MECHANICAL BEHAVIOUR OF TAPE SPRINGS USED IN THE DEPLOYMENT OF REFLECTORS AROUND A SOLAR PANEL

Florence Dewalque\(^*(*)\), Jean-Paul Collette\(^2\), Olivier Brüls\(^1\)
\(^1\)Department of Aerospace and Mechanical Engineering (LTAS), University of Liège, Liège, Belgium
\(^2\)Walopt, Embourg, Belgium
\(*\)Email: f.dewalque@ulg.ac.be

ABSTRACT
In order to increase the production of power on small satellites, large solar panels are commonly deployed and, in some cases, reflectors are added to improve the concentration factor on solar cells. In this work, reflectors are deployed by the means of compliant mechanisms known as tape springs. Their attractive characteristics are, among others, their passive behaviour, their self-locking capacity, their elastic deformations and their robustness. It is shown here through parametric studies that the behaviour of a tape spring is mainly governed by its geometry. Thus, for each specific application, its dimensions must be determined in order to minimise two critical features: the maximum stresses affecting the structure and the maximum amplitudes of motion during deployment. In this paper, an optimisation procedure is proposed to meet these requirements.

Keywords: Tape springs, nonlinear behaviour, shells, deployable structure, reflectors.

INTRODUCTION
In recent years, the development of small satellites has experienced an impressive rise. The intended purpose is the launch of numerous missions requiring both a low budget and a short period of design. In order to reach these goals, one of the most restraining conditions is the mass reduction of components. However, regarding the electronic equipment, if their mass decreases, their power consumption is also reduced, but generally by a less significant amount. Thus, even covering all the external surface of a satellite with solar cells might not provide enough power to meet the requirements of the electronic systems. The most common solution is to deploy solar panels on the sides of the satellite once it is jettisoned from the rocket. Furthermore, adding reflectors along the solar panels concentrates the sunlight on the solar cells. Thus, in theory, the geometric concentrator factor can be doubled and the area of the solar panels can be reduced. This system, however, requires the folding of the panels and the reflectors in order to integrate the satellite inside the confined space of the fairing.

In this work, the choice was made to exploit the characteristics of tape springs for the deployment of space structures. In particular, the analyses focus on the last deployment stage that, in this case, concerns the reflectors.

Tape springs belong to the category of compliant structures. By definition, some elastic energy is stored during folding and then naturally released when deployed to reach an equilibrium configuration. Although two equilibrium states exist [1], only the straight configuration is sought in the applications studied in this paper. During deployment, the system is then completely passive and self-actuated, requiring thus no external source of energy on the contrary of usual mechanisms such as hinges or prismatic joints. Compared to
these latter, other significant advantages can be pointed out. First, no lubricant is needed since the motion results from the deformation of structural components only and not from the sliding between contact surfaces. Thus, the risks of outgassing and contamination are reduced. Then, the structural simplicity of tape springs is a strong asset since it greatly improves the robustness and limits the possibilities of failure during folding and deployment.

The complexity of tape springs comes from their highly nonlinear mechanical behaviour. First of all, according to the sense of bending, different deformed configurations are encountered. For example, the equal sense is characterised by some torsion, while in the opposite sense, the structure is not affected by any transverse displacements. Then, for a critical value of the rotation angle, the tape spring is submitted to buckling which induces the formation of a fold and a sudden drop in its stiffness. Finally, due to the non-superposition of the loading and unloading paths, the dynamic evolution is regulated by some hysteresis which, after a certain number of cycles, leads to the self-locking of the structure.

Wüst [2], Rimrott [3] and Mansfield [4] were the first authors to derive the theoretical relationship between the applied rotation angle and the associated bending moment. More recently, new analytical models were developed: in [5], tape springs are represented as two rigid bodies of variable length interconnected by a mobile hinge; in [6], a variational approach expressed in terms of potential energy is used to perform quasi-static analyses; in [7,8], a one-dimensional planar rod model with a flexible cross-section is investigated. Numerous finite elements analyses exploiting shells were also simulated. For example, static analyses of a large variety of tape springs can be found in [9], while dynamic analyses focusing on the self-locking phenomenon, on the impact of the numerical and structural dampings, and on three-dimensional tape springs are available in [10], [11] and [12] respectively. Finally, experimental studies were performed to either validate analytical or finite element analyses. Static and dynamic tests can be found in [5,13] and [5,14,15] respectively.

