SIZE EFFECT ANALYSIS OF COMPRRESSIVE STRENGTH FOR RECYCLED CONCRETE USING THE BFEM ON MICROMECHANICS

Yijiang Peng(†), Jiwei Pu
The Key Laboratory of Urban Security and Disaster Engineering, Ministry of Education, Beijing University of Technology, Beijing, China
(†)Email: pengyijiang@bjut.edu.cn

ABSTRACT
In this paper, the base force element method (BFEM) on damage mechanics is used to analyze the size effect on compressive strength for recycled concrete (RC) at meso-level. The recycled concrete is taken as five-phase composites consisting of natural coarse aggregate, new mortar, new interfacial transition zone (ITZ), old mortar and old ITZ on meso-level. The random aggregate model is used to simulate the meso-structure of recycled concrete. The size effects of Mechanical Properties of RC under uniaxial compressive loading are simulated using the BFEM on damage mechanics. The simulation results agree with the test results. This analysis method is the new way for investigating fracture mechanism and numerical simulation of mechanical properties for recycled concrete.

Keywords: finite element method, base force element method (BFEM), micromechanics, recycled concrete (RC), compressive strength, size effect.

INTRODUCTION
Now a large amount of waste concrete has been produced. The amount of waste concrete not only takes up precious land resources, but also causes serious environmental and social problems. The most effective way to dispose the waste concrete is to recycle and reuse the abandoned concrete. The mechanical properties of recycled concrete (RC) play a key role in the application of the RC technology. Many tests have been done on the mechanical properties of the RC by different scholars. An overview of study on recycled concrete has been given by Xiao et al[1]. However, because of the complexity of recycled coarse aggregates, conclusions made by different researchers are usually not very accordant, even opposite sometimes. To remove effects of experimental conditions, some numerical researches on meso-level was considered. For example, numerical simulation on stress-strain curve of recycled concrete was taken by Xiao et al[2] with Lattice Model under uniaxial compression. A method on meso-mechanics analysis was proposed by Peng et al[3] and Zhou, Peng et al[4] using FEM for recycled concrete based on random aggregate model. However, the Numerical researches on the damage mechanism for recycled concrete material have just begun.

The composite behavior of concrete is exceedingly complex and up to now many details such as strain softening, microcrack propagation, failure mechanisms and size effects etc. are still far from being fully understood. Since it is difficult to look inside concrete to observe the actual crack propagation or to experimentally determine the microscopic stress field, it has become obvious that further progress based exclusively on experimental studies will be limited. In order to overcome this defect, the concept of numerical concrete was presented by Wittanmm et al[5] based on micro-mechanics. Subsequently, some scholars did some creative
works in this field, and made a number of models. Among them, the two important models are the lattice model and the random aggregate model. For example, Schlangen et al\cite{6,7} applied the lattice model to simulate the failure mechanism of concrete. Liu et al\cite{8} adopted the random aggregate model to simulate cracking process of concrete using FEM. Peng et al\cite{9} adopted the random aggregate model to simulate the mechanics properties of rolled compacted concrete on meso-level using FEM. Du et al\cite{10,11} researched a meso-element equivalent method for the simulation of macro mechanical properties of concrete and a meso-scale analysis method for the simulation of nonlinear damage and failure behavior of reinforced concrete members.

The finite element method (FEM) is one of the most important numerical methods developed from 1950s, and it has been the most popular and widely used numerical analysis tool for problems in engineering and science. Over the past 50 years, numerous efforts techniques have been proposed for developing finite element models\cite{15,16,17,18}. In recent years, a new type of finite element method - the Base Force Element Method (BFEM) has been developed by Peng et al\cite{19,20,21,22,23,24} based on the concept of the base forces by Gao\cite{25}. Further, the base force element method (BFEM) on potential energy principle was used to analyze recycled aggregate concrete (RAC) on meso-level\cite{26}.

In this paper, the base force element method (BFEM) on damage mechanics is used to analyze the size effect on compressive strength for recycled concrete (RC) at meso-level.

