POLYNOMIAL STRESS FUNCTIONS FOR GENERAL 2D PROBLEMS

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ABSTRACT
In this paper, the analytical trial function method for the 8-node hybrid element (ATF-Q8) is generalized to general 2D problems, such as piezoelectric material problems. Based on generalized Lekhnitskii’s formalism, systematic schemes are given to determine all the n-th independent polynomial stress functions for general 2D problems, and the results can be degenerated to the case of anisotropic problem and isotropic problem. Complete polynomial stress functions are introduced as the trial functions of the generalized ATF method for analysis of anisotropic problems. Unlike the isotropic case, the trail polynomials are related to the material constants. In the end, the new explicit polynomial stress functions are implemented in the ATF-Q8 element method to solve piezoelectric problems. Several numerical examples are provided to test the accuracy of this approach.

Keywords: polynomial stress function, general 2D problem, analytical trial function.

INTRODUCTION
Based on the polynomial stress functions, Fu, et al.(2010) proposed a hybrid-‘stress function’ plane element, denoted as analytical trial function (ATF-Q8), which is shown to provide excellent performance and to be capable of avoiding usual direction dependence and interpolation failure through several numerical examples. In their work 15 polynomial Airy stress functions are introduced as the trial functions of the 8-node element for analysis of plane problems.

The key to implement the ATF-Q8 element is to construct all the independent polynomials for arbitrary n-th order homogenous polynomial stress functions. Systematic schemes to determine the polynomials for isotropic elastic solids are proposed by Wang, et al.(2011). Their results show that there are three independent polynomials for the Airy stress function of the second order homogeneous polynomial, and four independent polynomials for the Airy stress function of the nth order homogeneous polynomial (n>3). Their conclusion is generalized to anisotropic case for plane problems by Zhao et al.(2011). The general expressions of all the n-th order polynomials for anisotropic plane problems are presented and tested by some numerical examples. But how to construct all the independent polynomials of the n-th order polynomials for piezoelectric problems has not been reported.

Based on the generalized Lekhnitskii’s theory(1968) of piezoelectric materials, general expressions are obtained in explicit forms for all the possible independent polynomials of arbitrary n-th order homogenous polynomial stress functions for piezoelectric plane problems, which may be used to develop the ATF hybrid-element.
RESULTS AND CONCLUSIONS

For general 2D problems, the stress function $U(x,y)$ of the n-th order ($n \geq 4$) homogeneous polynomial of piezoelectric material can be expressed in the following form, for $\mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$, as

$$U = 2 \text{Re} \left[ a_0 z_1^n + b_0 z_2^n + c_0 z_3^n + d_0 z_4^n \right]$$

$$= 2 \text{Re} \left[ (A + iB)(x_1 + \mu_1 x_2)^n + (C + iD)(x_1 + \mu_2 x_2)^n + (E + iF)(x_1 + \mu_3 x_2)^n + (G + iH)(x_1 + \mu_4 x_2)^n \right].$$

The expression shows that there are only eight constants, $A, B, C, D, E, F, G, H$, to be determined in stress function $U(x_1,x_2)$, and the polynomials multiplied by these eight constants can define the respective polynomials. In another words, an n-th order polynomial stress function has at most eight independent polynomial terms. Also one may find the polynomials have relations with material constants.

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REFERENCES


