# Coordinated path-following of multiple underactuated autonomous vehicles with bidirectional communication constraints

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*Abstract*— **This paper addresses the problem of steering a group of underactuated autonomous vehicles along given spatial paths, while holding a desired inter-vehicle formation pattern. For a general class of vehicles moving in either two or three-dimensional space, we show how Lyapunov-based techniques and graph theory can be brought together to yield a decentralized control structure where the dynamics of the cooperating vehicles and the constraints imposed by the topology of the inter-vehicle communications network are explicitly taken into account. Path-following for each vehicle amounts to reducing the geometric error to a small neighborhood of the origin. The desired spatial paths do not need to be of a particular type (e.g., trimming trajectories) and can be any sufficiently smooth curves. Vehicle coordination is achieved by adjusting the speed of each vehicle along its path according to information on the positions and speeds of a subset of the other vehicles, as determined by the communications topology adopted. We illustrate our design procedure for underwater vehicles moving in three-dimensional space. Simulations results are presented and discussed.**

## I. Introduction

Increasingly challenging mission scenarios and the advent of powerful embedded systems and communication networks have spawned widespread interest in the problem of coordinated motion control of multiple autonomous vehicles. The types of applications envisioned are manifold and include aircraft and spacecraft formation flying control [5], [11], [15], coordinated control of land robots [6], [14], and control of multiple surface and underwater vehicles [7], [13], [16].

In spite of significant progress in the area, however, much work remains to be done to develop strategies capable of yielding robust performance of a fleet of vehicles in the presence of complex vehicle dynamics, severe communication constraints, and partial vehicle failures. These difficulties are specially challenging in the field of marine robotics for two main reasons: i) the dynamics of marine vehicles are often complex and cannot be simply ignored or drastically simplified for control design proposes, and ii) underwater communications and positioning rely heavily on acoustic systems, which are plagued with intermittent failures, latency, and multipath effects.

Inspired by the developments in the field, this paper tackles a problem in coordinated vehicle control that departs slightly from mainstream work reported in the literature. Specifically, we consider the problem of *coordinated path-following* where *multiple vehicles are required to follow pre-specified spatial paths while keeping a desired inter-vehicle formation pattern in time*. This problem arises for example in the operation of multiple autonomous underwater vehicles (AUV) for fast acoustic coverage of the seabed. In this application, two or more vehicles are required to fly above the seabed at the same or different depths, along geometrically similar spatial paths, and map the seabed using identical suites of acoustic sensors. By requesting that the vehicles traverse identical paths so that the projections of the acoustic beams on the seabed exhibit some overlapping, large areas can be covered in a short time. These objectives impose constraints on the inter-vehicle formation pattern. A number of other scenarios can of course be envisioned that require coordinated motion control of marine vehicles.

We solve the coordinated path-following problem for a general class of underactuated vehicles moving in either two or three-dimensional space. The solution adopted is well rooted in Lyapunov-based theory and addresses explicitly the vehicle dynamics as well as the constraints imposed by the topology of the inter-vehicle communications network. The latter are tackled in the framework of graph theory [12], which seems to be the tool par excellence to study the impact of communication topologies on the performance that can be achieved with coordination [8]. The class of vehicles for which the design procedure is applicable is quite general and includes any vehicle modeled as a rigid-body subject to a controlled force and either one controlled torque if it is only moving on a planar surface or two or three independent control torques for a vehicle moving in three dimensional space. Furthermore, contrary to most of the approaches described in the literature, the controller proposed does not suffer from geometric singularities due to the parametrization of the vehicle's rotation matrix. This is possible because the attitude control problem is formulated directly in the group of rotations $SO(3)$.

With the set-up adopted, path-following (in space) and inter-vehicle coordination (in time) are essentially decoupled. Path-following for each vehicle amounts to reducing a conveniently defined error variable to zero. The desired spatial paths do not need to be of a particular type (e.g., trimming trajectories) and can be any sufficiently smooth curves. Vehicle coordination is achieved by adjusting the speed of

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each of the vehicles along its path, according to information on the relative position and speed of the other vehicles, as determined by the communications topology adopted. No other kinematic or dynamic information is exchanged among the vehicles.

This paper builds upon and combine previous results obtained by the authors on path-following control [2], [4] and coordination control [9], [10].

