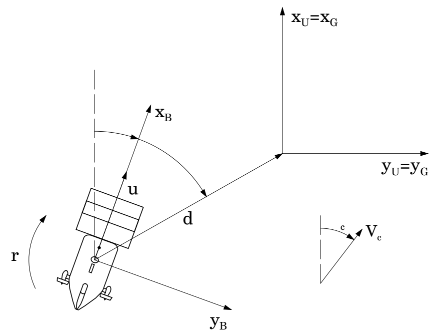

Fig. 2. Coordinate Transformation.

the proper quadrant for β. Let the ocean current disturbance be characterized by its intensity V_c and direction ϕ_c. The kinematics equations of motion of the AUV can be rewritten in the new coordinate system to yield

$$\dot{e} = -u_r\cos\beta - v_r\sin\beta - V_c\cos(\beta+\psi-\phi_c), \tag{4a}$$

$$\dot{\beta} = \frac{\sin\beta}{e}u_r - \frac{\cos\beta}{e}v_r - r + \frac{V_c}{e}\sin(\beta+\psi-\phi_c), \tag{4b}$$

$$\dot{\psi} = r. \tag{4c}$$

Notice that the coordinate transformation (3) is only valid for non zero values of the variable e, since for $e = 0$ the angle β is undefined. Throughout the paper the following assumptions are considered. See (Aguiar and Pascoal, 2001*a*) for a thorough discussion.

Assumption 1. The vehicle will not cross the singular position $e(t) = 0$ for any time $t \geq t_0$.

Assumption 2. The heading $\psi(t)$ will not converge to $\phi_c + 2\pi n, n = 0, \pm1, \pm2, \cdots$ as $t \to \infty$.

3.2 *Kinematic Controller*

At the kinematic level, the control objective consists of recruiting the linear and angular velocities u_r and r, respectively, to regulate the position (x, y) to zero. Consequently, it will be assumed that u_r and r are the control inputs. At this stage, the relevant equations of motion of the AUV are simply (4) and (2b). It is important to stress out that the dynamics of the sway velocity v must be explicitly taken into account, since the presence of this term in the kinematics equations (1) is not negligible (as is usually the case for wheeled mobile robots).

Returning now to the control problem, observe equations (4). The strategy for controller design consists basically of *i)* manipulating r to regulate β to zero (this will align x_B with vector d), and *ii)* actuating on u_r to force the vehicle position to $(x, y) = 0$. However, this must be done without driving formally e to zero, thus avoiding problems concerning the boundedness of the control inputs. This is done by defining the coordinates x_d and y_d adequately. See the statement of the theorem below. At this stage, it is assumed that the intensity V_c and the direction ϕ_c of the ocean current disturbance are known. The following result applies.

Theorem 1. Consider the nonlinear invariant system Σ_{kin} described by the AUV nonlinear model (1) and (2b) together with the control law

$$u_r = k_1 e - k_1\gamma - V_c\cos(\psi-\phi_c), \tag{5a}$$

$$r = k_1\sin\beta - k_1\frac{\gamma}{e}\sin\beta - \frac{v_r}{e}\cos\beta + \frac{V_c}{e}\sin(\psi-\phi_c)\cos\beta + k_2\beta, \tag{5b}$$

where k_1, k_2, and γ are positive constants such that

$$\frac{d_{v_r}}{m_u} > k_1, \quad k_1 > 2\frac{V_c}{\gamma}, \tag{6}$$

with β and e as given in (3), and

$$x_d = -\gamma\cos\phi_c, \quad y_d = -\gamma\sin\phi_c. \tag{7}$$

Let $\mathcal{X}_{kin}(t) = (x, y, \psi, v_r)' = \{\mathcal{X}_{kin} : [t_0, \infty) \to \mathbb{R}^4\}$, $t_0 \geq 0$, be a solution of Σ_{kin}. Let assumption 1 and 2 be satisfied. Then, for any initial conditions $\mathcal{X}_{kin}(t_0) \in \mathbb{R}^4$ the control signals and the solution $\mathcal{X}_{kin}(t)$ are bounded and the position (x, y) converges to zero as $t \to \infty$.