For space systems, the use of reflectors to concentrate the solar radiations on solar cells has been investigated since the nineties [16]. The V-trough concentrators were first integrated on the solar panels of the PanAmSat Galaxy XI satellite and, a few years later, they were exploited on the Boeing BSS-702 Satellite Bus with limited success due to the contamination of the reflecting surfaces and the shrinkage of the reflectors alignment system [17]. More recently, the JAXA launched the small spacecraft REIMEI equipped with single lateral reflectors on each solar panel which gave satisfactory results according to their last report [18].

Although the use of reflectors is not unusual to increase the performance of solar panels, spring hinges are more commonly used to deploy them than tape springs. A particular deployment procedure exploiting the self-actuated and the self-locking properties of tape springs can be found in [19] where they are used to provide the driving torques to classical hinges. In the present work, however, the hinges connecting the reflectors to the solar panels are only composed of tape springs and not combined to any additional classical hinges.

An extensive analysis is then required to prove their efficiency for this type of applications. During folding and then deployment, the most critical parameters are the maximum stress affecting the structure and the maximum amplitude of motion. Regarding the former, its value must be kept under the yield limit in order to remain in the elastic regime and prevent irreversible deformations, while for the latter, which is only relevant during deployment, any collision with the other elements of the spacecraft must be avoided. To fulfil these requirements, the geometry of the tape springs has to be adapted to each specific situation.
Since several parameters can be modified, it is proposed in this work to address this problem by the means of an optimisation procedure exploiting the results of finite element models.

The layout of this paper is as follows. In Section II, the problem to be solved is defined, along with the geometric characteristics and the material properties of the tape springs. Then, in Section III, their theoretical behaviour is recalled. In Section IV, the parameters of the finite element analyses and models are described. In Section V, parametric studies are performed by varying some geometric parameters. The impact of the thickness and the radius of curvature are assessed. In Section VI, the optimisation procedure is performed to minimise the stresses and the amplitudes of motion. Finally, the conclusions of this work are drawn in Section VII.

DEFINITION OF THE PROBLEM

The problem studied in this work is relative to the deployment of a reflector which, in its folded configuration, is located on the top of the adjacent solar panel. The reflective membrane is rectangular of size 200×200 [mm]² and, with its frame, has a mass of 0.4 kg. In its deployed configuration, the angle formed with the solar panel reaches 120⁰ as in [19]. The hinge is composed of two tape springs with the same orientation and initially bended in opposite sense.

The geometry of a tape spring is completely characterised by four parameters: its length L, its thickness t, its subtended angle α and its transverse radius of curvature R. Regarding the frame used in this work, the x-axis coincides with the longitudinal axis of the tape spring, the y-axis with its transverse axis and the z-axis is along the height. All these elements are represented in Figure 1. Furthermore, only straight tape springs, that is without any longitudinal curvature, are considered in this paper. However, applications with curved tape springs can be found in [14].

![Fig. 1 - Geometric characteristics of a tape spring with L the length, t the thickness, α the subtended angle and R the transverse radius of curvature.](image)

Since in space applications the available space is strictly limited and the one devoted to the hinges of the reflectors is most likely to be determined in an early stage of the spacecraft design, the tape springs length is fixed to 50 mm, while their width w = 2R sin α/2 and their height h = R(1 − cos α/2) cannot exceed 30 mm and 15 mm respectively. A schematic representation of the deployed reflector is given in Figure 2, along with the maximum dimensions accepted for the tape springs. Notice that in order to simplify the model, all the connections between the different components are considered as perfectly rigid.
The theoretical behaviour of tape springs is defined based on the evolution of the bending moment at the clamped extremity of the structure when the bending angle is controlled at the other one. The first characteristic feature clearly noticeable is the direction dependent behaviour according to the sense of bending. These two senses and the corresponding sign convention are illustrated in Figure 3.