## II. Problem statement

Consider an underactuated vehicle modeled as a rigid body subject to external forces and torques. Let $\{\mathcal{I}\}$ be an inertial coordinate frame and $\{\mathcal{B}\}$ a body-fixed coordinate frame whose origin is located at the center of mass of the vehicle. The configuration $(R, p)$ of the vehicle is an element of the Special Euclidean group $SE(3) := SO(3) \times \mathbb{R}^3$, where $R \in SO(3) := \{R \in \mathbb{R}^{3\times 3} : RR' = I_3, \det(R) = +1\}$ is a rotation matrix that describes the orientation of the vehicle by mapping body coordinates into inertial coordinates, and $p \in \mathbb{R}^3$ is the position of the origin of $\{\mathcal{B}\}$ in $\{\mathcal{I}\}$. Denoting by $v \in \mathbb{R}^3$ and $\omega \in \mathbb{R}^3$ the linear and angular velocities of the vehicle relative to $\{\mathcal{I}\}$ expressed in $\{\mathcal{B}\}$, respectively, the following kinematic relations apply:

$$\dot{p} = Rv, \tag{1a}$$

$$\dot{R} = RS(\omega), \tag{1b}$$

where

$$S(x) := \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}, \quad \forall x := (x_1, x_2, x_3)' \in \mathbb{R}^3.$$

We consider here underactuated vehicles with dynamic equations of motion of the following form:

$$\mathbf{M}\dot{v} = -S(\omega)\mathbf{M}v + f_v(v, p, R) + g_1 u_1, \tag{2a}$$

$$\mathbf{J}\dot{\omega} = -S(v)\mathbf{M}v - S(\omega)\mathbf{J}\omega + f_\omega(v, \omega, p, R) + G_2 u_2, \tag{2b}$$

where $\mathbf{M} \in \mathbb{R}^{3\times 3}$ and $\mathbf{J} \in \mathbb{R}^{3\times 3}$ denote constant symmetric positive definite mass and inertia matrices; $u_1 \in \mathbb{R}$ and $u_2 \in \mathbb{R}^3$ denote the control inputs, which act upon the system through a constant nonzero vector $g_1 \in \mathbb{R}^3$ and a constant nonsingular matrix[1] $G_2 \in \mathbb{R}^{3\times 3}$, respectively; and $f_v(\cdot)$, $f_\omega(\cdot)$ represent all the remaining forces and torques acting on the body. For the special case of an underwater vehicle, $\mathbf{M}$ and $\mathbf{J}$ also include the so-called hydrodynamic added-mass $M_A$ and added-inertia $J_A$ matrices, respectively, i.e., $\mathbf{M} = M_{RB} + M_A$, $\mathbf{J} = J_{RB} + J_A$, where $M_{RB}$ and $J_{RB}$ are the rigid-body mass and inertia matrices, respectively.

For an underactuated vehicle restricted to move on a planar surface, the same equations of motion (1)–(2) apply without the first two right-hand-side terms in (2b). Also, in this case, $(R, p) \in SE(2)$, $v \in \mathbb{R}^2$, $\omega \in \mathbb{R}$, $g_1 \in \mathbb{R}^2$, $G_2 \in \mathbb{R}$, $u_2 \in \mathbb{R}$, with all the other terms in (2) having appropriate dimensions, and the skew-symmetric matrix $S(\omega)$ is given by $S(\omega) = \left(\begin{smallmatrix} 0 & -\omega \\ \omega & 0 \end{smallmatrix}\right)$.

For each vehicle, the problem of following a predefined desired path is stated as follows:

[1] See [4, Remark 4] for the special case of $G_2 \in \mathbb{R}^{3\times 2}$.

**Path-following problem:** *Let $p_{d_i}(\gamma_i) \in \mathbb{R}^3$ be a desired path parameterized by a continuous variable $\gamma_i \in \mathbb{R}$ and $v_{r_i}(\gamma_i) \in \mathbb{R}$ a desired speed assignment for the vehicle i. Suppose also that $p_{d_i}(\gamma_i)$ is sufficiently smooth and its derivatives (with respect to $\gamma_i$) are bounded. Design a controller such that all the closed-loop signals are bounded, and the position of the vehicle* i) *converges to and remains inside a tube centered around the desired path that can be made arbitrarily thin, i.e., $\|p_i(t) - p_{d_i}(\gamma_i(t))\|$ converges to a neighborhood of the origin that can be made arbitrarily small, and* ii) *satisfies a desired speed assignment $v_{r_i}$ along the path, i.e., $|\dot{\gamma}_i(t) - v_{r_i}(\gamma_i(t))| \to 0$ as $t \to \infty$.*

We now consider the problem of coordinated path-following control. In the most general set-up, one is given a set of $n \geq 2$ autonomous underactuated vehicles and a set of $n$ spatial paths $p_{d_i}(\gamma_i)$; $i = 1, 2, ..., n$ and require that vehicle $i$ follow path $p_{d_i}$. As will become clear, the coordination problem will be solved by adjusting the speeds of the vehicles as functions of the "along-path" distances among them. Formally, the along-path distance between vehicle $i$ and $j$ is defined as $\gamma_{ij} := \gamma_i - \gamma_j$, and coordination achieved when $\gamma_{ij} = 0$ for all $i, j \in \{1, ..., n\}$ [10].

