INVESTIGATIONS OF STRUCTURAL STRENGTH AND OPERATING PROPERTIES OF WAVE-RING GASKETS

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ABSTRACT

The paper deals with the stress-strain analysis and estimation of operating properties of wave-ring gaskets. The investigations are carried out using simplified analytical approach. A cylindrical shell of constant mean thickness is introduced to simulate the gasket. It is assumed that the shell is simply supported at the inner surface of the seat. The influence of certain geometric and assembly parameters on the strength and leak tightness of the closure is analytically investigated. Certain selected parameters of the closure are examined versus applied operating pressure for the assumed materials of the gasketed members and fixed another several data of the gasket. The results of analysis are illustrated by a numerical example. The region of admissible solutions for the gasket parameters such as dimensions, applied assembly clearance and pressure is determined. The results are presented in dimensionless variables in order to generalize the conclusions. The analytical solution is verified by FEM calculations and compared with the experimental results.

Keywords: high-pressure closures, wave-ring gasket, stress-strain analysis, leak tightness.

INTRODUCTION

Modern power installations and advanced chemical equipment, operating usually at extremely high pressures, require reliable and hard-wearing sealing systems. Metal gaskets are sometimes used to seal the heads of reactors or boilers and pipe connections (Freeman, 1960). The metal gaskets give satisfactory sealing service and additionally they are chemical resistant, moisture-proof and stable to heat. Temporary closures with self-sealing metal wave-ring gasket are applied in heavy-duty systems, in particular in those of great diameter.

The paper follows earlier contributions of the authors devoted to the sealing systems with metal gaskets for the high-pressure applications. Several computational models of the wave-ring gasket were investigated (Rys et. al, 2006) with the aim to select the simplest and most effective one but sufficiently precise, which could be applied in the engineering approach. The results of the analysis confirm that the shell model of constant thickness simply supported at both ends at the inner surface of the seat is appropriate to describe the wave-ring gasket and leads to the good agreement with FEM modeling. However, the seat must be considered as a thick-walled cylinder loaded by shear forces and by the internal operating pressure.

The created analytical model was used to investigate the influence of several gasket parameters on its strength and sealing properties (Trojnacki, 2011). The results of analysis were compared with FEM (ANSYS\textsuperscript{*}) calculations and were verified by experimental results obtained under assembly conditions in the gasket installation state (Szybinski, Trojnacki, 2011, 2013, 2015).
ENGINEERING EXAMPLE AND SERVICE CONDITIONS OF THE SEALING

The wave-ring gasket is a certain type of self-sealing gaskets for very high-pressure equipment. Engineering example of the joint with wave-ring gasket between the vessel wall and the reactor head is shown in Fig. 1. The closure is successfully applied (Report TPP-5, 2000) in the heavy-duty chemical installation working at the pressure of 200 [MPa].

The yield stress of the wave-ring gasket material must be significantly lower than the yield stress of the seat material to ensure the proper effectiveness of the joint. The gasket is usually made of the low-carbon soft steel subjected to the heat refining to the yield limit of approximately 400 [MPa]. Sometimes the gasket is fabricated of copper, brass, or some other moderately soft metal. The high-quality chromium-nickel-molybdenum steel hardened to the yield limit of minimum 750 [MPa] is mostly used for the seats. The gasket must be made slightly oversized, so that an interference fit is obtained in the seat. The experience gathered with existing and properly running closures leads to the conclusion that the degree of the preferred radial interference between the external wave surface of the gasket and the cylindrical internal surface of the seat may vary from 0.5 [%] to 2.5 [%] and depends on the yield limit of the gasket material and on the operating pressure.

Fig. 1 - Engineering example of the joint between the vessel wall and the reactor head: 1 – head, 2 – sectional clamping rings, 3 – wave-ring gasket, 4 – locating pin, 5 – grips, 6 – cylindrical shell

Fig. 2 - Distribution of initial load at the contact region of the gasket

Under assembly conditions the initial contact pressure $q$ appears at the portion $e$ of a wave surface due to the assembly interference, thus making the initial seal just before the operating pressure $p$ is applied to the closure (Fig. 2). The working pressure is exerted on the entire inner surface, forcing a seal on the two outer radii. The initial assembly pressure $q$ increases as the stiffness of the gasket is much less than that of the seat. Because of its specific features such a sealing may be applied in equipment working at extremely high pressure, much more than 100 [MPa].

The difference in the yield limits of materials of the gasketed members and the value of the radial interference fit are the key parameters of the closure. They have an essential influence on the width of the contact zone of required size and on the related contact pressure value.
which provides the leak tightness of the connection under the operating pressure $p$. In the simple preliminary calculations of practical engineering applications of the wave-ring gaskets the non-leakage condition is usually formulated as

$$q_{m \text{ opr}} > R_{0.2} \geq 2p,$$

where the average contact pressure $q_{m \text{ opr}} = 2q_{\text{max \ opr}}/3$ as for the parabolic elastic distribution due to the Hertz theory. Because of highly approximate estimation of the contact stress distribution the average value of the distribution is introduced into equation (1). It means that the average contact pressure $q_{m \text{ opr}}$ in the contact zone under operating conditions must be greater than the yield limit $R_{0.2}$ of the gasket material and should at least twice exceed the operating pressure $p$. The magnitude of the yield limit of the gasket material and the size of the radial interference fit are related and depend strongly on the applied operating pressure.