**BASIC FORMULA**

Consider a two-dimensional material domain. $x^\alpha (\alpha = 1, 2)$ denote the Lagrangian coordinates. Let $P$ denotes the position vector of a point before deformation and $Q$ denotes the position vector of a point after deformation. The displacement $u$ of a point is

$$u = Q - P$$

(1)

The gradient of displacement $u_\alpha$ can be written as

$$u_\alpha = \frac{\partial u}{\partial x^\alpha} = Q_\alpha - P_\alpha$$

(2)

In order to describe the stress state at a point $Q$, a parallelogram with the edges $dx^1Q_1, dx^2Q_2$ is shown in Figure 1. Let $dT^\alpha$ denote the force acting on the $\alpha$ edge. We calculate the limit

$$t^\alpha = \lim_{dx^i \rightarrow 0} \frac{dT^\alpha}{dx^{\alpha+1}}$$

(3)

![Fig. 1 - The base forces](image)

-2168-
where we promise 3=1 for indexes. Quantities $\mathbf{t}^\alpha (\alpha = 1, 2)$ are called the base forces at point $Q$ in the two-dimensional coordinate system $x^\alpha$.

In order to further explain the meaning of $\mathbf{t}^\alpha$, let us compare $\mathbf{t}^\alpha$ with the stress vector $\mathbf{\sigma}^\alpha$ which represent the forces per unit area in the deformed body. That is

$$\mathbf{t}^\alpha = |Q_{\alpha \alpha 1}| \mathbf{\sigma}^\alpha$$  \hspace{1cm} (4)

where we promise 3=1 for indexes.

The base forces $\mathbf{t}^\alpha$ can also be understood as stress flux. For the Cartesian coordinate system, there is $|Q_{\alpha \alpha 1}| = 1$. In small deformation, the base forces are the stress vector.

According to the definitions of various stress tensors, the relation between the base forces and various stress tensors can be given. For example, the Cauchy stress is

$$\mathbf{\sigma} = \frac{1}{A} \mathbf{t}^\alpha \otimes Q_{\alpha \alpha}.$$  \hspace{1cm} (5)

where $\otimes$ is the dyadic symbol and the summation rule is implied.

Further, the elastic law can be given as follows

$$\mathbf{t}^\alpha = \rho A \frac{\partial W}{\partial \mathbf{u}_\alpha}$$  \hspace{1cm} (6)

in which $W$ is the strain energy density, $\rho$ is the mass density after deformation.

Equation (6) expresses the $\mathbf{t}^\alpha$ by strain energy directly. Thus, $\mathbf{u}_\alpha$ is just the conjugate variable of $\mathbf{t}^\alpha$. It can be seen that the mechanics problem can be completely established by means of $\mathbf{t}^\alpha$ and $\mathbf{u}_\alpha$.

For the small deformation case, the basis vector after deformation $Q_{\alpha \alpha}$ equals the basis vector before deformation $P_{\alpha \alpha}$, and the Green strain $\mathbf{\epsilon}$ can be written as

$$\mathbf{\epsilon} = \frac{1}{2} (\mathbf{u}_\alpha \otimes \mathbf{P}^\alpha + \mathbf{P}^\alpha \otimes \mathbf{u}_\alpha)$$  \hspace{1cm} (7)

where $\mathbf{P}^\alpha$ is the conjugate of $P_{\alpha \alpha}$.