In the first case, the deformed structure is characterised by longitudinal and transverse curvatures in opposite sense, hence named opposite sense bending. The bending moment $M$ and the bending angle $\theta$ are by definition positive. In the second case, the longitudinal and the transverse curvatures are in equal sense, hence named equal sense bending. Accordingly, the bending moment $M$ and the associated bending angle $\theta$ are negative. These definitions were first stated in [2].
The theoretical evolution of the bending moment compared to the bending angle given in Fig. (inspired from [9]) is commonly acknowledged. Comprehensive descriptions can be found in [5,9,14], while a simplified diagram is available in [20]. The major features are recalled here.

In opposite sense bending, starting from point \( O \), the evolution of the bending moment \( M \) is linear for small rotation angles. The corresponding deformed configuration is characterised by a smooth curvature along the longitudinal axis. As the angle of rotation increases, the region located at the middle of the tape spring flattens until the associated bending moment reaches a maximum \( M_*^{\text{max}} \) called the peak moment. Shortly after, the moment experiences a dramatic drop (\( AB \)) which is due to the buckling of the structure leading to the formation of a fold. In this configuration, only the middle part of the tape spring is deformed because of the fold, while the rest of the structure remains straight (Figure 3). Regarding the radius of curvature of the fold, it can be considered as a first approximation as equal to the transverse radius of curvature of the non-deformed structure [2, 21]. However, finite element analyses showed that a difference exists [22], although marginal. Once buckling has occurred, bigger rotation angles induce almost no changes in the bending moment (\( BC \)) which is defined as the steady-state moment \( M_*^s \). Regarding the deformed configuration, only the arc-length of the fold is
affected by the variations of the bending angle. All this description is representative of the loading or the folding of a tape spring.

During unloading, the first part of the curve (CB) is superimposed with the loading path. However, at point B, the red curve stays equal to $M_\alpha^*$ until point D is reached before jumping back on the loading curve at point E which corresponds to a deformed configuration where the fold has disappeared. For the last part (EO), a linear evolution is recovered.

The fact that the fold does not disappear at the same angle of rotation as the one at which it is formed is responsible for some dissipation as cycles of loading-unloading are performed. This hysteresis phenomenon will, in the end, lead to the self-locking of the structure in its straight configuration.

In the equal sense bending, starting again from $O$, the relationship between the bending moment and the rotation angle rotation is linear (OF) with the same slope as in the opposite sense. The peak moment is reached for a relatively small angle which implies that $|M^{\text{max}}| < |M_\alpha^*|$. It is directly followed by buckling which, in this case, is due to asymmetric torsional folds converging from the extremities of the tape spring to its middle where they combine to form a symmetric fold. Then, as in opposite sense, the behaviour after buckling (GH) is defined by a constant steady-state moment $M_\alpha^*$. Finally, it is commonly accepted that, in theory, the unloading path in equal sense coincides with the folding sequence.

**FINITE ELEMENT ANALYSES**

In this work, all the finite element analyses are performed by the means of the commercial software Samcef [23]. Among the various finite elements provided in the software, shells are selected since the geometry of a tape spring is such that its thickness is several orders of magnitude smaller than all the other dimensions. Their behaviour is based on the Mindlin-Reissner theory and elements with linear shape functions are chosen here. Indeed, tests were performed with quadratic shells, however, no significant modification on the results was noticed and it unnecessarily increased the computational cost.

The resolution of the problem follows two steps. Starting from the straight configuration, the tape spring is first folded until the rotation angle reaches the required value. This stage could be based on a static analysis, however, the associated continuation methods are quite sensitive and the determination of their numerical parameters cumbersome. In order to facilitate the folding analysis, a very slow dynamic analysis is preferred. As it was proven in [10], this choice has no significant impact on the results, except at the buckling instant where some numerical oscillations may appear in the solution. For that purpose, the bending velocity is constant and fixed to one degree per second. The solver exploited at this stage is the Newmark method [24] with first-order accuracy. The numerical damping parameters are chosen such that the corresponding spectral radius at infinite frequencies is equal to zero. Thus, the numerical damping is very high and the oscillations after buckling can be annihilated in a minimum number of time steps.