The influence of several parameters of the connection on its sealing properties in the operating conditions was investigated in dimensionless variables (Trojnacki, 2011). The dimensionless non-leakage parameter was defined as $\psi = q_{m \text{ opr}}/2p$ and the simplified analytical approach was applied to obtain the solution.

Wave-ring gaskets give satisfactory service where the vessel or piping need not to be opened very often. In the opposite case they are somewhat impractical as they sometimes become so tightly wedged that the vessel head can be removed only with extreme difficulty. When this jamming occurs, the gasket usually must be discarded, as the crests have been flattened and scarred. Where the closure must be opened and closed fairly often the gasket is sometimes made of hardened steel.

In conclusion it should be noted that there is no design objectives and constraints which are collected and can be recommended in the design procedures of the closures with wave-ring gaskets. The parameters of the new connections are selected basing on the experience gained during operating and improving of the existing sealing systems. Moreover, in each individual case of technical application a set of expensive and time-consuming calculations and experimental tests should be carried out to confirm the accuracy of the choice. The sudden unexpected decrease of the leak tightness of the very high-pressure installation operating with caustic dangerous packing may cause serious damages.

**SIMPLIFIED ANALYTICAL SOLUTION**

Geometry of the investigated closure is presented in Fig. 3. For the analytical calculations the gasket is replaced by a cylindrical shell of a constant thickness $t$ and of a mean radius $r$, where $t$ is defined as an arithmetic average of three extreme values of the gasket thickness. The analytical investigations of the gasket are then based on a simple shell model of length $2l$ simply supported around the circumference at a contact with the seat. The hinges are selected at the top points of the wave surface where the contact with the seat occurs. The spacing of the supports is $2h$ (Fig. 4). It is assumed that except a small region in the vicinity of supports the shell is purely elastic. The analytical calculations verified by FEM modeling lead to the conclusion that the influence of external parts of the gasket outside of the supports (broken line in Fig. 4) is negligible. The relative difference in maximum equivalent stress $\sigma_{eq}$ at the inner surface of the gasket is for this simplified model less than 2 [%] with respect to the complete shell model with attached external segments. The results of the analysis confirm that the shell model of constant thickness simply supported at both ends at the inner surface of the seat in the cross-sections of coordinates $x = -h$ and $x = h$ is appropriate to describe the wave-ring gasket and leads to the good agreement with FEM modeling. At the assembly conditions...
the shell is loaded by shear forces $Q_x$ at the supports only. At the service conditions the shell is additionally loaded by an operating pressure $p$ acting at the inner cylindrical surface and at the edge plain surfaces. The seat must be considered as a thick-walled cylinder loaded by shear forces and by the internal pressure $p$.

Fig. 3 - Geometry of the closure with wave-ring gasket in assembly conditions (operating pressure $p = 0$)
Fig. 4 - Simplified computational model of the wave-ring gasket in service conditions

The applied approach and permissible simplifications depend on the geometric proportions of the element (Wozniak, 2001, Zyczkowski, 1988). In this case the gasket must be solved on the basis of the bending shell theory and some terms in the differential equation of deflection could not be neglected. Under the assumptions as for the cylindrical axisymmetric shell of mean thickness $t$, mean radius $r$ and small radial deflections $w$ with respect to the thickness $t$, the differential equation of deflection takes the well-known form (Kozlowski, 1968, Timoshenko-Woinowsky-Krieger, 1962)

$$\frac{d^4w}{dx^4} + \alpha \frac{d^2w}{dx^2} + \beta^4w = \delta,$$

where $\alpha = v/r^2$, $\beta = \frac{4}{12}(1-v^2)/r^2 t^2$, $\delta = 12 p(1-v^2)/Et^3$ and $v$ and $E$ stand for the Poisson’s ratio and Young’s modulus, respectively. The constants of integration can be determined from the boundary conditions as for the simply supported shell and eventually the shell model of the gasket may be easily solved. The principle stresses $\sigma_z$, $\sigma_\phi$ and $\sigma_x$ can be expressed in terms of the shell parameters and the operating pressure $p$. The maximum equivalent von Mises stress $\sigma_{eq}$ occurs at the inner surface at the gasket midpoint and equals

$$\sigma_{eq} = \sqrt{\sigma_z^2 + \sigma_\phi^2 + \sigma_x^2 - \sigma_z \sigma_\phi - \sigma_z \sigma_x - \sigma_\phi \sigma_x}.$$

The sectional seats (Fig. 3) are usually executed directly in thick vessel walls like in the exemplary closure shown in Fig. 1. These walls are designed for very high pressure and their thickness ratio is of great value, nevertheless the resultant displacement $w_h$ (negative) at the support after assembly is different from the designed radial interference $\delta$. The resultant displacement $w_h$ was finally determined basing on the thick-walled cylinders theory applied to the shell model of the gasket and to the seat, respectively (Walczak, 1973)
\[ w_s = \frac{\delta}{2} \frac{\kappa_2^2 - 1}{\kappa_1^2 - 1} \left[ \kappa_1^2 (1 - \nu) + 1 + \nu \right], \]  

