PEAK LOAD ESTIMATION OF PRE-CRACKED PLAIN CONCRETE BEAMS IN MIXED-MODE FRACTURE

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ABSTRACT

A major difficulty in simulating load response of a concrete structure in mixed-mode fracture is that crack path is not known a priori. Predicting both the crack path and the associated load response involves advanced simulation techniques and novel numerical methodologies, and making these predictions remains a fundamental challenge to the community interested in investigating mixed-mode fracture. Here, an intrinsic cohesive crack model is employed to study mixed-mode fracture in a plain concrete beam with regular meshes. Simulations illustrate that this concise approach can provide a reasonable estimation of peak load of the pre-cracked plain concrete beams in mixed-mode fracture.

Keywords: mixed-mode fracture, cohesive crack, peak load.

INTRODUCTION

In a three-point-bending test of a concrete beam, a pre-crack, which shifts from load plane, leads to asymmetric loading and thus a mixed-mode fracture event (John and Shah, 1990; Guo et al., 1995). In addition, a double-edge-notched specimen in a Nooru-Mohamed test (1992), a three-dimensional four-edge-notched specimen in a Hassanzadeh test (1991), and a concrete plate with an embedded steel disk in a dowel disk test (1995) fracture in mixed mode.

In contrast to simulating a concrete structure in mode-I fracture, simulating load response of a concrete structure in mixed-mode fracture faces an extra difficulty because the crack path is not known a priori. There have been numerous attempts to predict the load response of a concrete structure in mixed-mode fracture. Galvez et al. (1998) found that the crack path from linear elastic fracture mechanics (LEFM) analysis was a satisfactory approximation of the actual crack path for concrete structures. Building on Galvez et al.’s finding, Cendon et al. (2000) and Galvez et al. (2002a, 2002b) developed a two-step approach including numerically predicting the crack path using the maximum tensile stress (MTS) criterion (Erdogan and Sih, 1963) and incorporating spring elements or cohesive elements.

Currently, in a framework of the finite element method (FEM), several available methodologies to predict both the crack path and the associated load response of a concrete structure in mix-mode fracture include the cohesive crack model, smeared crack model, discontinuity approach, and eXtended FEM (XFEM).

The cohesive crack model was proposed to characterize phenomenology of a failure process, based on a softening law (Dugdale, 1960; Barenblatt, 1962). Later, it was incorporated into the FEM as an application to concrete fracture, allowing for fracture processes to be resolved.
explicitly (Hillerborg et al., 1976; Bazant, 2002; Elices et al., 2002). Two approaches are available in the framework of the cohesive crack model. The intrinsic cohesive crack model usually embeds cohesive elements along the boundaries of volumetric elements as a part of the physical model (Needleman, 1987; Guo et al., 2012a, 2012b, 2012c, 2013), whereas the extrinsic model inserts cohesive elements into the model as fracture develops based on an extrinsic fracture initiation criterion (Camacho and Ortiz, 1996; Ruiz et al., 2001; Yu and Ruiz, 2006; Yu et al., 2008). Song et al. (2006) employed an intrinsic cohesive crack model with a visco-elastic bulk material model to investigate mixed-mode fracture of asphalt concrete. The predicted crack path was found to be in agreement with the experimental results, but no result for the associated load response was presented. Lens et al. (2009) investigated mixed-mode fracture of concrete and demonstrated that the cohesive model with a “plastic” potential defined by Coulomb’s law with adherence was able to cope with the spurious traction components. Using the extrinsic cohesive crack model, Ruiz et al. (2001) formulated mixed-mode fracture of concrete specimens subjected to dynamic loading, and accounted for micro-cracking, macroscopic crack development, and inertia effect. Yang and Deeks (2007) coupled the FEM and the scaled-boundary FEM (SBFEM) for cohesive crack propagation with a LEFM-based re-meshing procedure. The results showed that the SBFEM–FEM coupled method was capable of predicting both the crack path and the load response with a small number of degrees of freedom.

In the smeared crack model, infinite parallel cracks with infinitesimal openings are distributed over the finite element, and crack propagation is simulated by a reduction in the stiffness and strength of the material. The total strain is a sum of the strains resulting from the deformation of the un-cracked material and the cracking process. The constitutive laws show strain softening and introduce numerical difficulties (i.e., the system of equations may become ill-posed) and localization instabilities and spurious mesh sensitivity may occur (Bazant and Cedolin, 1979). Prisco et al. (2000) adopted multiple smeared crack models including Ottosen’s model, the gradient Rankine plasticity model, and the crush-crack model based on local and non-local approaches to investigate the mechanical behavior of plain and reinforced concrete in mixed-mode fracture. Ozbolt and Reinhardt (2002) used a micro-plane model, a type of the smeared crack model, for concrete in mixed-mode fracture, and predicted the structural response and crack patterns realistically.