Once the tape spring reaches its folded configuration, the rotation constraint is released and the deployment is simulated. In this case, a classical dynamic analysis is implemented. For more accuracy, the second-order time integration algorithm defined by the generalized-$\alpha$ method [25] is chosen.
For both solvers, an adaptive procedure is used to determine the time step. Thus, the nonlinear phenomena such as buckling, where small time steps are expected, can easily be captured, while the computational cost can be reduced when the evolution of the deformed structure is not significant.

Regarding the geometry of the model, besides the parameters described in Section II, two nodes coinciding with the centroids of the cross-sections at the extremities are added for the definition of the boundary conditions. They are rigidly connected to these cross-sections in order to facilitate the control of the bending angle and the collection of the results. The boundary conditions correspond to a clamp at one extremity and an imposed or free rotation angle at the other one. To represent the reflectors, lumped masses are located at their centre of mass and rigidly connected to the tape springs.

**PARAMETRIC STUDIES**

Before starting an optimisation procedure involving several parameters, it can be interesting to understand their impact on the tape spring behaviour separately. Quasi-static analyses are performed on a tape spring of length 200 mm which is bended to an angle of 60° in opposite or equal sense, then, while still controlling the rotation angle, the structure is brought back to its straight configuration. Thus, the pair of curves (loading and unloading paths) schematically given in Figure 4 are obtained. As opposed to the conditions described in Section IV, both the folding and the unfolding are controlled so that the Newmark method is used in both phases. Furthermore, no lumped mass is attached to the tape spring since the dynamic deployment is not analysed here.

The impact of two geometric parameters is assessed: the thickness \( t \) and the radius of curvature \( R \). Regarding the latter, in order to compare models, the width of the tape spring is kept constant and the subtended angle \( \alpha \) is modified accordingly.

**Impact of the thickness**

The impact of the thickness on the evolution of the bending moment when the rotation angle is controlled is summarized in Figure 5, where only four curves and the abscissae from -30° to 30° are represented for sake of visibility. The radius of curvature \( R \) is fixed to 20 mm and the subtended angle \( \alpha \) to 90°. The full lines correspond to the loading or folding sequence, while the dashed lines correspond to the controlled unloading of the tape spring.

In each case, the overall evolution in the opposite sense respects the theoretical description given in Section III with small discrepancies. The vertical peaks that are visible at the instants of buckling and disappearance of the fold are purely numerical and are due to the transient behaviour of the Newmark solver.

The behaviour in equal sense bending requires more explanations. During the folding, a first peak is encountered. In terms of deformation, it corresponds to the formation of torsional folds. Then, for higher bending angles, these folds combine themselves at the middle of the tape spring to form a symmetric one. This instant appears on the bending moment curve as oscillations of very small amplitude (not visible in Figure 5 due to the scale on the x-axis). During unloading, a similar procedure is followed in the reverse order: the symmetric fold divides itself into torsional folds, then these latter are removed as the bending angle is reduced. However, in this case, the division of the symmetric fold can create larger oscillations and a vertical jump in the bending moment. Furthermore, the associated bending
angles are not necessarily the same as those characterising the loading. All these elements can be responsible for the non-superposition of the loading and unloading paths. The equal sense bending behaviour obtained through finite element analyses is then more complex than what is commonly accepted in theory.

As can be expected, structures with a bigger thickness are stiffer. In Figure 5, this behaviour is illustrated by the larger slope at the origin which leads to higher peak moments in both senses. Accordingly, the phenomenon of buckling is triggered for higher absolute bending angles. It can be shown that the peak moments follow a polynomial evolution with the thickness of the third order, while it is almost linear for the bending angles. Regarding the steady-state moments, a similar evolution is observed.

