**PREDICTION OF UNSTEADY SHEET CAVITATION ON MARINE CURRENT TURBINES WITH A BOUNDARY ELEMENT METHOD**

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**ABSTRACT**

An iteratively coupled solution method for the calculation of unsteady sheet cavitation on marine current turbines with a potential-based Boundary Element Method is investigated. The solution of the linear system of equations is obtained with an iterative technique which avoids a new matrix inversion at each iteration step in the prediction of the cavity planform. The solution method explores the fact that only the source strengths on the panels beneath the cavity change due to the presence of the cavity. The advantage is that the complete system matrix is identical to the matrix of the wetted flow problem and needs only to be inverted once at each time step. The numerical studies are carried out for a marine current turbine, where a significant reduction in the computational time is obtained with the iteratively coupled technique in comparison with the classical approach to the cavitating problem.

**Keywords:** Marine Current Turbines, Sheet Cavitation, Boundary Element Methods, Iteratively Coupled Solution Method.

**INTRODUCTION**

The energy in flowing river streams, tidal currents or other artificial water channels is being considered as a viable source of renewable power and horizontal axis marine current turbines are among the most investigated energy conversion systems for hydrokinetic energy extraction (Khan et al., 2009). For marine current turbines under certain operating conditions cavitation on the blades may occur. The ability to predict the extent of sheet cavitation and the pressure distribution on the turbine blades is essential for the design and analysis of such systems. For the analysis of the flow on a marine current turbine with cavitation, a potential flow model may be adequate and cost-effective for the use in the design of such systems. The potential flow analysis may be carried out with a Boundary Element Method (BEM). This method has been widely used in the hydrodynamic analysis of marine propellers.

The modelling of sheet cavitation on marine propellers using BEM was first introduced by Fine (1992). The method is based on an integral equation for the velocity perturbation potential. The presence of a cavity in the blades is modelled as a free boundary problem. A thin cavity is assumed so that the boundary conditions on the cavity are linearised with respect to the wetted flow. This implies that the dynamic and kinematic boundary conditions are applied on the foil surface beneath the cavity. On the wetted surfaces only the kinematic boundary conditions is applied. Dipoles and sources are placed on the body surfaces either on the wetted part or beneath the cavities. The problem is closed by suitable specification of cavity detachment and closure, and a Kutta condition at the blade trailing edge. The wake surfaces are modelled by dipoles and in the presence of cavities extending into the wake with additional sources.
The solution of the problem for a given cavitation number is to iterate on the cavity length. However, for each iteration step on the cavity extension the method solves a complete system of equations for the unknown potentials on the wetted panels of the blade and for the unknown sources on the panels beneath the cavity. This requires the solution of a new system of equations because some of the elements of the system matrix were changed with the modification of the cavity planform. An alternative iterative technique has been proposed (Baltazar and Falcão de Campos, 2010) to solve the linear system of equations which avoids a new matrix inversion at each iteration step in the prediction of the cavity planform. The method has been applied to partial and super-cavitation on marine propellers (Baltazar and Falcão de Campos, 2012a). The BEM was first applied for the prediction of steady sheet cavitation on marine propellers by Baltazar and Falcão de Campos (2012b). In this paper the iterative technique is tested for the prediction of sheet cavitation on a marine current turbine in uniform and non-uniform inflow conditions.

**MATHEMATICAL FORMULATION**

**Governing Equations**

Consider the rotor of a horizontal axis turbine with radius $R$ placed in a fluid stream and rotating at constant angular velocity $\Omega$ around its axis. The turbine rotor is made of $K$ blades symmetrically distributed around a hub. We introduce a Cartesian coordinate system $(x_0, y_0, z_0)$ fixed in space and a Cartesian coordinate system $(x, y, z)$ rotating with the turbine rotor. Figure 1 shows the turbine rotor and the two coordinate system used to describe the flow field.