Sancho et al. (2007b) developed a strong discontinuity model with crack equilibrium at the element level, cohesive cracking with central forces, and limited local crack adaptability; they found that it could describe the crack propagation with adequate accuracy. Cohesive crack propagation was formulated based on a truly-mixed discretization with the stresses as main regular variables, while discontinuous displacements played the role of Lagrangian multipliers (Bruggi and Venini, 2009). The approach handled the cohesive cracks through the appropriate inclusion of cohesive energy terms that enrich the formulation. Generally, a major difficulty in the discontinuity approach is that the crack propagation may lock because of kinematical incompatibility among the cracks; therefore, special implementation should be taken to prevent crack locking (Sancho et al., 2007a).

In the XFEM, crack propagation through a finite element is modeled by the enrichment of the classical displacement-based finite element’s approximation by a discontinuous function obtained through the partition of unity (Babuska and Melenk, 1997). Areias and Rabczuk (2008) modeled set-valued softening laws in plate bending and plane crack propagation and presented an algorithm for the softening law to determine the crack path. Generally, a major difficulty with the XFEM lies in modeling spontaneous multiple crack initiation, branching, and coalescence (Song et al., 2008).
Overall, simulating both the crack path and the associated load response of a plain concrete structure in mixed-mode fracture requires advanced simulation techniques and even novel numerical methodologies and remains a fundamental challenge in the community interested in investigating mixed-mode fracture. Among the above methodologies in the framework of the FEM, the cohesive crack model is comparatively simpler in concept and easier in application and has come to be popular in the research on non-linear fracture mechanics. Compared with the intrinsic cohesive crack model, the extrinsic one involves some issues, both in the model implementation and in the results interpretation (Zhang et al., 2007). Therefore, in a previous study of Guo et al. (2012c), the intrinsic cohesive crack model was employed to investigate mode-I fracture of three-point-bending specimens and to extract the best-fit energy and stress ratios for both normal-strength and high-strength concrete. As an extension of Guo et al. (2012c), the intrinsic cohesive crack model with a bilinear softening law is employed here to study mixed-mode fracture in a plain concrete beam. Simulations with regular (not oriented) meshes illustrate that the approach can provide a reasonable estimation on the peak load of the pre-cracked plain concrete beam. Parametric studies are conducted to investigate effects of the energy and stress ratios and cohesive strength on the load response and energy terms.

COHESIVE CRACK MODEL WITH A BILINEAR SOFTENING LAW

The cohesive crack model is formulated on the virtual work (Roesler et al., 2007). The internal work done by the virtual strain tensor ($\delta \varepsilon$) in the domain ($\Omega$) and the internal work by the virtual crack opening displacement vector ($\delta w$) along the crack line ($\Gamma_c$) are equal to the external work by the virtual displacement vector ($\delta u$) at the traction boundary ($\Gamma$), i.e.,

$$\int_{\Omega} \delta \varepsilon^T \sigma \, d\Omega + \int_{\Gamma_c} \delta w^T T \, d\Gamma_c = \int_{\Gamma} \delta u^T P \, d\Gamma$$

(1)

where $\sigma$ is the stress tensor, $T$ is the traction vector in the cohesive zone, and $P$ is the external traction vector. The resulting cohesive crack model has been implemented in ABAQUS (2011).

A bilinear softening law has been gradually accepted as a reasonable approximation to the softening behavior in concrete. It can be fully characterized by four parameters: cohesive strength $T_m$, fracture energy $G_F$, initial fracture energy $G_f$, and kink stress $T_k$, a stress level corresponding to $w_k$ (a separation at the kink point). Equivalently, in line with the previous study of Guo et al. (2012), $T_m$, $G_F$, an energy ratio (the ratio of the fracture energy to the initial fracture energy $G_F/G_f$), and a stress ratio (the ratio of the kink stress to the cohesive strength $T_k/T_m$) are employed in this study. To restrict the inherent mesh dependence, the penalty stiffness is used to modify the bilinear softening law, as shown in Fig. 1. The cohesive strength $T_m$ is the softening stress at which the separation reaches $w_m$ and damage initiates. The fracture energy $G_F$ is the work required to create and fully separate a unit surface area of a cohesive element and is given by the area under the softening law, i.e.,