When comparing the areas subtended by the loading and unloading curves, it can be clearly seen in opposite sense that the higher the stiffness, the bigger their difference. It implies that the dissipation of energy by hysteresis has a stronger effect as the thickness increases (Figure 6). It can then be expected that the oscillations of a thick structure after deployment require a shorter period of time to be damped out. In equal sense, the evolution of the dissipated energy is less smooth due to the non-superposition of the loading and unloading paths after buckling especially for a thickness of 0.2 mm, however the general behaviour is preserved. Furthermore, for each considered thickness, more energy is dissipated in opposite sense than in equal sense.

Another critical parameter that will affect the choice of the tape spring geometry is the maximum Von Mises stresses reached in the structure throughout its folding and deployment. In the case of a quasi-static folding and unfolding, the results in opposite and equal senses are given in Figure 7, where it can be seen that the stresses increase almost linearly with the thickness. In practical applications, it is then necessary to fix an upper limit to this geometric parameter in order to keep the deformations in the elastic regime.
Fig. 6- Impact of the thickness on the energy dissipated by hysteresis.

Fig. 7 - Impact of the thickness on the maximum Von Mises stresses.

**Impact of the radius of curvature**

In order to understand the impact of the radius of curvature \( R \) on the behaviour of a tape spring, we consider that the width \( w \) is kept constant. It implies that the subtended angle \( \alpha \) has to be modified accordingly. The width is fixed to the value obtained in the previous section, that is the one associated to a radius of 20 \( \text{mm} \) and a subtended angle of 90°. Only a specific interval of radii can be studied in order to keep geometries representative of tape springs. Indeed, if the subtended angle exceeds 180°, the structure becomes tubular and some contacts on the inside surface occur when folded. Extensive analyses of tubular compliant hinges can be found in [26, 27]. On the other hand, if the radius of curvature is too high and the subtended angle too small, all the attractive properties in terms of dissipation, self-locking and self-actuation in tape springs are lost, since the structure tends to behave as a flat plate.

The evolution of the bending moment for four different radii of curvature is given in Figure 8, where the interval of the bending angle is limited to 10° in both senses for sake of clarity. As it was the case in Figure 5, the full lines represent the loading step, while the dashed ones represent the controlled unloading. Furthermore, the behaviour in opposite sense is the one expected by the theory, while in equal sense, the evolution is more complex.
Increasing the radius of curvature implies an absolute reduction of the peak moments. Indeed, since the structure becomes flatter, the structure is less stiff and the required moment to bend it naturally decreases. Nonetheless, the associated bending angles do not vary significantly. A slight decrease is visible in opposite sense, while it is the contrary in the equal sense. Regarding the steady-state moments, their absolute evolution is similar in each sense and the stiffer structures are logically characterised by higher moments.

Focusing on the opposite sense bending, since the bending angles corresponding to buckling and the disappearance of the fold are very similar for each model, the highest dissipation by hysteresis is encountered in the structure with the smallest radius of curvature (Figure 9). It can then be expected that the oscillations resulting from the deployment are damped out in a shorter period of time. On the other hand, as mentioned in the previous section, the complexity of the behaviour in equal sense bending does not allow an evolution as clear as in the opposite sense, but the general features are preserved. Furthermore, as it was already the case in the previous section, the dissipation by hysteresis is more significant in opposite sense than in equal sense.

![Fig. 8 - Impact of the radius of curvature on the evolution of the bending moment in opposite and equal senses.](image)

Finally, compared to the thickness (Figure 7), the radius of curvature has the opposite effect on the evolution of the maximum Von Mises stresses. Indeed, increasing this parameter leads to a reduction of the stresses as it can be seen in Figure 10. Let us mention that the curve in equal sense is not as smooth as in opposite sense due to numerical effects which artificially increase the stresses.
OPTIMISATION

In order to determine the most efficient geometry of tape springs which meets the requirements to deploy the reflector defined in Section 0, an optimisation procedure is proposed. As mentioned previously, only the thickness $t$, the radius of curvature $R$ and the subtended angle $\alpha$ are chosen as design variables, while the length $L$ and the material properties are fixed.