(4)

where \( \kappa_1 = (2r + t)/(2r - t) \) is the ratio between the outer and inner radii of the shell, \( \kappa_2 \) is the ratio between the outer and inner radii of the seat and \( \kappa \) stands for the thickness ratio of the entire unit. The pressure \( q \) at the contact surface of the cylinders corresponding to the interference \( \delta \) is

\[ q = \frac{E \delta}{2r + t} \frac{(\kappa_1^2 - 1)(\kappa_2^2 - 1)}{\kappa^2 - 1} \]  

(5)

and may be additionally used to estimate the shear force at the support

\[ Q_x = \frac{1}{2} 2q l. \]  

(6)

Special attention must be paid to the interaction conditions between the wave working surface of the gasket and the cylindrical surface of the seat (Fig. 2). Initial assembly interference \( \delta \) is usually of great value (more than 0.5 [‰]) and the difference in mechanical properties of the materials may cause the plastic process in the gasket. For this reason the Hertz theory can not be used directly to calculate the width of the contact region, where the sealing is obtained.

Hypothetical elastic distribution of the contact stress \( q(x) \) under operating pressure \( p \) is depicted in Fig. 5a. In these regions, where the stress \( q \) calculated from the initial elastic Hertz distribution is considerably beyond the yield limit \( R_{0.2} \) of the gasket material, the plastic process must appear. As a result a redistribution of the initial elastic stress \( q(x) \) must occur and finally a resultant stress distribution \( q_{pl}(x) \) must appear which allows for the plastic deformations (Fig. 5b).

Fig. 5 - Distribution of stress at the contact region of the gasket and the seat: (a) – hypothetical elastic (parabolic) with respect to the Hertz theory, (b) – changed (partially-linear) with respect to the plastic deformation

The simple and rough estimation of the width \( e \) of the contact region is derived under the assumption that the gasket material satisfies pure elastic-plastic stress-strain relationship and
that the seat material is perfectly rigid. Moreover, it is assumed that the plastic deformation begins when the gasket is subjected to the load \( Q(x) \) which produces the stress \( q_{\text{max}} = R_{0.2} \). Under the load which produces the stress \( q_{\text{max}} \) which is \( n \) times greater than the yield limit \( R_{0.2} \) \( (q_{\text{max}} = nR_{0.2}) \), the elastic parabolic distribution \( q_{\text{el}}(x) \) corresponding to the load \( Q_{\text{el}}(h) \leq Q_x(h) \) will exist in the contact surface, for which the maximum stress equals \( q_{\text{max}} \). The width of the contact region satisfying this elastic Hertz distribution \( q_{\text{el}}(x) \) with respect to the distribution \( q(x) \) is

\[
e_{\text{el}} = \frac{1}{n} e. \tag{7}
\]

The surplus shear load \( \Delta Q_x(h) = Q_x(h) - Q_{\text{el}}(h) \) produces the plastic process which leads under the applied assumptions to the plastic deformation. A new partially-linear stress distribution \( q_{\text{pl}}(x) \) is introduced to model the problem (Fig. 5b). The width of the additional plastic zone is determined from the condition that the entire shear force \( Q_x(h) \) does not change

\[
e_{\text{pl}} = \frac{2}{3} \left( n - \frac{1}{n} \right) e. \tag{8}
\]

The total width of the contact region is then a sum of the elastic contact \( e_{\text{el}} \) (7) and plastic contact \( e_{\text{pl}} \) (8)

\[
e_{\text{total}} = \left( 2n + \frac{1}{n} \right) \frac{e}{3}. \tag{9}
\]

The suggested simplified distribution of the contact stress \( q_{\text{pl}}(x) \) must be treated as a highly approximate one. The assumption of the pure elastic-plastic stress-strain curve of the gasket material leads to the overestimation of the total width of the contact region.

**NUMERICAL MODELLING (FEM)**

Three parts of the investigated structure are assembled with an interference fit existing between the external wave surface of the gasket and the inner cylindrical surface of both seats. The shape of the structure and relatively high radial interference, which should preserve the leak tightness of the junction under operating pressure, result in high stresses and stress gradients distributed over the small zones in the vicinity of contacting areas. The problem considered in the paper concerns the contact of two deformable bodies and belongs to the class of flexible-to-flexible contact, for which the analytical solutions are known only in a limited number of simple cases. The contact tasks are nowadays solved numerically, by means of the finite element approximation. The ANSYS® code (Release 8.0, 2003) was used to solve the problem in the present paper and to get the strain and stress distributions in all contacting bodies. Also the contact pressure distribution and the width of the contact zone were the results of that analysis.