![Fig. 1 - Rotor inflow and coordinate systems.](image)

The $x$ and $x_0$ axes of the two coordinate systems coincide with the turbine rotation axis and are reckoned positive pointing in the downstream flow direction. The $y_0$ and $z_0$ are at the turbine plane, with $y_0$ pointing upwards and $z_0$ completing the right-hand system. The $y$ axis is coincident with the turbine reference line, passing through the reference point at the root section of the key blade and $z$ completes the right-hand system. We introduce cylindrical coordinate systems $(x_0, r_0, \theta_0)$ and $(x, r, \theta)$ related to the Cartesian systems by
\[ y_0 = r_0 \cos \theta_0, \quad z_0 = r_0 \sin \theta_0, \quad y = r \cos \theta, \quad z = r \sin \theta. \]  

(1)

The relation between the two coordinate systems for a rotating right-handed rotor is

\[ x_0 = x, \quad r_0 = r, \quad \theta_0 = \theta - \Omega t, \]  

(2)

where \( \Omega = \left| \Omega \right| \) and \( t \) is the time. At \( t = 0 \) the key blade reference line coincides with the \( y_0 \) axis.

We will assume that the turbine rotor is subjected to a non-uniform inflow. This inflow is assumed to be steady in the Cartesian inertial frame \( (x_0,y_0,z_0) \) and we denote the inflow velocity to the turbine by \( \vec{U}_e (x_0,y_0,z_0) \). In the reference frame rotating with the turbine, the undisturbed inflow velocity is time dependent and is given by

\[ \vec{V}_\infty (x,r,\theta,t) = \vec{U}_e (x,r,\theta - \Omega t) - \vec{\Omega} \times \vec{x}, \]  

(3)

where \( \vec{x} = (x,y,z) \).

The fluid flow is assumed to be inviscid and incompressible with density \( \rho \) in an infinite exterior domain to the turbine blades and hub. The presence of the bottom surface and the free surface is disregarded. The perturbation velocity \( \vec{v}(x,y,z,t) \) to the inflow is assumed to be irrotational so that it may be written as the gradient of a scalar perturbation potential,

\[ \vec{v}(x,y,z,t) = \nabla \phi(x,y,z,t). \]  

(4)

In the reference frame rotating with the turbine the flow velocity is

\[ \vec{V}(x,y,z,t) = \vec{V}_\infty (x,y,z,t) + \nabla \phi(x,y,z,t). \]  

(5)

The perturbation potential satisfies the Laplace equation

\[ \nabla^2 \phi(x,y,z,t) = 0. \]  

(6)

The Bernoulli equation on the rotating frame reads

\[ \frac{\partial \phi}{\partial t} + \frac{p}{\rho} + \frac{1}{2} |\vec{V}|^2 + g_y = \frac{p_\infty}{\rho} + \frac{1}{2} |\vec{V}_\infty|^2, \]  

(7)

where \( p \) is the pressure, \( p_\infty \) a reference pressure at the turbine axis and \( y_0 \) is the vertical coordinate given by \( y_0 = r \cos(\theta - \Omega t) \) according to Eq. (1) and Eq. (2). We introduce the pressure coefficient \( C_{pn} \) and cavitation number \( \sigma_n \):

\[ C_{pn} = \frac{p - p_\infty}{\frac{1}{2} \rho n^2 D^2}, \quad \sigma_n = \frac{p_\infty - p_v}{\frac{1}{2} \rho n^2 D^2}, \]  

(8)

where \( p_v \) is the vapour pressure, \( n = \Omega / 2\pi \) is the rate of revolution and \( D = 2R \) the turbine diameter.
Boundary Conditions

The boundary of domain consists of the wetted blade and hub surfaces $S_B$ (without cavitation), the cavity surfaces $S_C$ and the wake surfaces $S_W$ behind the blades. The perturbation potential must satisfy the following boundary condition:

$$\nabla \phi \rightarrow 0, \text{ if } r \rightarrow \infty \text{ or } x \rightarrow -\infty$$

at infinity, and a Neumann boundary condition

$$\frac{\partial \phi}{\partial n} = \bar{n} \cdot \nabla \phi = -\bar{n} \cdot \bar{V}_\infty \text{ on } S_B,$$

where $\partial/\partial n$ denotes differentiation along the normal and $\bar{n}$ is the unit vector normal to the surface directed outward from the body.