Proof. The proof is organized as follows: First, it will be shown that β converges to zero. Then, resorting to LaSalle's invariance principle, convergence of e to γ is concluded under the assumption that the sway velocity v_r is bounded. Due to space limitations the proof of boundedness of v_r is omitted. See (Aguiar and Pascoal, 2001*a*) for details. Finally, if e converges to γ it follows from (3), (7) and Assumption 2 that $(x,y) \to 0$ as $t \to \infty$.

Consider the candidate Lyapunov function

$$V_{kin} = \frac{1}{2}\beta^2. \tag{8}$$

Computing its time derivative along the trajectories of the system Σ_{kin}, gives $\dot{V}_{kin} = -k_2\beta^2$. Thus, $\beta \to 0$ as $t \to \infty$. Now, consider the dynamic motion of e in closed loop given by

$$\dot{e} = -k_1 \cos\beta\, e + h_e(t) - v_r \sin\beta, \tag{9}$$

where $h_e(t) = k_1\gamma\cos\beta + V_c\big[\cos(\psi-\phi_c)\cos\beta - \cos(\beta+\psi-\phi_c)\big]$. Clearly, on the manifold $E = \{\mathcal{X}_{kin} : \beta = 0\}$, one can easily conclude that $e \to \gamma$ as $t \to \infty$ since $k_1 > 0$. Moreover, from (9), the solution $e(t)$ for $t \geq t_0$ can be expressed as

$$\begin{aligned} e(t) = \Phi_e(t,t_0)e(t_0) + \int_{t_0}^{t} \Phi_e(t,\sigma)h_e(\sigma)\,d\sigma \\ - \int_{t_0}^{t} \Phi_e(t,\sigma)v_r \sin\beta\, d\sigma, \end{aligned} \tag{10}$$

where $\Phi_e(t,t_0) = e^{-\int_{t_0}^{t}[k_1\cos\beta]d\sigma}$. Since β converges to zero, there exists a finite time $T_\beta \geq t_0 \geq 0$ such that $\cos\beta > 0$ for all $t \geq T_\beta$. Consequently, $\Phi_e(t,\sigma) \leq \gamma_e e^{-\lambda_e(t-T_\beta)}$, for $\sigma < T_\beta$, and $\Phi_e(t,\sigma) \leq e^{-\lambda_e(t-\sigma)}$, for $\sigma \geq T_\beta$, where $\gamma_e = e^{-\int_{t_0}^{T_\beta}[k_1\cos\beta]d\sigma}$ and $\lambda_e = \inf_{t\geq T_\beta} k_1\cos\beta$. Thus, if v_r is bounded, it follows from (10) that $e(t)$ is bounded. Resorting to LaSalle's invariance principle, one can conclude that $e(t)$ converges to the largest invariant set M contained in E. Notice in this case that the set M does not reduce to the singleton 0. However, it is well known (see (Khalil, 1996, Lemma 3.1)) that any bounded solution converges to its positive limit set L^+ which must be necessarily a subset of E. To characterize the set L^+, observe that on the invariant manifold E the trajectory $(e(t)-\gamma) \to 0$ as $t \to \infty$, and therefore L^+ is the origin. This in turn implies that $e(t)$ converges to γ as $t \to \infty$.

To prove that (x,y) converges to zero, it remains to analyze the evolution of (ψ, v_r). Since (e,β,ψ,v_r) is bounded and $(e-\gamma,\beta)$ converges to zero as $t \to \infty$, LaSalle's theorem guarantees convergence of (ψ, v_r) to the largest invariant set M contained in $E = \{(e,\beta,\psi,v_r) \in \mathbb{R}^4, e \neq 0 : e = \gamma, \beta = 0\}$. On the manifold E, consider the closed loop dynamics of $\{\psi, v_r\}$ and the candidate Lyapunov function