The gaskets were made of the soft 25CrMo4 (1.7218) chromium-molybdenum normalized steel and the seats were made of 42CrMo4 (1.7225) high-carbon chromium-molybdenum steel toughened to \( R_m = 1000 \) [MPa]. The mechanical properties of 25CrMo4 and 42CrMo4 steels were verified experimentally. Two cylindrical specimens were subjected to the same heat treatment as the corresponding elements, and prepared for the static tensile tests. The obtained real load-displacement curves \( F = f(\Delta l) \) for both materials are shown in Fig. 6. The

strength properties of both materials calculated as arithmetic means of the two tests are given in Table 1. Experimentally-verified Brinell hardness number of the sealing surfaces of the gaskets was of $250 \div 280$ [BHN] and Rocwell hardness number of the seats was of $45 \div 48$.

<table>
<thead>
<tr>
<th>Steel</th>
<th>$E$ [MPa]</th>
<th>$R_{0.05}$ [MPa]</th>
<th>$R_{0.2}$ [MPa]</th>
<th>$R_m$ [MPa]</th>
<th>$\varepsilon_{0.05}$ [%]</th>
<th>$\varepsilon_{0.2}$ [%]</th>
<th>$\varepsilon_{\max}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>25CrMo4 (N)</td>
<td>$2.014 \times 10^5$</td>
<td>253.59</td>
<td>260.30</td>
<td>523.38</td>
<td>0.185</td>
<td>0.359</td>
<td>15.338</td>
</tr>
<tr>
<td>42CrMo4 (T)</td>
<td>$2.064 \times 10^5$</td>
<td>809.12</td>
<td>812.46</td>
<td>918.50</td>
<td>0.460</td>
<td>0.711</td>
<td>8.802</td>
</tr>
</tbody>
</table>

Two approximations of the real stress-strain curves $\sigma = f(\varepsilon)$ of the materials used for the gasket and the seats were adopted in the paper. The parameters of the first multi-linear approximation (Fig. 7a) were calculated from the set of equations (10)

$$
\begin{align*}
R_m - R_{0.2} - E_{12}(\varepsilon_{\max} - \varepsilon_{0.2}) &= 0, \\
R_{0.2} - R_{0.05} - E_{11}(\varepsilon_{0.2} - \varepsilon_{0.05}) &= 0, \\
R_{0.05} - E\varepsilon_{\max} - E_{11}(\varepsilon_{0.05} - \varepsilon_s) &= 0, \\
S_s - E\varepsilon_s &= 0, \\
\end{align*}
$$

and are gathered in Table 2. The parabolic modelling was suggested beyond the yield limit in the second approximation (Fig. 7b). The parabola containing the point of coordinates $\varepsilon_{0.2}, R_{0.2}$ and reaching the maximum value at the point $\varepsilon_{\max}, R_m$ was applied to describe the tensile behaviour of the material. For the numerical calculations the parabola was replaced by several (twenty) segments of different slope but of equal length in the orthogonal projection at the $\varepsilon$ axis. Such approximation enables direct introduction of the nonlinear material properties in the software module ANSYS® which was used in the paper. Both approximations are conservative beyond the yield limit, although the second one is more precise. The similar
approximation was proposed for the material used for the seats. However, the plastic deformations were present in the softer part of the junction, namely in the gasket only.

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_t$ [MPa]</th>
<th>$\varepsilon_s$ [%]</th>
<th>$\varepsilon_s$ [%]</th>
<th>$E_1$ [MPa]</th>
<th>$E_2$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gasket 25CrMo4 (N)</td>
<td>251.35</td>
<td>0.1248</td>
<td>0.159</td>
<td>3854.02</td>
<td>1756.37</td>
</tr>
<tr>
<td>Seat 42CrMo4 (T)</td>
<td>808.26</td>
<td>0.3916</td>
<td>0.511</td>
<td>1329.88</td>
<td>1310.63</td>
</tr>
</tbody>
</table>

Moreover, it was assumed that the relationship between the equivalent stress $\sigma_{eq}$ and equivalent strain $\varepsilon_{eq}$ under complex stress states $\sigma_{eq} = f(\varepsilon_{eq})$ is the same as the stress-strain relationship under uniaxial tensile loading $\sigma = f(\varepsilon)$. The stress intensity is derived from the von Mises yield criterion and the strain intensity is defined as (Zyczkowski, 1988)

$$
\varepsilon_{eq} = \frac{2}{\sqrt{3}} \sqrt{(\varepsilon_z - \varepsilon_p)^2 + (\varepsilon_p - \varepsilon_x)^2 + (\varepsilon_x - \varepsilon_z)^2},
$$

where $\varepsilon_z$, $\varepsilon_p$ and $\varepsilon_x$ are the principal strains at a certain point of the cross-section.

The so-called surface-to-surface contact elements are recommended to use in the case of contact of deformable bodies which are assembled with certain interference. Such elements can be of higher order approximation with inside nodes introduced. This provides better results for many engineering applications and enables modelling complex, curved shapes of bodies being in touch. The above contact elements are defined on the surface geometry and need several constants and options to set prior to the analysis. The augmented Lagrangian method was used with contact detection points localized in nodal points in the considered problem. Also the contact stiffness updated at each iteration step based on the current mean stress was applied. The Coulomb friction law was used in the analysis.