Consider a cavity surface $S_C$ by,

$$s_j = \eta(s_1, s_2, t),$$

where $\eta$ is the cavity thickness defined in the local boundary fitted non-orthogonal reference system $(s_1, s_2, s_3)$ with unit base vectors $\bar{t}_1, \bar{t}_2$ tangent to the surface and $\bar{t}_3 = \bar{n}$. A thin cavity is assumed so that the boundary conditions in the cavity may be partially linearised with respect to the wetted flow. This implies that the dynamic and kinematic boundary conditions are applied on the blade surface $S'_C$ beneath the cavity.

The kinematic boundary condition states that the flow is tangent to the cavity surface. As shown in Kinnas and Fine (1992), the kinematic boundary condition in the non-orthogonal reference system $(s_1, s_2, s_3)$ renders the following equation for the cavity thickness $\eta$ on the blade surface $S'_C$:

$$\frac{\partial \eta}{\partial s_1} \left( V_{s_1} - V_{s_2} \cos \alpha \right) + \frac{\partial \eta}{\partial s_2} \left( V_{s_2} - V_{s_1} \cos \alpha \right) = \left( V_{s_1} - \frac{\partial \eta}{\partial t} \right) \sin^2 \alpha \text{ on } S'_C,$$

where $\alpha$ is the angle between the $s_1$ and $s_2$ directions. Eq. (12) is a partial differential equation for $\eta$.

The dynamic boundary condition states that the pressure on the cavity equals the vapour pressure, i.e. $p = p_v$, or

$$-C_{pm} = \sigma_a \text{ on } S'_C.$$

By considering on each blade surface a local orthogonal coordinate system $(u_1, u_2, u_3)$ with $u_1 = s_1$ and $u_3 = s_3$, from Bernoulli equation, Eq. (7), we have

$$V_{u_i} = \sqrt{(nD)^2 \sigma_a + \tilde{U}_\infty^2 - 2g \gamma \nu_0 - 2 \frac{\partial \phi}{\partial t} - V_{u_2}^2 - V_{u_2}^2} \text{ on } S'_C.$$

The term $V_{u_i}$ is neglected in the dynamic boundary condition following the formulations of Fine (1992) and Vaz (2005), since it deteriorates the robustness and hardly influences the
cavity. Solving Eq. (14) for the perturbation potential velocity \( \frac{\partial \phi}{\partial s_1} \) and integrating along \( s_1 \) from the detachment point \( s_{10} \), we get

\[
\phi = \phi(s_{10}) + \int_{s_{10}}^{s_1} \left[ \sqrt{(nD)^2 \sigma_n^2 + \left( \frac{\partial U}{\partial t} \right)^2 - 2g\psi_0 - 2\frac{\partial \phi}{\partial t} - V_{s_2}^2} \right] ds + \int_{s_{10}}^{s_1} \left[ -\frac{\partial U}{\partial t} \cdot \mathbf{t}_s \right] ds \quad \text{on} \quad S^c.
\] (15)

The boundary conditions on the wake surfaces \( S_w \) are the tangency of the fluid velocity on each side of the sheet:

\[
\vec{V}_w \cdot \vec{n} = \vec{V}^+ \cdot \vec{n} = \vec{V}^- \cdot \vec{n} \quad \text{on} \quad S_w,
\] (16)

and the continuity of the pressure across the vortex wake

\[
p^+ = p^- \quad \text{on} \quad S_w,
\] (17)

where \( \vec{V}_w \) is the velocity of the vortex sheet surface \( S_w \).