Let $J_i$ be the index set of the vehicles that vehicle $i$ communicates with. Assume that the underlying communication graph is undirected and connected (i.e., the communication links are bidirectional and there exists a path connecting every two vehicles). In this case the graph Laplacian $L \in \mathbb{R}^{n\times n}$ is symmetric, with a simple eigenvalue at zero and an associated eigenvector $\mathbf{1} = [1]_{n\times 1}$. The other eigenvalues are positive. See [12] for the definitions and the properties of graphs. The Laplacian can be decomposed as $L = MM'$, where $M \in \mathbb{R}^{n\times n-1}$, $\operatorname{Rank} M' = \operatorname{Rank} L = n - 1$ and $M'\mathbf{1} = \mathbf{0}$. Define the "graph-induced coordination error" as $\theta := M\gamma \in \mathbb{R}^{n-1}$, where $\gamma := [\gamma_i]_{n\times 1}$. From the properties of $M$, it can be easily seen that $\theta = \mathbf{0}$ is equivalent to $\gamma_i = \gamma_j, \forall i, j$. Consequently, if $\theta$ is driven to zero asymptotically, so are the coordination errors $\gamma_i - \gamma_j$ and the problem of coordinated path-following is solved.

**Coordination problem:** *Derive a control law for $\ddot{\gamma}_i$ as a function of $\gamma_j$ and $\dot{\gamma}_j$ where $j \in J_i$ such that $\theta$ approaches a small neighborhood of zero as $t \to \infty$. Each of the $n$ vehicles has access to its own states and exchanges information on its coordination state $\gamma_i$ and speed $\dot{\gamma}_i$ with some or all of the other vehicles defined by sets $J_i$.*

## III. Main results

### A. Path-following

In this section, we briefly discuss the results presented in [2], [4] to solve the path-following problem. Let $e_i := R_i'\left[p_i(t) - p_{d_i}(\gamma_i(t))\right]$ be the path-following error of the vehicle $i$ expressed in its body-fixed frame. Borrowing from the techniques of backstepping, in [2], [4] a feedback law for $u_{1_i}$, $u_{2_i}$ was derived that makes the time-derivative of the Lyapunov function

$$V_i := \frac{1}{2}e_i'e_i + \frac{1}{2}\varphi_i'\mathbf{M}_i^2\varphi_i + \frac{1}{2}z_{2_i}'\mathbf{J}_i z_{2_i}$$

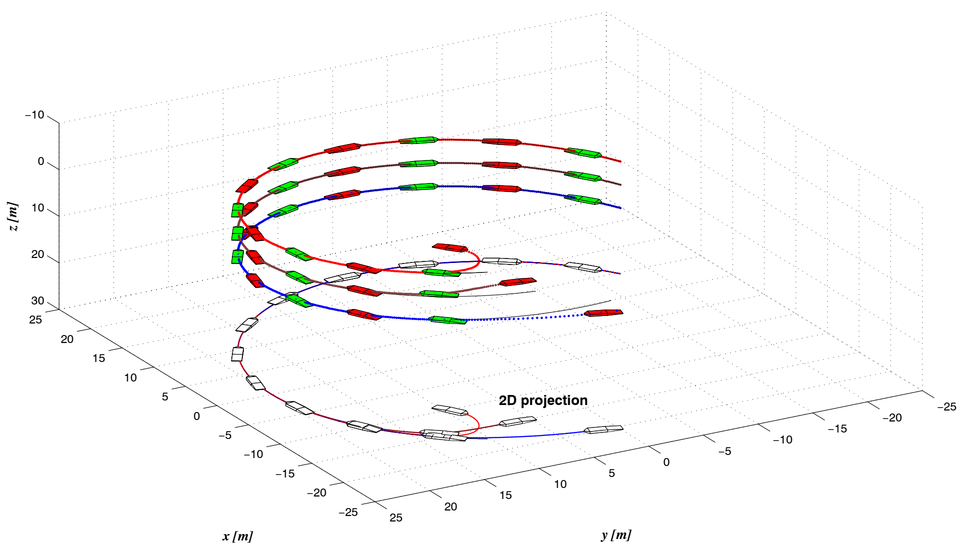


Fig. 1. Coordination of 3 AUVs in in-line formation.