The selection of the geometric parameters is first performed on a single tape spring connected to a lumped mass representing half of the reflector inertia. Nonetheless, as mentioned in Section 0, the final hinge in this practical application is composed of two tape springs with the same orientation in order to restrain the lateral displacements due to torsion and to keep a symmetric system. Since torsional and three-dimensional behaviours could be expected, the results obtained for one tape spring have then to be confirmed for the complete hinge a posteriori.
Model description

The optimisation procedure exploits the active set algorithm available in the Matlab software through the *fmincon* function which solves constrained nonlinear multivariable problems of medium scale [28]. The optimisation problem to be solved can be expressed as:

\[
\min_x f(x) \text{ such that } \begin{cases} 
  c(x) \leq 0 \\
  lb \leq x \leq ub
\end{cases}
\]

where \( f \) is the objective function to be minimised according to the vector of design variables \( x \), \( c(x) \) contains the nonlinear inequality constraints, and \( lb \) and \( ub \) are the vectors containing respectively the lower and upper bounds of the design variables.

These latter are defined in Table 2. Based on our experience acquired during the parametric studies, they were chosen in such a way that even in the extreme situations, the tape springs preserve their characteristic features. Regarding the limits on the width \( w \) and the height \( h \), they are expressed as nonlinear inequality constraints in \( c(x) \) since their expressions involve trigonometric functions:

\[
\begin{align*}
  c_1 &= w(\alpha, R) - w_{\max} \leq 0 \\
  c_2 &= h(\alpha, R) - h_{\max} \leq 0
\end{align*}
\]

with \( w_{\max} = 30 \text{ mm} \) and \( h_{\max} = 15 \text{ mm} \).

<table>
<thead>
<tr>
<th>( t ) [mm]</th>
<th>( R ) [mm]</th>
<th>( \alpha ) [rad]</th>
</tr>
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<tbody>
<tr>
<td>( lb )</td>
<td>0.08</td>
<td>10</td>
</tr>
<tr>
<td>( ub )</td>
<td>0.25</td>
<td>32.5</td>
</tr>
</tbody>
</table>

Starting from an initial geometry, the design variables \( x \) are introduced in the *Samcef* software to create the corresponding finite element model which is then solved. From the results, the objective function and the constraints are extracted and sent back to Matlab where the optimisation routine determines the next configuration. Several iterations of this cycle are carried out in an automatic way until the optimised geometry is reached.

In the *Samcef* software, each analysis follows the same pattern. First, the structure is folded in opposite sense until the bending angle reaches 120°. In order to keep a quasi-static evolution, this step is performed in 120 s during which the time is directly related to the bending angle. Then, the tape spring is deployed and its behaviour is studied for 30 s. Regarding the boundary conditions, the tape spring is clamped at one extremity, while the transverse displacements are locked at the other one, since as mentioned previously, for the hinge, these displacements should be limited.

Minimisation of the maximum Von Mises stress

In this first case, the optimisation procedure aims at minimising the Von Mises stresses. The results after 13 iterations are given in Table 3 along with the starting point. This new geometry leads to a maximum Von Mises stress of 666.25 MPa keeping thus the
deformations in the elastic regime (Table 1). As expected from the parametric studies, the optimised geometry is defined by the smallest allowed thickness.

With a width of 19.07 mm and a height of 2.55 mm, the optimal tape spring is relatively flat which implies that only a small amount of energy is dissipated by hysteresis during the deployment (Figure 11). This is confirmed by the evolution of the displacements along the x and z axes represented in Figure 12 where, in the worst case, the maximum motion amplitude $d_{\text{max}}$ of the tape spring extremity reaches 53.92 mm. This amplitude $d_{\text{max}}$ is defined as the maximum distance between the extremity of the tape spring in its initial configuration and a configuration reached during deployment. Furthermore, even after 30 s, the structure keeps oscillating with an amplitude of about 13 mm. Notice that in these two figures, time starts at 100 s in order to focus on the evolutions of the potential energy and the displacements during deployment.

Table 3 - Starting and optimised geometries of a tape spring which minimises the Von Mises stresses.