**MODEL OF THE BFEM**

Consider a triangular element, the stiffness matrix of a base force element based on potential energy principle can be obtained as \cite{26}

$$K^{IJ} = \frac{E}{2A(1 + \nu)} \left[ \frac{2\nu}{1 - 2\nu} \mathbf{m}^I \otimes \mathbf{m}^J + \mathbf{m}^I \mathbf{U}^J + \mathbf{m}^J \otimes \mathbf{m}^I \right] \hspace{1cm} (I = 1, 2, 3; \ J = 1, 2, 3)$$  \hspace{1cm} (8)

in which $E$ is Young’s modulus, $\nu$ is Poisson’s ratio, $A$ is the area of an element, $\mathbf{U}$ is the unit tensor $\mathbf{U} = P_{\alpha \alpha} \otimes P^\alpha = P^\alpha \otimes P_{\alpha \alpha}$, $\mathbf{m}^I = \mathbf{m}^I \cdot \mathbf{m}^J$ and $\mathbf{m}^I$ can be calculated from
\[ m^i = m^i_P \alpha = \frac{1}{2} (L_{ij} n^{ij} + L_{ik} n^{ik}) \] (9)

where \( L_{ij} \) and \( L_{ik} \) are the length of edges \( IJ \) and \( IK \) of an element, \( n^{ij} \) and \( n^{ik} \) denote the external normal of edges \( IJ \) and \( IK \), respectively.

When the element is small enough, the real strain \( \varepsilon \) can be replaced by the average strain \( \bar{\varepsilon} \). We can obtain the average strain \( \bar{\varepsilon} \) in the element as

\[ \bar{\varepsilon} = \frac{1}{A} \int_{A} \varepsilon \, dA \] (10)

in which \( A \) is the area of an element.

Substituting Equation (7) into Equation (10), we can easily derive the strain of an element as:

\[ \bar{\varepsilon} = \frac{1}{2A} (u_i \otimes m^i + m^i \otimes u_i) \] (11)

where \( u_i \) is the displacement of node \( I \) for a triangular element and the summation rule is implied.

When the element is small enough, the real stress \( \sigma \) can be replaced by the average stress \( \bar{\sigma} \). According to the generalized Hooke's law, the stress component expressions of an element can be obtained for the plane stress problem

\[ \sigma_x = \frac{E}{1-\nu^2} (\varepsilon_x + \nu \varepsilon_y) \] (12)

\[ \sigma_y = \frac{E}{1-\nu^2} (\varepsilon_y + \nu \varepsilon_x) \] (13)

\[ \tau_{xy} = \frac{E}{2(1+\nu)} \gamma_{xy} \] (14)

For the plane strain problem, it is necessary to replace \( E \) by \( \frac{E}{1-\nu^2} \) and \( \nu \) by \( \frac{\nu}{1-\nu} \) in Equations (12) - (14).

**RANDOM AGGREGATE MODEL FOR RC**

Based on the Fuller grading curve, Walraven J.C et al\(^{[27]}\) put the three dimensional grading curve into the probability of any point which located in the sectional plane of specimens, and its expression as follow:

\[ P_r (D < D_0) = P_e \left( 1.065 \left( \frac{D_0}{D_{\text{max}}} \right)^7 - 0.053 \left( \frac{D_0}{D_{\text{max}}} \right)^6 - 0.012 \left( \frac{D_0}{D_{\text{max}}} \right)^5 - 0.0045 \left( \frac{D_0}{D_{\text{max}}} \right)^4 - 0.0025 \left( \frac{D_0}{D_{\text{max}}} \right)^3 \right) \] (15)

Where \( P_e \) is the volume percentage of aggregate volume among the specimens, in general \( P_e = 0.75 \), \( D_0 \) is the diameter of sieve pore, \( D_{\text{max}} \) is the maximum aggregate size.

According to (15), the numbers of coarse aggregate particles with various sizes can be obtained. By Monte Carlo method, random to create the centroid coordinates of all kinds of coarse aggregate particles, namely to generate random aggregate model.
According to the projection method, we dissect the specimens of RC with different phases of materials. Then, the phase of recycled coarse aggregate, the phase of new hardened cement, the phase of old hardened cement, and the phase of new and old interfacial transition zone (ITZ) can be judged by a computer code as Figure 2:

![Fig. 2 - Attribute recognition figure](image)

**DAMAGE MODEL OF MATERIALS**

Components of RC such as recycled coarse aggregate, new mortar, old mortar, new interfacial transition zone (New ITZ) and old interfacial transition zone (Old ITZ) are basically quasi-brittle material, whose failure patterns are mainly brittle failure.