Fig. 7 - Approximations of the real stress-strain curves $\sigma = f(\varepsilon)$ (displayed in the stretched scale)
In general, the analysed structure should be modeled as a 3D object, which results in a very big numerical model and demands a lot of time to get the solution. The size of the task can be reduced when the ideal geometry of the junction is presumed. Then the structure is assumed to be axially-symmetric and only a half-part of the cross-section is considered. The high accuracy of numerical results was provided by application of the 8-node quadrilateral axisymmetrical finite elements, which are well-suited for irregular meshes and tasks with elastic and plastic deformations. The finite elements are accompanied with the contact elements introduced on these lines were the contact is expected. Like in the majority of nonlinear problems the number of applied finite elements should be rather high and the dense meshes should be used in order to keep the solution error within the acceptable range, in particular in the vicinity of the contact zones. The mesh in this area should be dense enough to give satisfying results, while the mesh on the outer unloaded surfaces can be rather rough (Wriggers, 2002).

In the first numerical approach (FEM 1) the interferences between the gasket and both sectional seats were arranged by means of the thermal method. For the calculation purpose the gasket was first cooled down and after inserting into the seats and expanding the appropriate interference fits were obtained in the closure. The thermal simulation of the assembly process leads to the same (symmetric) results in displacements, stresses and contact pressure in both half-parts of the axial cross-section.

The second numerical approach (FEM 2) follows the assembly process performed on the stand during the experiment. The nonlinear contact analysis was divided into two steps. In the first step the gasket was pressed into the bottom sectional seat (supported vertically), while in the second one the upper seat was pressed down until the edges of both seats have got in touch. In the second step the bottom edge of the bottom seat and the bottom edge of the gasket were blocked against the vertical displacement. The symmetry of results with respect to the middle surface disappears in this case.

![Example of the mesh of finite elements, division of the closure into parts and illustration of the boundary conditions](image1.png)

![Distribution of equivalent stress $\sigma_{eq}$ for the initial interference $\delta = 1.0$ [%] under the load $p = 100$ [MPa] at the contact surfaces of the gasket and the seat](image2.png)
Even with the introduced simplifying assumptions and restrictions concerning the geometry and loading the numerical solution is time consuming and difficult to obtain due to the numerical instability. The size of the finite elements in the anticipated contact zone and the size of the load step should be chosen with the particular care in order to avoid the convergence problems. Several numerical trials have been carried out to get the final mesh, which is shown in Fig. 8. As a final criterion for the choice of the element size the compromise between the calculation time and the approximation error has been established. The criterion used for the approximation error $\Delta$ is based on the comparison between the maximum absolute value of the radial stress $\sigma_{z\text{max}}$ and the maximum contact pressure $q_{\text{max}}$ and accepts the mesh for which the discrepancy is less than 5 [%] for each load step (Stein, 2003)

$$\Delta = \left| \frac{\sigma_{z\text{max}} - q_{\text{max}}}{q_{\text{max}}} \right| \times 100 \% \leq 5 \%.$$ (12)

Distribution of equivalent stress $\sigma_{eq}$ for the initial interference $\delta = 1.0$ [%] under the load $p = 100$ [MPa] at the contact surfaces of the gasket and the seat is depicted in Fig. 9.

The exemplary distribution of the contact pressure $q$ in the closure is presented in Fig. 10 for the same interference $\delta$ and the same operating pressure $p$. The numerical results are obtained applying FEM 2 procedure for the upper part of the wave-ring gasket which is assembled in the second step.

The friction in contact zones was included in the proposed finite element model with the coefficient of friction assumed $\mu = 0.25$ (Krukowski, Tutaj, 1987).

**DEFINITION OF DIMENSIONLESS PARAMETERS OF THE GASKET**

The dimensionless variables were applied to investigate the strength and operating properties of the wave-ring gaskets. The calculations were based on the simplified analytical shell model of the gasket. The critical section of the gasket is placed at its centre of symmetry where the equivalent stress $\sigma_{eq}$ is maximum at the inner radius under the operating pressure (point $A$ in
Fig. 3). The strength of the gasket were estimated using the dimensionless parameter \( \sigma \) defined as

\[
\sigma = \frac{\sigma_{eq}}{R_{0.2g}} \quad \text{– dimensionless strength parameter,}
\]

where \( R_{0.2g} \) [MPa] stands for the yield point of the gasket material. The leak tightness of the closure depends on the stress \( q \) acting at the toroidal wave working surface of the gasket (Fig. 2). Practically verified recommendations used in the design projects of the high-pressure chemical equipment was applied to investigate the leak tightness of the joint. It was assumed that the closure is leakproof if the average contact pressure \( q_{m\,op} \) satisfies condition (1). The dimensionless parameter \( \psi \)

\[
\psi = \frac{q_{m\,op}}{2p} \quad \text{– dimensionless leak tightness parameter,}
\]

was introduced to estimate the functional quality of the closure with respect to non-leakage condition. Dimensionless operating pressure was defined as

\[
\tau = \frac{p}{R_{0.2g}} \quad \text{– dimensionless operating pressure.}
\]

Another dimensionless variables subjected to change during the analysis were introduced as

\[
\begin{align*}
\chi &= \frac{t}{h} \quad \text{– dimensionless geometry ratio of the gasket,} \\
\delta &= \text{relative radial interference (in per milles),} \\
\gamma &= \frac{t}{r} \quad \text{– dimensionless average thickness of the gasket,} \\
\rho &= R_1/t \quad \text{– dimensionless radius of the working wave surface,}
\end{align*}
\]

where all quantities are defined in Figs. 3 and 4. The geometry of the real gasket is presented in Fig. 3 and the dimensions of the analytical model of the gasket used in the analytical approach are shown in Fig. 4.

