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OPTIMIZED DESIGN OF A DAM WITH RESPECT TO THE COEFFICIENTS OF POLYNOMIALS DEFINING ITS SHAPE

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ABSTRACT

When designing a dam, several factors are to be taken into account, an important one being safety requirements. The question arises of how to use the least quantity of material (concrete) while maintaining the strength of the resulting structure. This can be achieved by minimizing certain functionals depending on the characteristics of the dam, namely on its shape.

The mechanical behavior of the dam is modeled by the linearized equations of elasticity, where the elastic tensor is considered homogeneous and isotropic. Homogeneous Dirichlet boundary conditions are imposed on the parts of the dam which are in contact with the rock and Neumann boundary conditions are imposed on the rest of the boundary.

Several different functionals can be minimized, the easiest choice is to minimize the compliance of the dam, however, this choice is not realistic; the compliance is an overall quantity and a low compliance does not forbid the appearance of stress concentrations. A realistic approach should take into account local quantities (stress or strain). On the other hand, we recall that the concrete is a brittle material, which means that it does not resist well to traction stresses. This can be expressed mathematically by imposing that the stress tensor has only negative eigenvalues. The optimized design should only present compression stresses at each point of the dam.

Along the optimization process, the shape of the dam will vary. This variation is usually achieved through infinitesimal variations of the boundary in the normal direction. These variations are modeled by a vector field normal to the boundary. The optimization consists of minimizing a chosen functional and is based on the so called shape derivative which is the derivative of the objective functional with respect to the vector field defining the variation of the boundary, according to [Pironneau, 1984] and [Delfour & Zolezio, 2011].

The computation of the shape derivative is a lengthy process and produces an expression which depends in a highly implicit way on the infinitesimal variation of the boundary. This dependency is made explicit by using the adjoint method. This method, which is an essential ingredient in optimization, consists in introducing an auxiliary problem, called adjoint problem, of the same nature as the state equations of elasticity. The solution of this problem, called adjoint state, allows one to express the shape derivative in an explicit formula, as a boundary integral of a certain density multiplied by the normal component of the vector field defining the variation of the boundary. This density is expressed in terms of the solution of the state equation and of the adjoint state.

In order to use as few material as possible, a constraint on the total volume of material is added. This can be done through a penalty term or through a multi objective optimization approach. The shape derivative of the volume of the body is easy to compute.

The shape of a dam is often designed by using a polynomial parametrization of its boundary. A number of levels at given heights are chosen and the cross section of the dam corresponding to each level is described by two polynomials, one for the front and another for the rear part. One variable polynomials are used and symmetry is ensured by choosing only even powers of the variable. Our goal is to use the coefficients of these polynomials as optimization parameters. This means that instead of the shape derivative we need to compute the derivative of the objective functional with respect to the coefficients of the polynomials. This process can be regarded as the derivative of a composition of functionals.

Numerical simulations will be discussed.

REFERENCES

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