<table>
<thead>
<tr>
<th>Thickness $t$</th>
<th>Radius of curvature $R$</th>
<th>Subtended angle $\alpha$</th>
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</thead>
<tbody>
<tr>
<td>Starting geometry</td>
<td>0.08 mm</td>
<td>30 mm</td>
</tr>
<tr>
<td>Optimised geometry</td>
<td>0.08 mm</td>
<td>19.07 mm</td>
</tr>
</tbody>
</table>

Fig. 11 - Minimisation of the Von Mises stresses: evolution of the potential energy.
In order to check the absence of collisions, the deployment in the $x$ and $z$ axes is illustrated in Figure 13 where the straight blue line represents the solar panel, the straight green line represents the tape spring and the straight black line represents the reflector. The orientation of the frame is as defined in Section II, that is aligned on the tape spring. Regarding the two circular arcs, they show the position of the tape spring extremity and of the lumped mass located at the middle of the reflector during the different time steps of the dynamic analysis. Finally, the red dots correspond to the extremity of the reflector in two specific situations. In the second quadrant, the structure is in its folded configuration right before deployment, while in the third quadrant, it represents the configuration submitted to the maximum motion amplitude $d_{\text{max}}$. As it can be seen, despite the large displacements, no collision occurs with the solar panel.

**Minimisation of the maximum amplitude of motion**

In this second problem, the maximum amplitude of motion $d_{\text{max}}$ is minimised and no constraint is imposed on the Von Mises stresses.
The optimised geometry is obtained after 10 iterations. Its characteristics are given in Table 4, along with the starting point. The maximum amplitude of motion $d_{\text{max}}$ is this time reduced to 51.26 mm. Regarding the oscillations during deployment (Figure 14), compared to the previous results, it can be seen that they are characterised by a higher frequency and that they are more quickly damped out. Indeed, as it was shown in Section V, a bigger thickness and a larger radius of curvature are responsible for an increase of the dissipated energy by hysteresis.

Table 4 - Starting and optimised geometries of a tape spring which minimises the amplitudes of motion after deployment.

<table>
<thead>
<tr>
<th>Thickness $t$</th>
<th>Radius of curvature $R$</th>
<th>Subtended angle $\alpha$</th>
</tr>
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<tbody>
<tr>
<td>Starting geometry</td>
<td>0.1 mm</td>
<td>15 mm</td>
</tr>
<tr>
<td>Optimised geometry</td>
<td>0.244 mm</td>
<td>29.68 mm</td>
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Nonetheless, despite explicitly minimising the amplitudes of motion, their maximum stays relatively large and the reduction compared to the results in the previous section is not substantial. Furthermore, the maximum stress to which such structure is submitted during folding reaches 1856.18 MPa which is well above the yield limit of the beryllium copper (Table 1). The deformations are then such that, after deployment, a straight configuration might not be recovered due to irreversible deformations. In this work, plastic deformations are not taken into account in the models.

In Figure 15, as expected, the configuration of the reflector with the maximum motion amplitude, corresponding to the red dot in the fourth quadrant, does not collide with the solar panel.
Minimisation of both objective functions

In this case, the problem is set in order to minimise a hybrid objective function. This function is then defined as the weighted sum of the maximum Von Mises stress $\sigma_{\text{max}}$ and the maximum amplitude of motion $d_{\text{max}}$ such that:

$$f(x) = w_1 \sigma_{\text{max}} + w_2 d_{\text{max}}$$

where $w_1$ and $w_2$ are the weights respectively expressed in mm and MPa. In this problem, $w_1 = 0.1 w_2$ in order to scale down the maximum stress and thus have two terms with the same order of magnitude. Notice that the previous objective functions can be recovered by setting the weights to zero alternatively.

The resulting optimised geometry is given in Table 5. It leads to a maximum Von Mises stress of 877.75 MPa and a maximum motion amplitude $d_{\text{max}}$ of 52.08 mm. As it could be expected, these results are a compromise between those obtained by minimising the two objective functions separately. Furthermore, the time required to damp the oscillations is also between the two extreme situations as it can be seen in Figure 16.