The first condition, Eq. (16), implies that the vortex sheet moves with the fluid. If \( S_w(\vec{x},t) = 0 \) represents the equation of the vortex sheet surface \( S_w \), then

\[
\frac{\partial S_w}{\partial t} + \vec{V}_w \cdot \nabla S_w = \frac{\partial S_w}{\partial t} + \vec{V}^+ \cdot \nabla S_w = 0.
\] (18)

From the boundary condition, Eq. (17), and applying the Bernoulli equation at a given point on each side of the vortex sheet, we obtain

\[
\frac{\partial (\Delta \phi)}{\partial t} + \vec{V}_m \cdot \nabla (\Delta \phi) = 0,
\] (19)

where \( \vec{V}_m = \frac{1}{2} \left( \vec{V}^+ + \vec{V}^- \right) \) is the mean velocity and \( \nabla (\Delta \phi) = \vec{V}^+ - \vec{V}^- \) is the surface gradient of the potential discontinuity, which is equal to the velocity discontinuity on the wake surface. Eq. (19) shows that the potential-jump remains constant following a fluid particle moving on the wake with the velocity \( \vec{V}_m \).

In the general case, the instantaneous location of the wake has to be derived from Eq. (18) and the dipole strength from Eq. (19), which requires following the motion of the vortex sheet \( S_w \) in the unsteady flow velocity field. Such calculation is rather involved and beyond the scope of this paper. A considerable simplification is achieved if we assume that \( \vec{V}_m \) is constant and equal to the undisturbed time averaged axisymmetric inflow. In the cylindrical coordinate system \( (x,r,\theta) \),

\[
\vec{V}_m = (\vec{U}_r(r),0,\Omega r),
\] (20)

where \( \vec{U}_r(r) \) is the zero harmonic of the axial inflow at the given radius, and Eq. (19) becomes

\[
\frac{\partial (\Delta \phi)}{\partial t} + \Omega \frac{\partial (\Delta \phi)}{\partial \theta} = 0.
\] (21)
The solution of Eq. (21) is of the form, $\Delta \phi (r, \theta, t) = \Delta \phi (r, t')$ with $t' = t'(\theta, t)$ being a characteristic convection time. Considering that at $t = 0$, $\theta = \theta_{TE}$ we obtain,

$$\Delta \phi (r, \theta, t) = \Delta \phi \left(r, t - \frac{\theta - \theta_{TE}}{\Omega}\right).$$

(22)

The initial condition in the wake is

$$\Delta \phi (r, \theta, 0) = \Delta \phi_{TE} (r, 0) = -\Gamma (r, 0),$$

(23)

where $\Gamma$ is the flow circulation for a circuit around the blade intersecting the wake at the blade trailing edge.

The wake geometry compatible with the assumption for the constant mean convection velocity is a helicoidal wake with pitch angle $\beta = \tan^{-1}(U_r/\Omega r)$. This wake geometry may be empirically modified to account for the turbine induced velocities. In such case $|\vec{V}_{\infty}|$ is no longer constant in Eq. (19) and the solution is of the form, Eq. (22), if the tangential induced velocities are assumed zero so that the convection velocity along the wake has the tangential component $\Omega r$. As initial condition a steady flow solution obtained with the time-averaged axisymmetric inflow may be given.