$$V = V_c^2 \frac{m_u}{m_v}\big[1+\cos(\psi-\phi_c)\big] + \frac{1}{2}v_r^2.$$

Computing its time derivative, gives $\dot{V} = -\zeta'\, Q\, \zeta$, where $\zeta = \big(V_c\sqrt{\frac{m_u}{m_v}}\sin(\psi-\phi_c), v_r\big)'$, $Q_{11} = \frac{V_c}{\gamma}$, $Q_{12} = Q_{21} = -\frac{V_c}{2\gamma}\sqrt{\frac{m_u}{m_v}}\big[1+\cos(\psi-\phi_c)\big]$, and $Q_{22} = \frac{d_{v_r}}{m_v} + \frac{V_c}{\gamma}\frac{m_u}{m_v}\cos(\psi-\phi_c)$. Notice that the symmetric matrix Q is positive definite if the inequalities $\frac{V_c}{\gamma} > 0, \quad d_{v_r} > 2\frac{V_c}{\gamma}m_u$, hold. This is clearly true in view of conditions (6). Therefore, after some algebraic manipulations one obtains

$$\dot{V} \leq -\lambda_{min}(Q)\big[1-\cos(\psi-\phi_c)\big]V \leq 0,$$

where $\lambda_{min}(Q)$ denotes the minimum eigenvalue of the positive matrix Q. Hence, it can be concluded that $\lim_{t\to\infty}\dot{V}(t) = 0$ which implies that $\{\sin(\psi-\phi_c), v_r\}$ converges to zero as $t \to \infty$. Thus, from (3b), (3c), (7), and under Assumption 2 one conclude that $(x,y) \to 0$ as $t \to \infty$.

This concludes the proof of Theorem 1. □

3.3 *Observer Design*

Let v_{c_x} and v_{c_y} denote the components of the ocean current disturbance expressed in $\{U\}$. Then, the kinematic equation (1a) can be rewritten as $\dot{x} = u_r\cos\psi - v_r\sin\psi + v_{c_x}$. A simple observer for the component v_{c_x} of the current is

$$\begin{aligned} \dot{\hat{x}} &= u_r\cos\psi - v_r\sin\psi + \hat{v}_{c_x} + k_{x_1}\tilde{x}, \\ \dot{\hat{v}}_{c_x} &= k_{x_2}\tilde{x}, \end{aligned}$$

where $\tilde{x} = x - \hat{x}$. Clearly, the estimate errors $\tilde{x}$ and $\tilde{v}_{c_x} = v_{c_x} - \hat{v}_{c_x}$ are asymptotically exponentially stable if all roots of the characteristic polynomial $p(s) = s^2 + k_{x_1}s + k_{x_2}$ associated with the system

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{v}}_{c_x} \end{bmatrix} = \begin{bmatrix} -k_{x_1} & 1 \\ -k_{x_2} & 0 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{v}_{c_x} \end{bmatrix}$$

have strictly negative real parts.

The observer for the component v_{c_y} can be written in an analogous manner.

Define the variables $\hat{V}_c$ and $\hat{\phi}_c$ as the module and argument of the vector $[\hat{v}_{c_x}, \hat{v}_{c_y}]$, respectively. The next theorem shows convergence of the kinematic control loop when the observer is included. See (Aguiar and Pascoal, 2001*a*) for a formal proof.

Theorem 2. Consider the nonlinear time invariant system $\Sigma_{kin+Obs}$ consisting of the nonlinear AUV model (1), (2b), the current observer, and the control law

$$u_r = k_1 e - k_1\gamma - \hat{V}_c\cos(\psi-\hat{\phi}_c), \tag{12a}$$

$$\begin{aligned} r = k_1\sin\beta - k_1\frac{\gamma}{e}\sin\beta - \frac{v_r}{e}\cos\beta \\ + \frac{\hat{V}_c}{e}\sin(\psi-\hat{\phi}_c)\cos\beta + k_2\beta, \end{aligned} \tag{12b}$$

where k_1, k_2, and γ are positive constants that satisfy conditions (6). Let variables β and e be given as in (3) where x_d and y_d are now redefined as

$$x_d = -\gamma\cos\hat{\phi}_c, \quad y_d = -\gamma\sin\hat{\phi}_c. \tag{13}$$

Let $\mathcal{X}_{kin+Obs}(t) = (x,y,\psi,v,\tilde{v}_{c_x},\tilde{v}_{c_y})' = \{\mathcal{X}_{kin+Obs} : [t_0,\infty) \to \mathbb{R}^6\}$, $t_0 \geq 0$, be a solution to $\Sigma_{kin+Obs}$. Then, for any initial conditions $\mathcal{X}_{kin+Obs}(t_0) \in \mathbb{R}^6$ and under assumptions 1-2, the control signals and the solution $\mathcal{X}_{kin+Obs}(t)$ are bounded, and the position (x,y) converges to zero as $t \to \infty$.