take the form

$$\dot{V}_i = -k_{e_i}\, e_i'\mathbf{M}_i^{-1}e_i + e_i'\delta_i - \varphi_i' K_{\varphi_i}\varphi_i - z_{2_i}' K_{z_2 i} z_{2_i} + \mu_i\eta_i$$

where $\varphi_i$ and $z_{2_i}$ are linear and angular velocity errors (see [2], [4] for details), $k_{e_i}$, $K_{\varphi_i}$, $K_{z_2 i}$ are positive definite matrices, $\delta_i$ is a small constant vector, and $\mu_i$ captures the terms associated to the speed error $\eta_i := \dot{\gamma}_i - v_{r_i}$. At this point we remark that if all that is required is to solve a pure path-following problem then one can augment $V_i$ with the quadratic term $\frac{1}{2}\eta_i^2$ and utilize the freedom of assigning a feedback law to $\ddot{\gamma}_i$ in order to make $\dot{V}_i$ negative definite (see details in [2], [4]). This strategy must be modified to address coordination as shown below.

### B. Coordinated path-following

This section presents a solution to the coordinated path-following problem. Let $\eta := \dot{\gamma} - v_L\mathbf{1}$ be the speed vector error, where $v_L$ is a desired speed profile assigned to the formation. Consider the composite (coordination + path-following) Lyapunov function

$$V_c := \frac{1}{2}\theta'\theta + \frac{1}{2}z'z + \sum_{i=1}^{n} V_i$$

where $z := \eta + A_1^{-1}\mu + A_1^{-1}M\theta$. Computing the time-derivative of $V_c$ and assigning the following feedback law for $\ddot{\gamma}$

$$\ddot{\gamma} = -A_1^{-1}\dot{\mu} - A_1\eta - A_1^{-1}L\eta - A_2 z, \qquad (3)$$

where $A_1$, $A_2$ are diagonal positive definite matrices, we obtain

$$\dot{V}_c = -\eta' A_1\eta - z' A_2 z - \sum_{i=1}^{n}\big[k_{e_i}\, e_i'\mathbf{M}_i^{-1}e_i - e_i'\delta_i + \varphi_i' K_{\varphi_i}\varphi_i + z_{2_i}' K_{z_2 i} z_{2_i}\big].$$

It is now straightforward to prove the following result:

*Theorem 1:* The feedback laws for $u_{1_i}$, $u_{2_i}$ for each vehicle $i$ obtained in [2], [4] together with (3) solve the coordination and the path-following problems.

## IV. An illustrative example

This section illustrates the application of the previous results to underwater vehicles moving in three-dimensional space.

### A. Path-following and coordination of underwater vehicles in 3-D space

Consider an ellipsoidal shaped underactuated autonomous underwater vehicle (AUV) not necessarily neutrally buoyant. Let $\{\mathcal{B}\}$ be a body-fixed coordinate frame whose origin is located at the center of mass of the vehicle and suppose that we have available a pure body-fixed control force $\tau_u$ in the $x_{\mathcal{B}}$ direction, and two independent control torques $\tau_q$ and $\tau_r$ about the $y_{\mathcal{B}}$ and $z_{\mathcal{B}}$ axes of the vehicle, respectively. The kinematics and dynamics equations of motion of the vehicle can be written as (1)–(2), where

$$\mathbf{M} = \mathrm{diag}\{m_{11}, m_{22}, m_{33}\}, \qquad u_1 = \tau_u$$
$$\mathbf{J} = \mathrm{diag}\{J_{11}, J_{22}, J_{33}\}, \qquad u_2 = (\tau_q, \tau_r)'$$

$$D_v(v) = \mathrm{diag}\{X_{v_1} + X_{|v_1|v_1}|v_1|, Y_{v_2} + Y_{|v_2|v_2}|v_2|, Z_{v_3} + Z_{|v_3|v_3}|v_3|\}$$
$$D_\omega(\omega) = \mathrm{diag}\{K_{\omega_1} + K_{|\omega_1|\omega_1}|\omega_1|, M_{\omega_2} + M_{|\omega_2|\omega_2}|\omega_2|, N_{\omega_3} + N_{|\omega_3|\omega_3}|\omega_3|\}$$

$$g_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}, \qquad G_2 = \begin{pmatrix}0&0\\1&0\\0&1\end{pmatrix},$$
$$\bar{g}_1(R) = R'\begin{pmatrix}0\\0\\W-B\end{pmatrix}, \qquad \bar{g}_2(R) = S(r_{\mathcal{B}})R'\begin{pmatrix}0\\0\\B\end{pmatrix}$$