**ANALYSIS OF STRENGTH AND OPERATING PROPERTIES OF THE GASKET**

The detailed analytical calculations were carried out for the dimensions of the gasket, which were applied in the earlier investigations (Szybinski, Trojnacki, 2015), i.e. \( t = 8.73 \) [mm], \( r = 56.87 \) [mm], \( R_1 = 14 \) [mm], \( 2h = 20 \) [mm], \( 2l = 35 \) [mm] and \( = R_{0.2g} = 260.30 \) [MPa] like for the normalized 25CrMo4 material. The average thickness \( t \) of the gasket was calculated as the arithmetic mean of its three extreme values and the average radius \( r \) was a result of the assumed thickness \( t \) and of the inner radius \( \Omega A \) of the seat (Fig. 3). The thickness coefficient of the seats was set at the maximum admissible value \( \beta = 2.00 \).

The strength of the gasket and its leak tightness were investigated versus dimensionless operating pressure \( \tau \) in the range \([0, 1]\), i.e. the maximum operating pressure was assumed to be equal to the yield limit of the gasket material. The analysis was based on the variation of one of the earlier defined geometric or assembly dimensionless parameters of the gasket \( \chi, \delta, \gamma \) and \( \rho \) in the technical acceptable range while the remaining parameters were kept at a constant mean level. Dimensionless geometry ratio of the gasket was changed in the range \( \chi = 0.06 \div 1.60 \), relative radial interference was changed in the range \( \delta = 0.5 \div 3.0 \) [%], dimensionless average thickness of the gasket was modified in the range \( \gamma = 0.05 \div 0.30 \) and dimensionless radius of the working wave surface was subjected to variation in the range \(\rho = 0.5 \div 3.0 \). The mean values of dimensionless parameters refer to the geometry of the closure used in the experimental investigations \( \chi = 0.8730, \delta = 1.0 \) [%], \( \gamma = 0.1535 \) and \( \rho = 1.6037 \).
Initial analytical calculations based on the standard thin-walled shell theory (Kozlowski, 1968, Timoshenko-Woinowsky-Krieger, 1962) and preliminary FEM simulation lead to the conclusion that the maximum stress appears at the cross-section placed at the gasket centre of symmetry. The maximum equivalent stress $\sigma_{eq}$ appears at this cross-section which can be seen in Fig. 11 where the dimensionless strength parameter $\sigma$ is presented at four characteristic points versus the dimensionless operating pressure $\tau$: at the inner ($\sigma_i$) and at the outer ($\sigma_o$) radii of the gasket, in the central cross-section ($x = 0$) and in the extreme cross-section ($x = h$).

Special attention must be paid to the strength of the gasket at the assembly conditions and for relatively low pressure $\tau$ which occurs during starting the high-pressure equipment. At the assembly conditions the gasket is loaded by the shear forces $Q_x$ only which are acting at the supports (Fig. 4). The shear forces are usually of great value as they are caused by the initial radial assembly interference $\delta$. The interference $\delta$ must be of great value too because it should ensure the initial leak tightness of the closure necessary while the installation is being filled with working medium. During the assembly a local plasticity processes occur at the wave surfaces of the gasket. It should be noted that when the contribution of shear forces $Q_x$ in the loading of the gasket is large and the contribution of the operating pressure $\tau$ is small, the maximum strength parameter $\sigma$ appears at the central cross-section at the outer radius $\sigma_o(0)$.

The above effect can be seen in Fig. 11 where the maximum strength is moving to the inner radius $\sigma_i(0)$ only for the pressure $\tau > 0.08$.

The magnitude of pressure $\tau$ for which the maximum strength parameter $\sigma$ changes its localization depends strongly on the applied initial assembly interference fit $\delta$. The relation between the strength parameter at the inner radius $\sigma_i(0)$ (bold lines) and the strength parameter at the outer radius $\sigma_o(0)$ (fine lines) and the pressure $\tau$ is fragmentary presented in Fig. 12 for several interference fits. It appears, however, that even for the maximum practical value of the interference fit on the order of 2.5 [%] the maximum strength parameter $\sigma$ moves from the outer to the inner radius for the pressure $\tau > 0.23$. For this reason the equivalent stress acting at the inner radius was used to estimate the gasket strength.

Dimensionless strength $\sigma$ at this point for different values of the dimensionless average thickness $\gamma$ of the gasket and fixed values of $\chi$, $\delta$, and $\rho$ versus the pressure $\tau$ is shown in Fig.
13. Permissible range of parameters is for $\sigma \leq 1$ because the loading of the gasket must be lower as the yield limit $R_{0.2g}$ of its material. It appears, that the influence of the investigated parameter $\gamma$ on the gasket strength $\sigma$ is small in the assumed range of other parameters. The closure may be loaded with the pressure $\tau < 0.25$ for which the condition $\sigma \leq 1$ is satisfied.