In order to specify uniquely the circulation around the blades it is necessary to impose the Kutta condition at the trailing edge. The Kutta condition states that the velocity must remain bounded at a sharp edge

$$|\nabla \phi| < \infty.$$  

(24)

**Integral Equation**

Applying Green's second identity and using the Morino's formulation (Morino and Kuo, 1974), we obtain the integral representation of the perturbation potential at a point $P$ on the body surface,

$$2\pi \phi (p, t) = \iint_{S_b \cup S_c \cup S_w} \left[ G(p, q) \frac{\partial \phi (q, t)}{\partial n_q} - \phi (q, t) \frac{\partial G(p, q)}{\partial n_q} \right] dS_q + \iint_{S_w} \Delta \phi(q, t) \frac{\partial G(p, q)}{\partial n_q} dS_q, \quad p \in S_b \cup S_c,$$

(25)

where $G(p, q) = -1/R(p, q), R(p, q)$ is the distance between the field point $P$ and the point $q$ on the boundary $S_b \cup S_c \cup S_w$, $\vec{n}_q$ is the unit vector normal to the integration surface and $\Delta \phi$ is the potential-jump across the wake surface $S_w$.

Since a thin cavity is assumed and the boundary conditions on the cavitating surfaces are applied on the blade surface beneath the cavity $S'_c$, Eq. (25) becomes:

$$2\pi \phi (p, t) = \iint_{S_b \cup S'_c} \left[ G(p, q) \frac{\partial \phi (q, t)}{\partial n_q} - \phi (q, t) \frac{\partial G(p, q)}{\partial n_q} \right] dS_q + \iint_{S_w} \Delta \phi(q, t) \frac{\partial G(p, q)}{\partial n_q} dS_q, \quad p \in S_b \cup S'_c.$$  

(26)
Dipole and Source Strengths Decomposition

The potential and source distributions can be decomposed in the wetted flow solution (subscript $w$) and in the cavity perturbation to the wetted flow solution (subscript $c$):

$$\phi = \phi_w + \phi_c, \quad \sigma = \sigma_w + \sigma_c. \quad (27)$$

From the integral equation, Eq. (26), and using the decomposition, Eq. (27), we write

$$2\pi \left[ \phi_w (p,t) + \phi_c (p,t) \right] = \int_{S_w \cup S'_c} \left[ G(p,q) \left[ \sigma_w (q,t) + \sigma_c (q,t) \right] - \left[ \phi_w (q,t) + \phi_c (q,t) \right] \frac{\partial G(p,q)}{\partial n_q} \right] dS,$$

$$-\int_{S_w} \left[ \Delta \phi_w (q,t) + \Delta \phi_c (q,t) \right] \frac{\partial G(p,q)}{\partial n_q} dS, \quad p \in S_b \cup S'_c, \quad (28)$$

where

$$\sigma = \sigma_w + \sigma_c = \left( \frac{\partial \phi}{\partial n} \right)_w \left( \frac{\partial \phi}{\partial n} \right)_c, \quad (29)$$

is the decomposition of the source distribution in the body surface $S_b \cup S'_c$.

Since the wetted solutions cancel and $\sigma_c = 0$ in the wetted part $S_b$, a new integral equation is obtained for the cavity perturbation to the wetted solution of the potential and source distributions:

$$2\pi \phi_c (p,t) + \int_{S_w} \phi_c (q,t) \frac{\partial G(p,q)}{\partial n_q} dS - \int_{S'_c} \left[ G(p,q) \right] \sigma_c (q,t) dS = -\int_{S_w} \phi_c (q,t) \frac{\partial G(p,q)}{\partial n_q} dS - \int_{S'_c} \Delta \phi_c (q,t) \frac{\partial G(p,q)}{\partial n_q} dS, \quad p \in S_b \cup S'_c, \quad (30)$$

The solution of Eq. (30) determines $\phi_c$ on $S_b$ and $\sigma_c$ on $S'_c$, with $\phi_c$ on $S'_c$ given from Eq.(15) and Eq. (27). The wetted potential distribution $\phi_w$ on $S_b$ is obtained from Eq. (26) without the cavitation surface $S'_c$ and with $\sigma_w$ on $S_b$ known from Eq. (10). The Kutta condition, Eq. (24), yields the additional relationship between the dipole strength $\Delta \phi_c (q,t)$ in the wake surfaces $S_w$ and the dipole strength at the blade trailing edges.