3.4 *Nonlinear Dynamic Controller Design*

This section indicates how the kinematic controller is extended to the dynamic case. This is done by resorting to backstepping techniques (Krstić *et al.*, 1995). Following this methodology, let u_r and r in equations (12a) and (12b) be virtual control inputs and α_1 and α_2 the corresponding virtual control laws. Introduce the error variables $z_1 = u_r - \alpha_1$, $z_2 = r - \alpha_2$, and consider the Lyapunov function (8) augmented with the quadratic terms z_1 and z_2, that is,

$$V_{dyn} = V_{kin} + \frac{1}{2}m_u z_1^2 + \frac{1}{2}m_r z_2^2.$$

The time derivative of V_{dyn} for $\hat{V}_c = V_c$ and $\hat{\phi}_c = \phi_c$ can be written as

$$\dot{V}_{dyn} = -k_2\beta^2 + z_1\Big[\tau_u + m_v v_r r - d_{u_r}u_r - m_u\dot{\alpha}_1 + \frac{\sin\beta}{e}\beta\Big] + z_2\Big[\tau_r + m_{uv}u_r v_r - d_r r - m_r\dot{\alpha}_2 - \beta\Big].$$

Let the control law for τ_u and τ_r be chosen as

$$\tau_u = -m_v v_r r + d_{u_r}u_r + m_u\dot{\alpha}_1 - \frac{\sin\beta}{e}\beta - k_3 z_1,$$
$$\tau_r = -m_{uv}u_r v_r + d_r r + m_r\dot{\alpha}_2 + \beta - k_4 z_2,$$

where k_3 and k_4 are positive constants. Then,

$$\dot{V}_{dyn} = -k_2\beta^2 - k_3 z_1^2 - k_4 z_2^2,$$

which is negative definite. This result plays a key role on the development of the dynamic controller.

3.5 *Adaptive Nonlinear Controller Design*

So far, it was assumed that the AUV model parameters are known precisely. This assumption is unrealistic. In this section the control law developed is extended to ensure robustness against uncertainties in the model parameters.

Consider the set of all parameters of the AUV model (2) concatenated in the vector $\Theta = \Big[m_u, m_v, m_{uv}, m_r, X_u, X_{|u|u}, N_r, N_{|r|r}, m_r\frac{m_u}{m_v}, m_r\frac{Y_v}{m_v}, m_r\frac{Y_{|v|v}}{m_v}\Big]'$, and define the parameter estimation error $\tilde{\Theta}$ as $\tilde{\Theta} = \Theta - \hat{\Theta}$, where $\hat{\Theta}$ denotes a nominal value of Θ. Consider the augmented candidate Lyapunov function

$$V_{adp} = V_{dyn} + \frac{1}{2}\tilde{\Theta}^T\Gamma^{-1}\tilde{\Theta},$$

where $\Gamma = \text{diag}\{\gamma_1, \gamma_2, ..., \gamma_{11}\}$, and $\gamma_i > 0$, $i = 1, 2, ...11$ are the adaptation gains and V_{dyn} is given above.

Motivated by the choices in the previous sections, choose the control laws

$$\tau_u = -\hat{\theta}_2 v_r r - \hat{\theta}_5 u_r - \hat{\theta}_6|u_r|u_r + \hat{\theta}_1\dot{\alpha}_1 - \frac{\sin\beta}{e}\beta - k_3 z_1, \tag{14a}$$

$$\tau_r = -\hat{\theta}_3 u_r v_r - \hat{\theta}_7 r - \hat{\theta}_8|r|r + \hat{\theta}_4\dot{\alpha}_{2_a} + \hat{\theta}_9\frac{u_r}{e}r\cos\beta + \hat{\theta}_{10}\frac{v_r}{e}\cos\beta + \hat{\theta}_{11}|v_r|\frac{v_r}{e}\cos\beta + \hat{\theta}_4\frac{v_r}{e}\Big(\frac{\dot{\hat{e}}}{e}\cos\beta + \dot{\hat{\beta}}\sin\beta\Big) + \beta - k_4 z_2, \tag{14b}$$