$$G_F = \int_0^{w_m} T(w) \, dw$$

(2)

where $T$ is an effective traction, $w$ is an effective separation, and $w_m$ is the critical crack opening beyond which the traction becomes zero and the cohesive element fails completely.
The effective traction and the effective separation can be defined as
\[ T = \sqrt{T_n^2 + \eta^2 T_s^2} \quad \text{and} \quad w = \sqrt{w_n^2 + \eta^2 w_s^2} \] (3)
where \( T_n \) and \( T_s \) are the cohesive tractions in the normal and tangential directions, respectively, \( w_n \) and \( w_s \) are the normal and tangential opening displacements across the cohesive surfaces, respectively, and \( \eta \) is the tension-shear coupling constant representing the effect of the mixed mode (Zhang et al., 2007).

The softening law in Fig. 1 has two stages. In stage O-A, the cohesive element deforms elastically. Damage initiates at the critical point A. Note that \( w_m \) should be much smaller than \( w_k \) so that the penalty stiffness can be sufficiently large. During stage A-B-C, the separation of the cohesive element increases, and the cohesive element is softened, i.e., damage evolves. The softening stress can be formulated as
\[
T = \begin{cases} 
T_m - \frac{(T_m - T_k)w_m}{w_k} & \text{for } w_m < w \leq w_k \quad (A \rightarrow B); \\
T_k - \frac{T_k (w - w_k)}{(w_c - w_k)} & \text{for } w_k < w \leq w_c \quad (B \rightarrow C).
\end{cases}
\] (4)

Same with Song et al. (2006), the tension-shear coupling constant \( \eta \) is taken as 1, i.e., the quadratic strain criterion is employed for damage initiation and evolution in the cohesive elements.

**Fig. 1 - Softening law with a penalty stiffness to restrict mesh dependence.**

**MIXED-MODE FRACTURE SIMULATION OF PRE-CRACKED PLAIN CONCRETE BEAMS**

Here, the four-point, single-edge notched shear beams tested by Arrea and Ingraffea (1982) are studied. Their testing results, especially load versus crack mouth sliding displacement (CMSD) curves, have become a benchmark for mixed-mode crack propagation analysis. The geometry and boundary conditions of the series B beams in their test are shown in Fig. 2. The analytical configuration adopted has a span of 914 mm, a depth of 308.5 mm, a thickness of
152 mm, and a pre-crack length $a_0$ of 82 mm. It is assumed that the condition of plain strain prevailed. The specimen is discretized into the volumetric and cohesive elements. Uniform linear cross-triangular meshes (3 mm in size) are used in the potential fractured zone. As in the experiments, displacement control is used in the simulations.

![Image](image.png)

Fig. 2 - An analytical configuration for a four-point single-edge notched shear beam.

A linear elastic constitutive law is applied to the concrete itself. The available material constitutive parameters in Arrea and Ingraffea (1982) include Young’s modulus of 24.8 GPa, Poisson’s ratio of 0.18, and compressive strength of 45.5 MPa, which are also employed in the present simulations. The fracture behavior predicted by the cohesive crack model depends heavily on the cohesive strength and the fracture energy employed (Roesler et al., 2007; Park et al., 2008). Generally, although the mode-I fracture energy $G_{fi}$ of concrete can be measured by the work-of-fracture method, proposed by RILEM TC 50-FMC (1985), an approach to measuring the mode-II fracture energy $G_{fii}$ is still under the way. In the extrinsic cohesive crack model, the tensile strength has generally been taken as the cohesive strength (Camacho and Ortiz, 1996; Ruiz et al., 2001; Yu and Ruiz, 2006; Yu et al., 2008). Theoretically, in the intrinsic cohesive crack model, the cohesive strength is unknown and should be calibrated, for example, from a direct tensile test (Cornece et al., 2003; Guo et al., 2010, 2012c). Direct tensile tests of concrete are difficult to perform and the results always scatter extensively. Generally, the result of a splitting-tension test can provide an estimate of the tensile strength (Guinea et al., 1994). In line with prior studies employing the intrinsic cohesive crack model for concrete (Roesler et al., 2007; Guo et al., 2012b), the tensile strength of concrete, $f_t$, is taken here to be its mode-I cohesive strength $T_{mi}$. Furthermore, in line with prior studies employing the extrinsic cohesive crack model for concrete, the tensile strength of concrete $f_t$ is also taken here to be its mixed-mode cohesive strength $T_m$.