If $\sigma_c$ is known on the cavity surface $S'_c$ and $\Delta \phi_c$ is known on $S_w$, Eq. (30) defines a Fredholm integral equation of second kind on $S_b$ with the same kernel and domain as Eq.(26) for the wetted from problem. The present approach solves iteratively this coupled pair of integral equations, exploring the prior inversion of the wetted flow integral operator in the larger domain $S_b$.

**NUMERICAL METHOD**

**Solution of the Integral Equation**

For the numerical solution of the integral equation, Eq. (30), we discretise the surfaces $S_b \cup S'_c$ and the wake surfaces $S_w$ in bi-linear quadrilateral panels. The numerical solution
of the integral equation, Eq. (30), is obtained in the time domain at the time steps \( n = t / \Delta t \), where \( \Delta t \) is the constant time step. The integral equation, Eq. (30), is solved in space by the collocation method with the element centre point as collocation point. On the blade and hub surfaces \( S_g \), the dipole and source distributions are assumed to be constant on each panel. On the wake surface \( S_w \) piecewise linear or constant dipole distributions are assumed, depending on the specific location of the panel. The influence coefficients are determined analytically using the formulations of Morino and Kuo (1974). The solution in the rotating frame is periodic in time with a period, in general, equal to the time of a turbine revolution. By considering only one blade, referred as key blade, a reduction in the dimension of the system of equations is obtained. Hence, the contributions of the other blades are assumed to be known when solving for the key blade. The linear system of equations is solved with a direct solver (LU factorisation). The value of the dipole strength at the blade trailing edge is determined by the application of an iterative pressure Kutta condition, requiring that the pressure is equal on the collocation points of the blade panels adjacent to the trailing edge on each side. The non-dimensional time step \( \Delta \theta = \Omega \Delta t \) is introduced. We denote the total number of time steps by \( N_t = N_{rev} \times N_\theta \), where \( N_{rev} \) is the number of revolutions (or periods) for the time integration and \( N_\theta = 2\pi / \Delta \theta \). The details for the solution of the integral equation are found in Vaz (2005).

### Detachment and Reattachment Conditions

The initial detachment and reattachment positions are first obtained based on the fully wetted pressures. To allow for the occurrence of mid-chord cavitation, the following criterion proposed by Muller and Kinnas (1999) is used for the determination of the detachment point:

- If the cavity thickness is negative, then the detachment point is moved downstream towards the trailing edge of the blade.
- If the pressure upstream of the detachment point is below the vapour pressure, then the detachment point is moved upstream towards the leading edge of the blade.

A simple method is used for the determination of the reattachment position. If the cavity thickness is positive, then the reattachment point is moved downstream. In the case of a negative thickness, the reattachment point is moved upstream.

### Solution Method

The procedure starts by searching for cavitation inception on the blades, i.e. \( \min \{ -C_{ps} \} > \sigma_n \). Then, the solution of the problem for a given cavitation number at each time step is to iterate on the cavity length. For the solution of the cavitating flow, a reduced system of equations is solved on the cavitating panels for the unknown source strengths due to the cavity perturbation to the wetted flow:

\[
[S_c] \{ \sigma_c \} = [D_c] \{ \phi_c \},
\]

where \( [S_c] \) is \( N_c \times N_c \) matrix, \( [D_c] \) is a \( N_c \times N \) matrix, \( \{ \sigma_c \} \) is a \( N_c \) vector of the unknown source strengths on the cavity due to the cavity perturbation to the wetted flow and \( \{ \phi_c \} \) is a \( N \) vector of the known potentials due to the cavity perturbation to the wetted flow, where \( N_c \) is the number of panels with cavitation and \( N \) the total number of panels.
To handle the non-linearity of the pressure Kutta condition, the solution of this system is iteratively coupled with the solution of the complete cavitating flow problem $\phi = \phi_n + \phi_c$ with known source strengths:

$$[D]\{\phi\} = [S]\{\sigma\},$$

rather than to the solution of the cavity perturbation problem $\phi_c$. The larger matrix for this problem $N \times N$, Eq. (32), is identical to the matrix of the wetted flow problem and needs only one inversion. The iteration between the reduced system, Eq. (31), and the complete system, Eq. (32), is performed simultaneously with the iteration on the cavity extension.