where $\hat{\theta}_i$ denotes the i-th element of vector $\hat{\Theta}$, $\alpha_{2_b} = -\frac{v_r}{e}\cos\beta$, $\alpha_{2_a} = \alpha_2 - \alpha_{2_b}$,

$$\dot{\hat{e}} = -u_r\cos\beta - v_r\sin\beta - \hat{V}_c\cos(\beta + \psi - \hat{\phi}_c) - \gamma\,\dot{\hat{\phi}}_c\sin(\beta + \psi - \hat{\phi}_c), \quad \text{and}$$

$$\dot{\hat{\beta}} = \frac{\sin\beta}{e}u_r - \frac{\cos\beta}{e}v_r - r + \frac{\hat{V}_c}{e}\sin(\beta + \psi - \hat{\phi}_c) - \dot{\hat{\phi}}_c\frac{\gamma}{e}\cos(\beta + \psi - \hat{\phi}_c).$$

Then,

$$\dot{V}_{adp} = -k_2\beta^2 - k_3 z_1^2 - k_4 z_2^2 + \tilde{\Theta}^T\Big[Q - \Gamma^{-1}\dot{\hat{\Theta}}\Big],$$

where Q is a diagonal matrix given by $Q = \text{diag}\Big\{-\dot{\alpha}_1 z_1, z_1 v_r r, z_2 u_r v_r, -z_2\dot{\alpha}_{2_a} - z_2\frac{v_r}{e}\Big(\frac{\dot{\hat{e}}}{e}\cos\beta + \dot{\hat{\beta}}\sin\beta\Big), z_1 u_r, z_1|u_r|u_r, z_2 r, z_2|r|r, -u_r r\frac{z_2}{e}\cos\beta, \frac{v_r}{e}z_2\cos\beta, \frac{v_r}{e}|v_r|z_2\cos\beta\Big\}$. Notice in above equation how the terms containing $\tilde{\Theta}_i$ have been grouped together. To eliminate them, choose the parameter adaptation law as

$$\dot{\hat{\Theta}} = \Gamma Q, \tag{15}$$

to yield $\dot{V}_{adp} = -k_2\beta^2 - k_3 z_1^2 - k_4 z_2^2 \leq 0$.

The above results play an important role in the proof of the following theorem that extends Theorem 2 to deal with vehicle dynamics and model parameter uncertainty, see (Aguiar and Pascoal, 2001*a*).

Theorem 3. Consider the nonlinear invariant system Σ_{adp} consisting of the nonlinear AUV model (1) and (2), the current observer, and the adaptive control law (14), (15), where the adaptation gain Γ is a (11×11) diagonal positive definite matrix. Assume the control gains k_i, $i = 1, 2, 3, 4$ and the control variable γ are positive constants and satisfy conditions (6). Let variables β and e be given as in (3) where x_d and y_d are defined in (13).

Let $\mathcal{X}_{adp}(t) = (x, y, \psi, u, v, r, \tilde{v}_{c_x}, \tilde{v}_{c_y}, \tilde{\Theta}')' = \{\mathcal{X}_{adp} : [t_0, \infty) \to \mathbb{R}^{19}\}$, $t_0 \geq 0$, be a solution to Σ_{adp}. Then, for any initial conditions $\mathcal{X}_{adp}(t_0) \in \mathbb{R}^{11}$ and under assumptions 1 and 2, the control signals and the solution $\mathcal{X}_{adp}(t)$ are bounded, and the position (x, y) converges to zero as $t \to \infty$.

4. SIMULATION RESULTS

In order to illustrate the performance of the proposed control scheme in the presence of parametric uncertainty and a constant ocean current disturbance, computer simulations were carried out with a model of the SIRENE AUV. The vehicle dynamic model can be found in (Aguiar and Pascoal, 1997).