The influence of the initial assembly interference fit $\delta$ on the strength of the gasket is examined in Fig. 14. The increase of the interference fit $\delta$ produces a simultaneous decrease of the operating pressure $\tau$ for which $\sigma \leq 1$. The negative interference fit has an unloading effect on the stress distribution caused by the pressure $\tau$. The large magnitude of the interference fit $\delta$ in connection with the large value of the thickness coefficient $\beta$ of the seats causes that for $\delta = 2.0$ [‰] the strength of the gasket exceeds $\sigma = 1$ at certain places already during the mounting process of the closure.

![Fig. 13](image1.png)

Fig. 13 - Dimensionless maximum strength $\sigma$ of the gasket versus dimensionless operating pressure $\tau$ for different dimensionless parameters $\gamma$. Dimensionless parameters $\chi$, $\delta$ and $\rho$ being fixed at the average values

![Fig. 14](image2.png)

Fig. 14 - Dimensionless maximum strength $\sigma$ of the gasket versus dimensionless operating pressure $\tau$ for different dimensionless parameters $\delta$. Dimensionless parameters $\chi$, $\gamma$ and $\rho$ being fixed at the average values

The ratio of cross-sectional dimensions $\chi$ has a distinct influence on the strength of the gasket. The examination of the relationships in Fig. 15 leads to the conclusion that the increase of the parameter $\chi$ from 0.6 to 1.6 results in the significant increase of the gasket load capacity $\tau$.

![Fig. 15](image3.png)

Fig. 15 - Dimensionless maximum strength $\sigma$ of the gasket versus dimensionless operating pressure $\tau$ for different dimensionless parameters $\chi$. Dimensionless parameters $\delta$, $\gamma$ and $\rho$ being fixed at the average values
It is assumed that because of the highly approximate assumption of the stress distribution in the contact region the leak tightness is preserved for the parameter $\psi \geq 2$. The average stress $q_m$ calculated on the basis of purely elastic Hertz approach is in this case four times greater than the applied operating pressure $p$. The influence of the dimensionless parameters $\chi$, $\delta$, $\gamma$ and $\rho$ on the leak tightness parameter $\psi$ is presented in Figs. 16 ÷ 19. The general tendency of the leak tightness is to decrease with the increase of the operating pressure.

The increase of the parameter $\gamma$ (the increase of the gasket thickness) has a negative effect (Fig. 16). The examination of the relationships in Figs. 17 and 18 exhibits that the variation of the parameters $\delta$ and $\chi$ change distinctly the leak tightness parameter $\psi$ of the joint.

The variation of the parameter $\rho$ connected with the wavy working surface of the gasket produces significant changes in the leak tightness (Fig. 19). Too much value of this parameter ($\rho > 3.0$) may be the cause of the lost of the leak tightness, in particular for greater values of the operating pressure $\tau$. 
All distributions of the dimensionless leak tightness parameter $\psi$ versus dimensionless operating pressure $\tau$ presented in Figs. 16 ÷ 19 are placed considerably beyond of the admissible value $\psi = 2$ in the assumed range of the closure parameters. It appears that the non-leakage condition (1) of the joint is satisfied and the leak tightness is preserved in particular for low values of the operating pressure $\tau$.

**COMPARISON OF THE ANALYTICAL APPROACH WITH FEM SIMULATION AND TEST RESULTS**

The experimental results were obtained in the installation state with no operating pressure applied to the joint. The dimensions of the investigated closure are collected in Fig. 3. The gaskets were pressed into their seats and the circumferential and axial strains were measured at the inner surfaces of the gaskets. For this reason the comparison of the analytical approach with FEM modelling and test results may be carried out in the assembly conditions only.

Examination of the dimensionless strength parameter $\sigma$ distributed along the width of the gasket presented in Fig. 20 confirms that the change of loading of the gaskets and application of the numerical model FEM 2 similar to real test conditions causes that the numerical results are closer to the test results, in particular for the small nominal radial interference $\delta = 0.5 \%$ [%]. Most of the test results are placed in the vicinity of the range received by FEM 2 method and the analytical approach. The greater interferences $\delta$ (1.0 [%] and 2.0 [%]) result in respectively greater differences between FEM 1 and FEM 2 methods and analytical solutions whereas the test results are located between FEM 1 and analytical results. Investigation of experimental and theoretical strength distributions depicted in Fig. 20 leads to the conclusion that FEM 1 method is overestimated and analytical calculations are underestimated with respect to the experiment. The difference increases with the increase of the initial interference.

The dimensionless maximum strength parameter $\sigma$ at the point $A$ (Fig 3) versus dimensionless operating pressure $\tau$ is compared in Fig. 21 for the analytical approach and the FEM 1 simulation. The numerical calculations were carried out for the thickness coefficient of the seats $\beta = 2.00$. For the loading $\tau < 0.3$ the nature of both relationships is similar, although analytical results are considerably greater. Above the yield limit ($\sigma > 1$) the difference rapidly
increases with the increase of the loading. The width of the contact region $e$ versus the dimensionless operating pressure $\tau$ for the above solutions is presented in Fig. 22. In this case the relative difference is even greater.