<table>
<thead>
<tr>
<th>Thickness $t$</th>
<th>Radius of curvature $R$</th>
<th>Subtended angle $\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting geometry</td>
<td>0.08 mm</td>
<td>10 mm</td>
</tr>
<tr>
<td>Optimised geometry</td>
<td>0.0804 mm</td>
<td>30 mm</td>
</tr>
</tbody>
</table>
Thus, this last analysis leads to a structure which stays in the elastic regime during the folding sequence, avoids any collisions with the solar panel (Figure 17) and is stabilised after a reasonable amount of time. Nonetheless, after all these simulations, the relevance of forcing the minimisation of the amplitudes of motion can be questioned. Indeed, for each considered geometry, even though no collision is detected, the maximum amplitude stays very large and such motion is only possible because no other component is located under the solar panel. For example, if the same analysis was performed for the deployment of a solar panel itself, it would most likely interfere with the body of the spacecraft. Thus, it might be interesting in further studies to consider the use of mechanical stops to restrain the displacements of space structures deployed by the means of tape springs. Another aspect that could be considered is the impact of the chosen sense of bending during the folding of the structure.

Deployment of the reflector

The optimised geometry obtained in the previous section is now exploited to define the tape springs composing the hinge described in Section II. The representation of the finite element model is given in Figure 18. The solar panel is assumed already deployed and fixed. The tape springs can then be considered as clamped at those extremities. Furthermore, compared to the
previous models, the transverse displacements at the other sides are not constrained. The red dot represents the reflector as a lumped mass located at its centre of mass. Finally, the distance between the tape springs is chosen such that they are close to the lateral extremities of the reflector with a margin of 5 mm on each side.

The evolution of the displacements at the extremity of one of the tape springs is illustrated in Figure 19. Compared to the results obtained with a single tape spring, it can be seen that the curves have the same characteristic features, but they are slightly shifted to the left, while their peaks are slightly smaller. It means that for the hinge, the maximum amplitude of motion $d_{\text{max}}$ and the time required to damp out the oscillations are reduced of a small amount. Furthermore, it can be checked that the associated lateral displacements of the lumped mass connected to the hinge are very small, proving thus that the effects of the torsion are restrained. These results allow us then to validate our choice to perform the optimisation procedure on a single tape spring connected to a lumped mass with half the mass of the reflector and for which the transverse displacements at the unclamped extremity are locked. Nonetheless, the complete model with two tape springs provides a good validation of the previous results along with the confirmation of the absence of collision with the solar panel (Figure 20).
CONCLUSIONS

The purpose of this work is to use an optimisation procedure in order to determine the geometric parameters of tape springs used to deploy reflectors. Throughout the analyses, only the thickness, the radius of curvature and the subtended angle are considered as design variables, while the length and the material properties are fixed in an earlier stage of design.

The impact of the geometry is first assessed through quasi-static parametric studies. It is shown that for a chosen width, a small thickness and a large radius of curvature lead to low Von Mises stresses in the structure and little energy is dissipated by hysteresis. The opposite is true for a tape spring defined by a large thickness and a small radius of curvature.

An optimisation procedure is then performed in order to deploy a reflector of 0.4 kg. Constraints are defined as upper and lower bounds for each design variable and nonlinear constraints are added to limit the maximum width and height of the tape spring. The two objective functions to be minimised are the maximum Von Mises stress and the maximum amplitude of motion. If only the former is taken into account, the optimised geometry is characterised by a small thickness, an intermediate radius and a small subtended angle. On the other hand, if only the lowest amplitudes of motion are sought, the optimised geometry is defined by a large thickness, an intermediate radius and a small subtended angle. In the last optimisation analysis, a linear combination of these two objective functions is minimised and, as expected, the new geometry is between the two previous ones. Finally, two tape springs defined by this last geometry are used to form an hinge which deploys the reflector and the absence of collision with the solar panel is confirmed by simulation.

Nonetheless, the amplitudes of motion are relatively large for each of the optimised geometry. It might then be interesting in further works to perform analyses where the displacements are restrained by mechanical stops. This, however, will require taking into account more complex behaviours such as contacts and shocks. Furthermore, different orientations of the tape springs could be tested, along with the impact of the number of tape springs composing the hinge. Finally, to complete the results obtained here, it would be necessary to include the material properties in the set of design variables.
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REFERENCES


