THE $T^2$ MULTIVARIATE STATISTICAL CONTROL CHARTS CAPACITY WHEN APPLIED ON EQUIPMENTS CONTROL

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ABSTRACT

The use of on-line monitoring on diesel engines can be decisive on critical equipment and when condition based maintenance is implemented. A parameters database is crucial to equipment control and performance. The monitoring of oil and fuel pressure, temperature and vibration allow the early detection of a probable failure so the technicians could decide when to do an intervention, on the right time, with the right environment and with the needed components and tools. The $T^2$ multivariate statistics when applied on equipment control can enhance the data veracity, allowing despite the false alarms. In this paper we will work with independent data, and four variables will be used. The used database has real data from a diesel engine on Phase 1. On Phase 2 the data is simulated with MATLAB. This paper will apply $T^2$ Modified Chart on equipment monitoring considered the data independence and its capacity will be tested.

Keywords: Control Charts, Multivariate, Condition Maintenance, On-line Monitoring.

INTRODUCTION

The condition based maintenance allows the definition of a maintenance based on the equipment state. The use of on-line monitoring will allow the definition of maintenance in the present time.

The Statistical Process Control (SPC) is known for control processes stability (Phaladiganon et al, 2011), in this paper we propose a Statistical Equipment Control (SEC) to conduct a equipment on-line monitoring in order to define the performance quality and know its state.

The $T^2$ modified control charts will have 2 phases (Lampreia et al, 2013). On phase 1 the stabilization parameters will be verified on the phase 2 the data with anomalies will be simulated on MATLAB.

The defined variables to be study should allow the design of Modified $T^2$ chart in phase 1 (Oliveira et al, 2009)), and be significant for the engine control.

The capacity of the $T^2$ modified chart appliance will be tested.

$T^2$ MODIFIED CONTROL CHARTS

On phase 1 the process parameters and the limits are estimated consider that the data is Normal, independent and the observations are individual.
In phase 2, the online data is monitored by Hotelling Modified $T^2$, and the control limits are calculated based on the sample number and process parameters estimated during phase 1 (Pereira & Requeijo, 2012).

**Phase 1 - $T^2$ multivariate control charts**

The selected variables to be monitored should contribute to a clear result in the Modified $T^2$ chart in phase 1 (Oliveira et al, 2009), and be significant for a diesel engine control.

In this study the data from the diesel engine is independent, so only the independent subject will be presented.

In phase 1, $m$ individual observations $X_{jk}$ ($j = 1,2,\ldots,p$ and $k = 1,2,\ldots,m$) are collected. Based on the collected data the sample means, the sample variances and the sample covariance’s are estimated. The mean vector $\overline{X}$, given by the equation (1) and the covariance matrix $S$, given by equation (2), are calculated. (Pereira & Requeijo, 2012).

\[
\overline{X} = (\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_p)'
\]

\[
S = \begin{bmatrix}
S_{11} & S_{12} & S_{13} & \ldots & S_{1p} \\
S_{21} & S_{22} & S_{23} & \ldots & S_{2p} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
S_{p1} & S_{p2} & S_{p3} & \ldots & S_{pp}
\end{bmatrix}
\]

Both the phase 1 and phase 2 uses $T^2$ control charts, based on the statistic, defined for each $k$ by:

\[
(T^2)_k = (X_i - \overline{X})' S^{-1} (X_i - \overline{X})
\]

The ($LCL$) and the ($UCL$) to the both phases are set as in Table 1. For more specifications Pereira & Requeijo (2012) must be consulted.

<table>
<thead>
<tr>
<th>Chart</th>
<th>$LCL$</th>
<th>$UCL$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 1</td>
<td>0</td>
<td>(\frac{(m-1)^2}{m} \beta_{\alpha/p^2(m-p-1)/2})</td>
</tr>
<tr>
<td>Phase 2</td>
<td>0</td>
<td>(\frac{p(m+1)(m-1)}{m(m-p)} F_{\alpha/p,m-p})</td>
</tr>
</tbody>
</table>
Phase 2 – Modified $T^2$ multivariate control charts

In phase 2 the vector $\bar{X}$ and the matrix $S$ are calculated using the data collected in phase 1. The modification of $T^2$ statistics is based, for each variable $j$, on the maximum acceptable value $T_{L_j}$. This value is calculated based on the maximum value admitted by the fabricant $(T_{L_j})_{\text{Standard}}$, by the standard deviation and by a safety factor $k$ (assuming normal distributed data, we suggest $k = 3$):

$$(T_{L_j})_j = (T_{L_j})_{\text{Standard}} - k\sigma_j$$

(4)

Then

$$(T^2)_k = (X'_k - T_k)^\top S^{-1}(X'_k - T_k)$$

(5)

where

$$(X'_k - T_k) = \max(0; X'_k - T_k)$$

(6)

For the multivariate Modified $T^2$ chart, two types of limits exist: Alert Level ($AL$), defined for $\alpha_j = 1\%$ and the Upper Control Limit ($UCL$), defined for $\alpha_j = 0.2\%$, for more detail see Table 1. (Lampreia et al, 2013)

Capacity of $T^2$ Control Charts for equipment control.

The capacity analysis is based in the technical specification range and the process range for $\alpha = 0.27\%$ comparison. The potential capacity index, $C_p$, compares the technical specification range ($USL - LSL$) (USL–Upper specification level, LSL–Lower specification level) with the range normally considered to the process dispersion ($6\sigma$), given to a $X$ normal distributed in a univariate study. (Pereira & Requeijo, 2012)

In a capacity multivariate study the comparison between areas, volumes or other dimensional are done, considered a multivariate normal distributed $N_p(\mu, \Sigma)$.

