ANALYTICAL BUCKLING LOADS OF CONCRETE-FILLED STEEL TUBULAR COLUMNS WITH INTERLAYER SLIP

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ABSTRACT
This paper presents analytical results for critical buckling loads of circular concrete-filled steel tubular (CFST) columns with interlayer slip between the concrete core and steel tube. The analytical buckling loads of CFST columns are compared to the buckling loads proposed by various design codes and to the experimental results as well. In addition, a parametric study is conducted by which an influence of several geometric and material parameters on buckling loads of CFST columns is investigated.

Keywords: stability, buckling, analytical, slip, concrete-filled steel tubes, column.

INTRODUCTION
The use of concrete-filled steel tubular (CFST) columns has increased tremendously in recent years due to their excellent structural performance characteristics, which among many include high strength, high ductility, and large energy absorption capacity. The CFST columns gained increasing popularity in buildings, bridges, and other structural applications.

It is well known that the reduced thickness of these columns has an important impact on the local stability (local buckling) of the steel section and the compressive strength capacity of the concrete. If the CFST column is slender, the overall load capacity is susceptible on the global buckling of CFST column.

Slender CFST columns have been the subject of much research, which, for example, has included studies by Choi and Xiao (2010), Goode et al. (2010), Liang (2009), and many others. More recently, tests on slender CFST columns have been conducted by Han (2000).

To the best of the authors’ knowledge there exists very little work on the partial interaction in CFST columns and no analytical solution of critical buckling loads of CFST columns with interlayer slip between the constituents. The present paper aims to fill this gap.

GOVERNING EQUATIONS
Considered is a straight planar CFST column of undeformed length \( L \) shown in Fig. 1. The CFST column is made from concrete core “layer \( c \)” and steel tube “layer \( s \)”. \( D \) and \( t \) denote the outer diameter and the wall thickness of the steel tube, respectively. The CFST column is placed in the \( X, Z \) plane of the spatial Cartesian coordinate system with coordinates \( X \), \( Y \) and \( Z \). The undeformed reference axis of the CFST column is common to both layers. The CFST column is subjected to a conservative compressive load \( P \) which acts along the neutral axis of the CFST column in a way that homogeneous stress-strain state of the column is achieved in its primary configuration.
The linearized governing equations of the CFST column with compliant interface are derived using the following assumptions: 1) the material is linear elastic; 2) each layer is modelled by Reissner planar beam theory (Reissner, 1972); 3) the layers are continuously connected and slip and uplift moduli are constant; 4) shear deformations are not taken into account; 5) the cross-sections are symmetrical and remain unchanged in the form and size during deformation; and 6) the interlayer slip and uplift are small. The linearized governing equations are composed of kinematic, equilibrium and constitutive equations along with natural and essential boundary conditions for each of the layers. Moreover, there are also constraining equations which assemble each individual layer into a composite structure. The linear equations of CFST column are thus as follows:

\[
\begin{align*}
\delta u' - \delta e &= 0, \\
\delta w' + (1 + \varepsilon)\delta \varphi &= 0, \\
\delta \varphi' - \delta k &= 0, \\
\delta R_X' - \delta p_X &= 0, \\
\delta R_Z' - \delta p_Z &= 0, \\
\delta M_Y' + R_X \delta w - (1 + \varepsilon)\delta R_Z - \delta m_Y &= 0, \\
\delta R_X - C_{11} \delta e &= 0, \\
\delta M_Y - C_{22} \delta k &= 0, \\
\delta u^s - \delta e^s &= 0, \\
\delta w^s + (1 + \varepsilon)\delta \varphi^s &= 0, \\
\delta \varphi^s - \delta k^s &= 0, \\
\delta R_X^s - \delta p_X &= 0, \\
\delta R_Z^s - \delta p_Z &= 0, \\
\delta M_Y^s + R_X^s \delta w^s - (1 + \varepsilon)\delta R_Z^s + \delta m_Y &= 0, \\
\delta R_X^s - C_{11}^s \delta e^s &= 0, \\
\delta M_Y^s - C_{22}^s \delta k^s &= 0,
\end{align*}
\]
\[
\begin{align*}
\delta \Delta - \delta u^c + \delta u^s + r \sin \alpha (\delta \varphi^c - \delta \varphi^s) &= 0, \\
\delta d - \delta w^c + \delta w^s &= 0,
\end{align*}
\]
where
\[
\varepsilon = \varepsilon^c = \varepsilon^s = -\frac{P}{C_{11}^c + C_{11}^s},
\]
\[
\delta p_x = \delta p_x^c = \delta p_x^s = \int_0^{2\pi} K \delta \Delta \, r \, d\alpha = 2\pi r K (\delta u^c - \delta u^s),
\]
\[
\delta p_z = \delta p_z^c = \delta p_z^s = \int_0^{2\pi} C \delta d \, r \, d\alpha = 2\pi r C (\delta w^c - \delta w^s),
\]
\[
\delta m_y = \delta m_y^c = \delta m_y^s = \\
= \int_0^{2\pi} \left( 0, -r \cos \alpha, -r \sin \alpha \right) \times (K \delta \Delta, 0, C \delta d) \, r \, d\alpha = 2\pi r^2 K (\delta \varphi^c - \delta \varphi^s).
\]