In this paper, according to the characteristics of recycled concrete on meso-structure, the damage degradation of recycled concrete is described by the bilinear damage model, and the failure principal is the criterion of maximum tensile strain. Damage constitutive model is defined as  

\[
\bar{E} = E_0 (1 - D) \quad (0 \leq D \leq 1)
\]

shown in Figure 3, where the damage factor \( D \) can be expressed as follow

\[
D = \begin{cases} 
0 & \varepsilon < \varepsilon_0 \\
\frac{1 - \lambda \varepsilon_0}{\eta - 1} + \frac{1 - \lambda}{\eta - 1} & \varepsilon_0 < \varepsilon \leq \varepsilon_u \\
1 - \lambda \varepsilon_0 & \varepsilon_u < \varepsilon \leq \varepsilon_u \\
1 & \varepsilon > \varepsilon_u 
\end{cases}
\]

where \( f_t \) is the tensile strength of material, the residual tensile strength is defined as \( f_{tr} = \lambda f_t \), the residual strength coefficient \( \lambda \) ranges from 0 to 1, the residual strain is \( \varepsilon_r = \eta \varepsilon_0 \), \( \eta \) is the residual strain coefficient, the ultimate strain is defined as \( \varepsilon_u = \xi \varepsilon_0 \), where \( \xi \) is ultimate strain coefficient, \( \varepsilon \) is principal tensile strain of element.

![Fig. 3 - Bilinear damage model](image)
The flowchart of the BFEM code on meso-damage analysis for recycled concrete materials is as Figure 4.

**Fig. 4 - The flowchart of the BFEM code on meso-damage analysis for RC**

**NUMERICAL EXAMPLE**

Based on the uniaxial compression experiments, uniaxial compressive strengths for RAC specimens are investigated using the BFEM as shown in Figure 5.

According to the test results got from the experiment, material parameters of recycled concrete are selected, which is completely coherent with model material used in the experiment. Material parameters of numerical simulation are shown in table 1. The recycled concrete specimens were loaded by displacement steps.
Table 1 - Material parameters of uniaxial tensile tests

<table>
<thead>
<tr>
<th>Materials</th>
<th>Elastic modulus/GPa</th>
<th>Position ratio</th>
<th>Tensile strength/MPa</th>
<th>$\lambda$</th>
<th>$\eta$</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural coarse aggregate</td>
<td>70</td>
<td>0.16</td>
<td>10</td>
<td>0.1</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Old ITZ</td>
<td>13</td>
<td>0.2</td>
<td>2</td>
<td>0.1</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>Old cement mortar</td>
<td>25</td>
<td>0.22</td>
<td>2.5</td>
<td>0.1</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>New ITZ</td>
<td>15</td>
<td>0.2</td>
<td>2</td>
<td>0.1</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>New cement mortar</td>
<td>30</td>
<td>0.22</td>
<td>3</td>
<td>0.1</td>
<td>4</td>
<td>10</td>
</tr>
</tbody>
</table>

For the size 100mm×100mm×100mm of compression specimen, the numbers of coarse aggregate particles can be obtained according to Equation (15). By Monte Carlo method, random to create the centroid coordinates of all kinds of coarse aggregate particles, namely to generate random aggregate model as Figure 6:
The uniaxial compressive stress - strain curve of recycled concrete is getting as shown in Figure 7. The compressive strengths of the four specimens were 20.58MPa, 20.48MPa and 20.74MPa. The uniaxial compressive strength average of the specimen group is 20.60MPa. The result BFEM on meso-damage analysis for RC is consistent with the test results\cite{28}.

![Fig. 7 - Stress-strain curve of RC with 100mm×100mm×100mm](image)

The propagation process of cracks of the RC specimen with 100mm×100mm×100mm by uniaxial compression is shown in Figure 8.