Neither the fracture energy $G_F$ nor the tensile strength $f_t$ was reported in Arrea and Ingraffea (1982). A wide range of values were used for the series B beams by different analysts, as listed in Table 1, where $T_{mII}$ is the mode-II cohesive strength. Here, $G_{fi} = 107$ N/m and $f_t = 3.7$ MPa, which are the same as those in Galvez et al. (2002a, 2002b), are used, and $G_{fii}$ is taken to be the same as $G_{fi}$ such that the total fracture energy is $G_F = G_{fi} + G_{fii} = 2G_{fi}$. Unless otherwise stated, the mode-II cohesive strength is evaluated as the critical internal shear stress when shear failure occurs in a simple compression, i.e., $c = f_{cu} \sin \phi \cos \phi$. The compressive strength $f_{cu}$ was measured to be 45.5 MPa (Arrea and Ingraffea, 1982), and the internal friction angle $\phi$ can be taken as 30°, and thus, $c = 19.7$ MPa. The mode-I cohesive
strength $T_{mI}$ and the mode-II cohesive strength $T_{mII}$ are used in the penalty and damage initiation parts in Fig. 1 while the mixed-mode cohesive strength $T_m$ is used in the damage evolution part in Fig. 1.

Table 1 Available approaches to simulate experiments in Arrea and Ingraffea (1982) and the fracture energy $G_f$ and tensile strength $f_t$ employed.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Approach</th>
<th>Fracture energy $G_f$ (N/m)</th>
<th>Tensile strength $f_t$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rots &amp; De Borst (1987)</td>
<td>Smeared crack model</td>
<td>75</td>
<td>2.7</td>
</tr>
<tr>
<td>Xie &amp; Gerstle (1995)</td>
<td>Energy-based extrinsic cohesive crack model</td>
<td>150</td>
<td>4.0</td>
</tr>
<tr>
<td>Saleh &amp; Aliabadi (1995)</td>
<td>Boundary element model</td>
<td>100</td>
<td>2.8</td>
</tr>
<tr>
<td>Cendon et al. (2000)</td>
<td>LEFM + spring model</td>
<td>125</td>
<td>4.0</td>
</tr>
<tr>
<td>Galvez et al. (2002a, 2002b)</td>
<td>LEFM + intrinsic cohesive crack model</td>
<td>$G_f = 107$</td>
<td>3.7</td>
</tr>
<tr>
<td>Yang &amp; Deeks (2007)</td>
<td>Scaled boundary FEM + extrinsic cohesive crack model + LEFM-based re-meshing</td>
<td>$G_f = 100$ N/m, $G_{fII} = 0.1 G_f$, $f_t = 3.0$ MPa, $T_{mII} = f_t / 3 = 1.0$ MPa</td>
<td></td>
</tr>
<tr>
<td>Yu et al. (2008)</td>
<td>Extrinsic cohesive crack model</td>
<td>99.4</td>
<td>2.8</td>
</tr>
<tr>
<td>The present authors</td>
<td>Intrinsic cohesive crack model</td>
<td>$G_f = 107$ N/m, $G_{fII} = G_f$, $T_{m} = f_t = 3.7$ MPa, $T_{mII} = c = 19.7$ MPa</td>
<td></td>
</tr>
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</table>

As a feature and also a limitation due to its basic assumption, in the cohesive crack model, crack path is confined to bulk-element boundaries, which are not necessarily coincident with the actual crack path. Fig. 3 shows a typical crack path in the present simulation with an energy ratio of 2.0 and a stress ratio of 0.25. It can be found that the main crack propagation included two line segments, namely, the first one starting from the pre-crack tip and oriented $45^\circ$ and the second one kinked $45^\circ$ and pointing upward. The crack path is very close to that in Oliver et al. (2002) via the strong discontinuity approach. There is an obvious local mixed mode when the crack propagates from the pre-crack, as the kink between the pre-crack and the first line segment shows. Upon careful observation of the kink region between the two line segments, multiple micro-cracks initiate there and actually formed a transitional region. The deviation of the crack path having two kinked line segments from the actual crack path being a smooth curve has a potential effect on energy release rate and the associated load response.