The capacity index are $C_{pM}, PV$ and $LI$.

The $C_{pM}$ (7) index compares the volume between the specification and the process, it is a potential capacity index.
For a process:

\[
C_{pM} = \left( \prod_{j=1}^{p} \frac{(USL_j - LSL_j)}{(UPL_j - LPL_j)} \right)^{\frac{1}{2}}
\]

For an equipment:

\[
C_{pM} = \left( \prod_{j=1}^{p} \frac{(USL_j - LSL_j)}{(UEL_j - LEL_j)} \right)^{\frac{1}{2}}
\]

Where:

\[
UEL_i = \mu_i + \sqrt{\frac{\sum_{i}^{X_{n,p} \text{det} (\sum_{i}^{-1})}}{\sum_{i}^{X_{n,p} \text{det} (\sum_{i}^{-1})}}} \quad i = 1, 2, ..., p \quad \text{and} \quad LEL_i = \mu_i - \sqrt{\frac{\sum_{i}^{X_{n,p} \text{det} (\sum_{i}^{-1})}}{\sum_{i}^{X_{n,p} \text{det} (\sum_{i}^{-1})}}} \quad i = 1, 2, ..., p
\]

The PV index compares the average process vector, \( \mu \), with the specification nominal values of \( T \) vector.

\[
PV = P \left( T^2 > \frac{p(N-1)F_{p,N-p}}{N-p} \right)
\]

The statistic \( T^2 \) is defined by:

\[
T^2 = n(\mu - T)' \sum_{i=1}^{-1}(\mu - T) = n\left( \frac{X}{n} - T \right)'S^{-1}\left( \frac{X}{n} - T \right)
\]

The LI (10) index analyses the regions between the specifications and the process, verifying if the process region (P) are in the specification region (S), necessary condition to the process be capable. And is defined by:

\[
LI = \begin{cases} 1 - P_{\mu S} & \text{if} \ P_{\mu S} < 1 \\ 0 & \text{other} \end{cases}
\]

When p characteristics are analyzed, a process is capable if \( C_{pM} \geq 1.33 \), and PV is approximately one, and \( LI = 1 \).

For more detail Pereira & Requeijo (2012) should be consulted.

The capacity of the \( T^2 \) modified chart appliance will be tested for the chosen variables.

**METHODOLOGY**

a. In phase 1 the data is collected considering the good functioning of the engine.

b. Considered the variables independent, normal, and checking the equipment stability, the \( T^2 \) chart should be built and the mean vector and covariance matrix are estimated.

c. Test the \( T^2 \) chart Capability.

d. In phase 2 built the Modified \( T^2 \) to monitor the equipment performance:
- Estimate the Upper Control Limit (UCL) and the Alert Value (AL) basis on the limits from fabricant.

- Establish rules for action on the system. The next are suggested:
  - Execute an intervention to detect any anomalous situation when 4 consecutive points above the AL are observed.
  - Proceed to a maintenance intervention when 2 consecutive points above UCL are observed.

RESULTS

The application of the $T^2$ Modified Multivariate Control Charts can be seen of Fig. 1. Where all the observations are under control, so the conditions to define the functioning parameters are establish. In the $T^2$ Modified the limits are defined based on fabricants and by normative instead of the mean.

The data from the engine are independent and it was taken from 1 hour intervals.

Phase 1

The X vector and the $S$ matrix are:

$$X = \begin{bmatrix} 5.7599 \\ 2.0108 \\ 564.14 \\ 560.18 \end{bmatrix}$$

$$S = \begin{bmatrix} 0.02396 & 0.00138 & 0.02143 & 0.16616 \\ 0.00138 & 0.25060 & -0.28737 & -0.17897 \\ 0.02143 & -0.28737 & 59.4610 & -3.39725 \\ -0.16616 & -0.17897 & -3.39725 & 56.02524 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 42.6 & -0.156 & -0.009 & 0.125 \\ -0.1559 & 4.0 & 0.0203 & 0.0136 \\ -0.0090 & 0.0203 & 0.02 & 0.001 \\ 0.125 & 0.01362 & 0.001 & 0.02 \end{bmatrix}$$

Fig. 1 - $T^2$ Modified multivariate Control Chart
Capacity of $T^2$ Charts

For the capacity index was obtained: $C_{pM}=281.446$, $PV=0.7393$ and $LI=1$. So we conclude that the process is capable because $C_{pM} \geq 1.33$, $PV$ obtained a result close to 1 e LI as a value of 1.

Phase 2

On phase 2 the data was simulated in MATLAB, since the engine is from an operational platform. For the first simulation, corresponding to data without anomalies non-observation was registered.

For the second progression we can see four observations above the AL, so an investigation of the state of equipment should be made.
For the third progression two points above the UCL were detected so a maintenance action allied to other diagnosis technics should be considered.

![Graph](image)

**CONCLUSIONS**

This paper had applied the $T^2$ Modified Multivariate Control Charts, where the conjunction of four variables increase the sensitivity data giving an integrate result of a diesel engine performance.

This paper shows the capacity of Modified Traditional Multivariate Control Chart.

The $T^2$ Modified Multivariate Control Charts can detect the tendency of a diesel engine functioning parameters.

The use of Statistical Equipment Control methodology can contribute to avoid unplanned stops and allows the maintenance planning.

**ACKNOWLEDGMENTS**

Portuguese Naval School and CINAV are kindly acknowledged for the use of diesel engine database.

**REFERENCES**