![Graph 1](image1)

**Fig. 21** - Dimensionless maximum strength $\sigma$ of the gasket versus dimensionless operating pressure $\tau$. Comparison of the analytical approach with FEM simulation

![Graph 2](image2)

**Fig. 22** - Width of the contact region versus dimensionless operating pressure $\tau$. Comparison of the analytical approach with FEM simulation

The leak tightness of the joint depends in particular on the applied initial assembly interference $\delta$. Visual inspection of the gaskets (Fig. 23) after the disassembly indicates that the designed interference must be less than $2.0$ [%] for the unchanged another parameters of the investigated closure. Too much radial interference fit with respect to the yield limit $R_{0.2}$ of the gasket material was probably the main reason of the serious damage of the working surface. In this case the applied interference fit together with the large value of the friction coefficient lead during the assembly (and disassembly) operation under load in the absence of adequate lubricant to the adhesive wear (Norton, 2006). The original adhesion theory postulated that all asperity contacts would result in yielding and adhesion due to the high stresses present.

![Image 3](image3)

**Fig. 23** - The gaskets after disassembly. Interferences $\delta$: (a) $-0.5$ [%]; (b) $-1.0$ [%]; (c) $-2.0$ [%]. Note the contact traces

When clean surfaces are pressed against one another under load, some of the asperities in contact will tend to adhere to one another due to the attractive forces between the surface atoms of the two materials. As sliding between the surfaces is introduced, these adhesions are broken, either along the original interface or along a new plane through the material of the asperity peak. In the latter case, a piece of one part is transferred to another part, causing
surface disruption and damage. Sometimes, a particle of one material will be broken free and become debris in the interface, which can then scratch the surface and plough furrows in both parts. This damage is called scoring or scuffing of the surface.

**FINAL REMARKS**

The simplified analytical approach based on the shell theory was applied to investigate the influence of certain geometric, material and assembly parameters on the strength and leak properties of high-pressure closure with the wave-ring gasket. The analytical solution was verified by the FEM modelling and by the experiment. On the basis of obtained results certain general conclusions and recommendations may be formulated with respect to the safety of the joint.

Because of specific working features of the wave-ring gasket the yield point of its material must be relatively lower than that of the seat. Reactor vessel walls and its heads as well as the pipelines designed for the high-pressure applications are fabricated of high-carbon chromium-nickel-molybdenum steel. The yield stress of such material usually exceeds $R_{0.2} > 750$ [MPa] and possible Rockwell hardness number is on the order of $45 \div 48$. The corresponding wave-ring gasket should be made of the material which Brinell hardness number is less than $250 \div 280$ [BHN] and the yield stress is approximately $R_{0.2} = 400$ [MPa]. Acceptable operating pressure should be on the order of $p = 250 \div 300$ [MPa]. Application of the higher operating pressure must be preceded with the detailed stress-strain analysis of the gasket and earlier considerations with respect to the leak tightness.

The strength of the wave-ring gasket can not be improved significantly through the change of the dimensionless parameter $\gamma$, however, greater loading capacity may be obtained through the increase of the parameter $\chi$. The increase of parameter $\chi$ to 1.6 produces the increase of the parameter $\tau$ to 0.43.

Special attention should be paid to the magnitude of the initial assembly interference fit $\delta$. The interference fit less than 0.5 [%] may cause the leakage of installation at the start, in particular for the joints of large diameters for which it is hard to provide the precise dimensional accuracy. On the other hand, the interference fit exceeding 2.5 [%] appears too large because it insignificantly improves the leak tightness but it produces an essential damage of the working surface during the assembly of the joint. The interference fit $\delta = 1.0 \div 2.0$ [%] is recommended to the closures with wave-ring gaskets for the assumed data.

The analytical computational model of the closure may be convenient to proceed an initial analysis. A large number of simple calculations can be carried out for different geometry of the gasket, different material properties and assembly requirements. It should be noted that the comparison of experimental and theoretical results reveals that the analytical calculations are underestimated with respect to the test. The final parameters of the closure may be then determined in the detailed FEM verification which is closer to the experimental results.

The experimental investigations point out that the gasket installation and replacement operations should be carry out with extreme precision. The attention must be paid to the exact alignment of the gasket with respect to the seat, above all in closures with great diameter. As a conclusion of the experimental investigations it should be noted that the assembly process realized by means of the expanding technique (thermal method) is advantageous with respect to the mounting process in which the gasket is pressed into the seats. The inaccurate sliding process may result in scaring of the wave surface of the gasket.

Moreover, the manufacturing process of wave-ring gaskets must ensure the high dimensional accuracy, in particular with respect to the wave working surface, as the effective interference
fit depends on this accuracy. The average nominal diameters of approximately 200 [mm] require the IT6 ÷ IT7 grade of tolerance and the roughness number of $R_a = 0.16$ [µm] seems to be sufficient.

REFERENCES