Fig. 3 - Crack path in the simulation with an energy ratio of 2.0 and a stress ratio of 0.25 when the CMSD is 0.133 mm (a deformation scale factor of 100 is employed for clear illustration).
EFFECTS OF THE ENERGY RATIO ON THE LOAD RESPONSE AND ENERGY TERMS

Bazant and Becq-Giraudon (2002) confirmed a value of 2.5 as the optimal energy ratio, with a variation of approximate 40% from a database involving 238 test series. Bazant (2002) reviewed the available research on concrete fracture and found that the energy ratio could be taken as 2.5 and that the stress ratio varied from 0.15 to 0.33. In this section, three levels of energy ratio (1.5, 2.0, and 2.5) and a stress ratio of 0.25 are chosen. Load-CMSD curves in the present simulations, together with the lower and upper bounds of experimental results in Arrea and Ingraffea (1982), are illustrated in Fig. 4. As the energy ratio increases, the initial fracture energy decreases, and the concrete is damaged more easily; thus, both the peak load and the peak-load CMSD decrease. By comparing the simulation results with the experimental results in Fig. 4, it can be found that the width of the peak region larger than that in the experimental results. After the peak region, the simulation results are larger than the experimental ones, due to the difference between the crack path in the simulations and the actual crack path, as shown in Fig. 3. The maximum energy release rate criterion states that the actual crack propagates in a direction that maximizes the release of stored energy in the structure. Therefore, the crack path in the simulation releases stored energy at a slower pace.

Energy terms are extracted from the simulations for further analysis. For the specimen here, the damage dissipation, external work, and strain energy versus the CMSD are illustrated in Fig. 5. Both the damage dissipation and the external work increase monotonically with the CMSD, i.e., with the fracture process. As shown in Fig. 5a, the damage dissipation increases with an increasing energy ratio. After damage initiation, as the energy ratio increases, the initial fracture energy decreases, i.e., the softening stress in this region also decreases. Therefore, the concrete is damaged more easily, and a comparatively larger fracture process zone (FPZ) occurs, leading to larger cumulative damage dissipation. Fig. 5b shows that, after damage initiation, the external work decreases with the energy ratio because less work is required to overcome the initial fracture energy for the case of the larger energy ratio. Before the peak-load occurs, the strain energy increases in a parabolic manner, as shown in Fig. 5c. After the strain-energy peak, as the FPZ increases, the unloading in the undamaged zone occurs around the FPZ, which explains why the strain energy decreases when extensive
damage is present.

![Fig. 5](image-url)

**Fig. 5** - (a) Damage dissipation, (b) external work, and (c) strain energy versus the CMSD with three levels of energy ratio and a stress ratio of 0.25.

**EFFECTS OF THE STRESS RATIO ON THE LOAD RESPONSE AND THE ENERGY TERMS**

In this section, an energy ratio of 2.5 and three levels of stress ratio (0.15, 0.25, and 0.35) are chosen. The predicted load-CMSD curves are illustrated in Fig. 6. For a specimen with a median size, when the load is at the peak position, the FPZ around the pre-crack tip is small; thus, the softening stress at the crack tip is almost defined by the initial linear segment of the bilinear softening law. Therefore, the peak load is dominated by the initial fracture energy $G_f$.

When the energy ratio is fixed, the peak of the load-CMSD curves remains almost the same, as shown in Fig. 6, and is in agreement with many available numerical simulations, as reviewed in Bazant (2002). Because the peak of the load-CMSD curve is mainly dominated by the initial fracture energy and depends less on the stress ratio, the stress ratio of 0.15 recommended by the CEB-FIP Model Code (1993) predicts the same peak load as the other stress ratios. After the peak, with the FPZ around the pre-crack developing further, the crack tip displacement (the vector sum of crack tip opening and sliding displacements) is larger than $w_k$, and thus, the softening stress in the FPZ is larger for a larger stress ratio. As the stress ratio increases from 0.15 to 0.25, the load level in the post-peak region remains almost identical; as the stress ratio further increases from 0.25 to 0.35, the load level in the post-peak region increases significantly.