Figure 3 shows the resulting vehicle trajectory in the xy-plane for two simulations using the nonlinear adaptive control law (14), (15). The control parameters were selected as following:

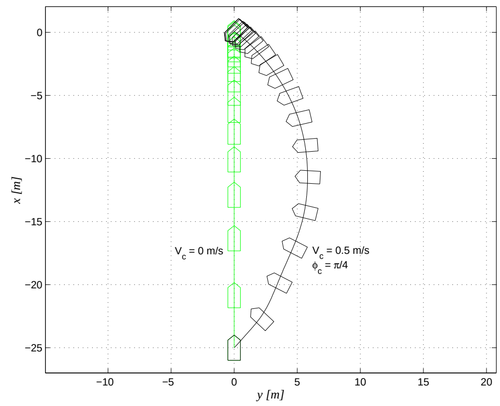

Fig. 3. Simulation resulting paths of the AUV.

$k_1 = 0.04$, $k_2 = 0.8$, $k_3 = 2 \times 10^3$, $k_4 = 500$, $k_{x_1} = 1.0$, $k_{x_2} = 1.0$, $k_{y_1} = 1.0$, $k_{y_2} = 1.0$, $\gamma = 15$, and $\Gamma = \text{diag}(10, 10, 10, 1, 1, 2, 2, 2, 1, 0.1, .1) \times 10^3$. The parameters satisfy constraints (6). The initial estimates for the vehicle parameters were disturbed by 50% from their true values. In both simulations, the initial conditions for the vehicle were $(x, y, \psi, u, v, r) = (-25\,m, 0, 0, 0, 0, 0)$. In one simulation there is no ocean current. The other simulation captures the situation where the ocean current (which is unknown from the point of view of the controller) has intensity and direction $V_c = 0.5\,m/s$, and $\phi_c = \frac{\pi}{4}$ rad, respectively.

In the figure it can be seen that the vehicle converges to the desired position (the origin, in this case). Notice how in the presence of ocean current the vehicle automatically recruits the yaw angle that is required to counteract that current at the target point. Thus, at the end of the maneuver the vehicle is at the goal position and faces the current with surge velocity u_r equal to V_c. This is clearly illustrated in the Figures 4-5 that show the time responses for the case where ocean current is different from zero.

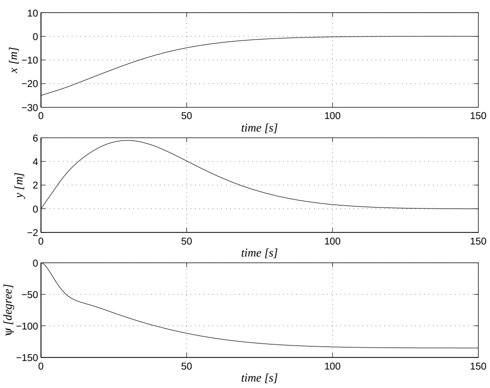

Fig. 4. Time evolution of $x(t)$, $y(t)$, and $\psi(t)$.

5. CONCLUSIONS

A solution to the problem of dynamic positioning of an underactuated AUV (in the horizontal plane) in the presence of a constant unknown ocean current disturbance and parametric model uncertainty was proposed. Convergence of the resulting nonlinear regulation system was analyzed and simulations were performed to illustrate the behaviour of the proposed control scheme. Future research will address the application of the new control strategy developed to the operation of a prototype marine vehicle.

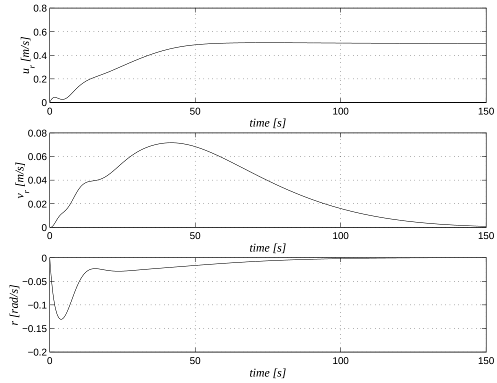

Fig. 5. Time evolution of the relative linear velocity (surge) $u_r(t)$, (sway) $v_r(t)$, and the angular velocity $r(t)$.

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