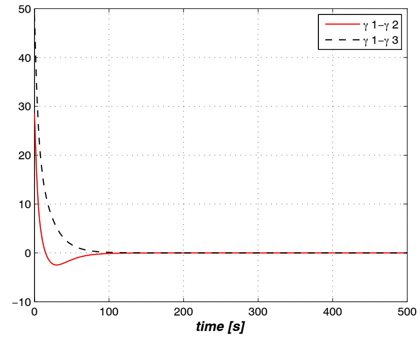


Fig. 2. Time evolution of the coordination errors $\gamma_{12} := \gamma_1 - \gamma_2$ and $\gamma_{13} := \gamma_1 - \gamma_3$.

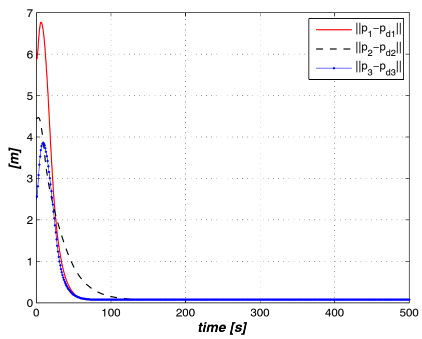


Fig. 3. Time evolution of the path-following errors $\|p_i - p_{d_i}\|$, $i = 1, 2, 3$.

$$f_v = -D_v(v)v - \bar{g}_1(R), \quad f_\omega = -D_\omega(\omega)\omega - \bar{g}_2(R).$$

The gravitational and buoyant forces are given by $W = mg$ and $B = \rho g \nabla$, respectively, where $m$ is the mass, $\rho$ is the mass density of the water and $\nabla$ is the volume of displaced water. The numerical values used for the physical parameters match those of the *Sirene* AUV, described in [1], [3].

## B. Simulation results

This section contains the results of simulations that illustrate the performance obtained with the coordinated path-following control laws developed in the paper. Figures 1– 3 illustrate the situation where three underactuated AUVs are required to follow paths of the form

$$p_{d_i}(\gamma_i) = \left[a_1 \cos(\frac{2\pi}{T}\gamma_i + \phi_d), a_1 \sin(\frac{2\pi}{T}\gamma_i + \phi_d), a_2\gamma_i + z_{0_i}\right],$$

with $a_1 = 20\,m$, $a_2 = 0.05\,m$, $T = 400$, $\phi_d = -\frac{3\pi}{4}$, and $z_{0_1} = -10\,m, z_{0_2} = -5\,m, z_{0_3} = 0\,m$. The initial conditions of the AUVs are $p_1 = (x_1, y_1, z_1) = (10\,m, -10\,m, -5\,m)$, $p_2 = (x_2, y_2, z_2) = (5\,m, -15\,m, 0\,m)$, $p_3 = (x_3, y_3, z_3) = (0\,m, -20\,m, 5\,m)$, $R_1 = R_2 = R_3 = I$, and $v_1 = v_2 = v_3 = \omega_1 = \omega_2 = \omega_3 = \mathbf{0}$. The vehicles are required to keep a formation pattern whereby they are aligned along a vertical line. In the simulation, vehicle 1 is allowed to communicate with vehicles 2 and 3, but the last two do not communicate between themselves directly. The reference speed $v_L$ was set to $v_L = 0.5\,s^{-1}$. Notice how the vehicles adjust their speeds to meet the formation requirements. Moreover, the coordination errors $\gamma_{12} := \gamma_1 - \gamma_2$ and $\gamma_{13} := \gamma_1 - \gamma_3$ and the path-following errors converge to a small neighborhood of the origin.

# V. Conclusions

The paper addressed the problem of steering a group of underactuated autonomous vehicles along given spatial paths, while holding a desired inter-vehicle formation pattern (coordinated path-following). A solution was derived that builds on recent results on path-following control [2], [4] and state-agreement (coordination) control [9], [10] obtained by the authors. The solution proposed builds on Lyapunov based techniques and addresses explicitly the constraints imposed by the topology of the inter-vehicle communications network. Furthermore, it leads to a decentralized control law whereby the exchange of data among the vehicles is kept at a minimum. Simulations illustrated the efficacy of the solution proposed. Further work is required to extend the methodology proposed to address the problems of robustness against temporary communication failures.

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