![Fig. 6](image-url)

**Fig. 6** - Simulated load-CMSD curves with an energy ratio of 2.5 and three levels of stress ratio.
Next, the effect of the stress ratio on the energy terms is studied. For the specimen here, the damage dissipation, the external work, and the strain energy versus the CMSD are illustrated in Figs. 7. All three energy terms are almost independent of the stress ratio before the peak-load CMSD. Both the damage dissipation and the external work increase monotonically with the fracture process. As shown in Fig. 7a, the damage dissipation has negligible dependence on the stress ratio. Fig. 7b shows that the external work increases with the stress ratio after the peak-load CMSD because, when extensive damage occurs, more work is required to overcome the larger softening stress in the case of the larger stress ratio. Before the peak-load CMSD, the strain energy increases in a parabolic manner, as shown in Fig. 7c. After the peak-load CMSD, for the case of the stress ratios 0.15 and 0.25, the strain energy decreases because of the unloading in the undamaged zone; for the stress ratio 0.35, the strain energy remains constant and then increases.

**Fig. 7** - (a) Damage dissipation, (b) external work, and (c) strain energy versus the CMSD with an energy ratio of 2.5 and three levels of stress ratio.

**EFFECTS OF THE COHESIVE STRENGTH ON THE LOAD RESPONSE AND THE ENERGY TERMS**

Fig. 8 illustrates the simulated load-CMSD curves with an energy ratio of 2.5, a stress ratio of 0.25, and three levels of the mode-I cohesive strength $T_{mI}$, i.e., $0.80f_y$ (2.96 MPa), $0.90f_y$ (3.33 MPa), and $f_y$ (3.70 MPa). Fig. 8 shows that the peak load increases clearly with $T_{mI}$ while the increasing rate of the peak load is smaller than that of $T_{mI}$. Furthermore, the peak-load CMSD decreases as $T_{mI}$ increases. The associated damage dissipation, the external work, and the strain energy versus the CMSD are illustrated in Fig. 9. The damage dissipation increases negligibly with $T_{mI}$, as shown in Fig. 9a. Fig. 9b shows that the external work always increases with the CMSD. Fig. 9c shows that the strain energy increases with the CMSD before the peak-load CMSD and then decreases after the peak-load CMSD due to unloading in the undamaged zone around the FPZ. Figs. 9b and c show that the external work and the strain energy increase with $T_{mI}$.
Fig. 8 - Load-CMSD curves with an energy ratio of 2.5, a stress ratio of 0.25, and three levels of $T_{ml}$.

Fig. 9 - (a) Damage dissipation, (b) external work, and (c) strain energy versus the CMSD with an energy ratio of 2.5, a stress ratio of 0.25, and three levels of $T_{ml}$.

Fig. 10 illustrates load-CMSD curves with an energy ratio of 2.0, a stress ratio of 0.25, and three levels of the mode-II cohesive strength $T_{ml}$, namely, 0.50c (9.85 MPa), 0.75c (14.775 MPa), and c (19.70 MPa). Fig. 10 shows that the load curves essentially coincide and show nearly no dependence on $T_{ml}$. The associated damage dissipation, the external work, and the strain energy versus the CMSD are illustrated in Fig. 11. After the peak-load CMSD, the damage dissipation in the case of 0.50c is larger than that in the case of 0.75c and c by almost a constant, as shown in Fig. 11a. On careful observation of Fig. 11a, the constant amount of damage dissipation difference is accumulated during the initiation stage of the crack propagation from the pre-crack. Figs. 11b and c show that the external work and the strain energy are essentially independent of the $T_{ml}$. These observations are in agreement with the current understanding that the crack grows mainly in a stable manner in local mode-I under a global mixed-mode loading, as implied by the MTS criterion (2002), and that a mixed-mode fracture mechanism dominates at crack initiation, though the specimens finally fracture in mode-I manner (Ozbolt and Reinhardt, 2002). Furthermore, Figs. 8-11 show that $T_{ml}$ had much larger effects on the load response and the energy terms than $T_{ml}$. 
Fig. 10 - Load-CMSD curves with an energy ratio of 2.0, a stress ratio of 0.25, and three levels of $T_{mII}$.

CONCLUSIONS

In this study, the intrinsic cohesive crack model with a bilinear softening law is employed to investigate mixed-mode fracture in a plain concrete beam. In contrast to the available research, the present approach requires neither preliminary results from LEFM simulations nor a re-meshing procedure or special implementation to prevent crack locking. The simulations with regular meshes show that the approach can provide a reasonable estimation of the peak load of the plain concrete structure. The parametric studies illustrate that the energy ratio in the bilinear softening law have larger effects on the load response and energy terms than the stress ratio. The simulations also confirm that the crack initiates in a mixed-mode manner but grows in local mode-I under a global mixed-mode loading and that the mode-I cohesive strength has much larger effects on the load response and the energy terms than the mode-II cohesive strength.

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