![Fig. 8 - Propagation process of cracks of the RC specimen with 100mm×100mm×100mm](image)
For the size 150mm × 150mm × 150mm of compression specimen, the numbers of coarse aggregate particles can be obtained according to Equation (15). By Monte Carlo method, random to create the centroid coordinates of all kinds of coarse aggregate particles, namely to generate random aggregate model as Figure 9:

![Random aggregate model with 150mm × 150mm × 150mm](image)

Fig. 9 - Random aggregate model with 150mm × 150mm × 150mm

The uniaxial compressive stress - strain curve of recycled concrete is getting as shown in Figure 10. The compressive strengths of the four specimens were 19.65MPa, 19.49MPa and 19.58MPa. The uniaxial compressive strength average of the specimen group is 19.57MPa.

![Stress-strain curve of RC with 150mm × 150mm × 150mm](image)

Fig. 10 - Stress-strain curve of RC with 150mm × 150mm × 150mm

The propagation process of cracks of the RC specimen with 150mm × 150mm × 150mm by uniaxial compression is shown in Figure 11.

For the size 300mm × 300mm × 300mm of compression specimen, the numbers of coarse aggregate particles can be obtained according to Equation (15). By Monte Carlo method, random to create the centroid coordinates of all kinds of coarse aggregate particles, namely to generate random aggregate model as Figure 12:
The uniaxial compressive stress - strain curve of recycled concrete is getting as shown in Figure 13. The compressive strengths of the four specimens were 18.77MPa, 18.68MPa and 18.83MPa. The uniaxial compressive strength average of the specimen group is 18.76MPa.

The propagation process of cracks of the RC specimen with 300mm×300mm×300mm by uniaxial compression is shown in Figure 14.
The size effects of mechanics properties of RC under uniaxial compressive loading are shown in table 2.

Fig. 13 - Stress-strain curve of RC with 300mm×300mm×300mm

Fig. 14 - Propagation process of cracks of the RC specimen with 300mm×300mm×300mm
Table 2 - Different sizes of recycled concrete compressive strength under uniaxial compressive loading

<table>
<thead>
<tr>
<th>Different sizes of recycled concrete</th>
<th>Compressive strength/MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>100mm×100mm×100mm</td>
<td>20.60</td>
</tr>
<tr>
<td>150mm×150mm×150mm</td>
<td>19.57</td>
</tr>
<tr>
<td>300mm×300mm×300mm</td>
<td>18.76</td>
</tr>
</tbody>
</table>

CONCLUSION

In the direct computation of the recycled concrete effective properties the base force element method (BFEM) on damage mechanics is used at meso-level. The recycled concrete is taken as five-phase composites consisting of natural coarse aggregate, new mortar, new interfacial transition zone (ITZ), old mortar and old ITZ on meso-level in this paper. The random recycled aggregate model is used for the numerical simulation of uniaxial compressive performance of recycled concrete.

The results obtained from the numerical simulations are then compared with experimental data found in the literature. The results of the BFEM computations show a good agreement to the experimental data. It can be seen that the damage occurs in the old interfacial transition zone between the old mortar material and the natural aggregate particles, and the new interfacial transition zone between the old mortar material and the new mortar material. The coalescence of the damaged areas, which finally results in the failure of the material, is also to be clearly observed within the interfacial transition zone. The compressive strength of recycled concrete will decrease with increasing specimen size.

It shows that the BFEM with the meso-damage model is feasible and effective to study failure process and mechanical parameter of RC. The presented method provides further insight into the compressive behaviour and the size effects of recycled concrete, with an emphasis on the strength prediction of the composites. Reliable prediction of material strength is of great use in the civil engineering field, allowing one to reduce experimental testing in expensive wallets and to avoid the usage of conservative empirical formulae.

ACKNOWLEDGMENTS

This work is supported by the National Science Foundation of China, No. 10972015, 11172015 and the pre-exploration project of Key Laboratory of Urban Security and Disaster Engineering, Ministry of Education, Beijing University of Technology, No. USDE201404.

REFERENCES