In above equations, \(\delta u^c, \delta u^s, \delta w^c\) and \(\delta w^s\) are the components of the displacement vector of layers \(c\) and \(s\), \(\delta \varphi^c\) and \(\delta \varphi^s\) are rotations of the layer’s reference axis; \(\delta \varepsilon^c\) and \(\delta \varepsilon^s\) are the extensional strains; \(\delta k^c\) and \(\delta k^s\) are the pseudocurvatures; \(\delta R_x^c, \delta R_x^s, \delta R_z^c, \delta R_z^s, \delta M_y^c,\) and \(\delta M_y^s\) are the generalized equilibrium internal forces; \(\delta p_x, \delta p_z,\) and \(\delta m_y\) are the interlayer contact tractions; \(\delta \Delta\) is the interlayer slip; \(\delta d\) is the interlayer uplift; \(C_{11}^c, C_{11}^s, C_{22}^c,\) and \(C_{22}^s\) are the material and geometric constants of the cross sections; \(K\) is the longitudinal stiffness of the contact; \(C\) is the transverse contact stiffness; \(t\) denotes the derivative with respect to material coordinate \(x\); and \(\delta\) is the first variation.

**ANALYTICAL SOLUTION**

The system of linearized governing equations of CFST column with compliant interface can be written as a homogeneous system of 12 first order linear differential equations as
\[
Y'(x) = A \, Y(x) = 0,
\]
and
\[
Y(0) = Y_0 = 0,
\]
where \(Y(x)\) is the vector of unknown functions, \(Y(0)\) the vector of unknown integration constants, and \(A\) is the real 12x12 matrix. The exact solution of the problem is given
\[
Y(x) = e^{Ax} \, Y_0 = 0.
\]

The unknown integration constants are determined from the boundary conditions. As a results a system of 12 homogeneous linear algebraic equations for 12 unknown constants is obtained
\[
K \, Y_0 = 0,
\]
where \( K \) denotes the tangent stiffness matrix. A nontrivial solution is obtained from the condition of vanishing determinant of the matrix \( K \)

\[
\det K = 0.
\]

This condition represents a linear eigenvalue problem. Its solution corresponds to critical buckling loads \( P_{cr} \) and buckling modes. The exact solution for the lowest buckling load and corresponding buckling mode can easily be determined but are generally too cumbersome to be presented as closed-form expressions.

**RESULTS AND CONCLUSIONS**

In this numerical example the proposed analytical buckling loads are compared to the experimental and empirical ones proposed in the literature. First, the critical buckling loads of the P-P (pinned-pinned) circular CFST column are compared to the experimental results obtained by Han (2000). The details of each tested column are shown in Fig. 2 and Table 1.

![Fig. 2 - Geometric and material properties of CFST column](image)

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Slenderness</th>
<th>( N_{cr,e} )</th>
<th>( P_{cr} (K=0) )</th>
<th>( P_{cr} (K=1/100) )</th>
<th>( P_{cr} (K=1/10) )</th>
<th>( P_{cr} (K=\text{inf.}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC154-2</td>
<td>154</td>
<td>292</td>
<td>177.712</td>
<td>186.544</td>
<td>240.955</td>
<td>300.829</td>
</tr>
<tr>
<td>SC141-1</td>
<td>141</td>
<td>350</td>
<td>212.006</td>
<td>220.886</td>
<td>279.354</td>
<td>358.900</td>
</tr>
<tr>
<td>SC130-1</td>
<td>130</td>
<td>400</td>
<td>249.420</td>
<td>258.338</td>
<td>320.144</td>
<td>422.259</td>
</tr>
</tbody>
</table>

A comparison of analytical critical buckling loads of CFST simply-supported columns with experimental ones shown in Table 1 is made for different interlayer contact stiffness \( K \) and \( C=0 \). It can be seen that a good agreement between the analytical and experimental results exists if the longitudinal interface stiffness \( K \) is high. The distribution of the normalized critical buckling loads \( P_{cr}^* \) for various \( Ks \) and \( Cs \) is given in Fig. 3.

Fig. 3 shows that the critical buckling loads may reduce significantly by the interface compliance. It can also be seen that critical buckling loads are almost identical to the case where full connection between the constituents is present if at least one (longitudinal and transverse) interface stiffness is high.
Fig. 3 - The distribution of normalized critical buckling loads of circular P-P CFST column for various $K_s$ and $C_s$

Moreover, the Fig. 4 shows the comparison of the critical buckling loads of the circular P-P CFST column with the empirical ones.

Fig. 4 - Comparison of the analytical critical buckling loads of CFST simply-supported columns with the empirical and experimental ones for various column lengths

The comparison of the analytical critical buckling loads of CFST pinned-pinned column with empirical ones for various column lengths (slenderness) shows that the discrepancy of the analytical critical buckling loads with the ones calculated with various design codes is considerable.

We can thus conclude that in some cases the interface compliance can have a significant influence of buckling behaviour of CFST column with compliant interfaces.
REFERENCES