**RESULTS**

**General**

The model scale turbine rotor tested by Bahaj et al. (2007) is considered, where a considerable set of experimental data for an 800 mm diameter model in straight and yawed inflow conditions is available. The standard geometry has a pitch angle at the blade root equal to 15 degrees, corresponding to 0 degrees pitch setting at the tip. In the present work, the 5 degrees pitch setting angle at the tip is considered. The detailed geometry of the turbine rotor is given in Bahaj et al. (2007). For the purposes of modelling an approximate hub geometry is assumed.

Figure 2 shows a typical panel arrangement of the turbine rotor. Note that only one wake grid is shown. Each turbine is discretised with 80 and 31 panels in the chordwise and radial directions, respectively. The blade wake is discretised with 630 panels along the streamwise direction and 30 panels along the radial direction. The pitch of the helicoidal lines is constant and was obtained from the lifting line theory with optimum circulation distribution (Falcão de Campos, 2007).

For the turbine rotor, the non-dimensional quantities used to express the general performance characteristics are the tip-speed-ratio $TSR$, the axial force coefficient $C_T$ and the power coefficient $C_P$:

$$TSR = \frac{\Omega R}{U}, \quad C_T = \frac{T}{\frac{1}{2} \rho U^2 \pi R^2}, \quad C_P = \frac{\Omega Q}{\frac{1}{2} \rho U^3 \pi R^2}$$

where $T$ is the axial force and $Q$ the rotor torque.
Steady Cavitating Flow

The method is first tested for the prediction of steady sheet cavitation in uniform inflow conditions. Calculations are carried out at $\text{TSR} = 7.0$ with $\sigma_n = 6.5$ with the effect of gravity neglected. Fig. 3 illustrates the convergence history of the non-dimensional maximum cavity length $I_{c\text{ max}}/R$, the cavity area per expanded blade area $A_c/A_0$ and a characteristic cavity thickness $\eta^* = 10V_c/A_cI_{c\text{ max}}$, where $I_{c\text{ max}}$ is the maximum cavity length, $A_c$ the cavity area and $V_c$ the cavity volume. Convergence of the cavity is attained when the relative differences in $I_{c\text{ max}}/R$, $A_c/A_0$ and $\eta^*$ are less than $10^{-3}$. The complete coupled procedure converged after 11 iterations and the iteratively coupled procedure converged after 17 iterations. With the iteratively coupled procedure, a relative computational time of 19% per iteration is obtained in respect to the complete coupled procedure in this case.

Unsteady Cavitating Flow

The effect of a tidal velocity profile on the unsteady turbine loading is presented. A power law of the form:

$$\frac{u(d)}{u_0} = \left(\frac{d}{d_0}\right)^{\frac{3}{7}},$$

based on the calculations of Young et al. (2010) is assumed for the tidal velocity profile, where $d$ is the vertical distance measured from the bottom of the seabed and $u_0$ is the reference velocity defined at the centre of the hub axis with $d_0=15$m. Calculations are performed for the turbine at TS = 4.28 in a velocity profile based on a 10 m radius turbine with 30 m depth and tidal velocity of 3.5 m/s. Assuming the vapor pressure to be $p_v=1230$Pa, the cavitation number $\sigma_n$ defined at the hub axis is 21.47. For the calculations, $N_{\theta}=90$ time steps per revolution are considered, which leads to an angular step of 4 degrees. For the number of revolutions $N_{rev}=5$ is used.
Both procedures converged for the solution of the cavitating flow at each time step. Convergence of the cavity is attained when the relative differences in $\frac{I_{c_{max}}}{R}, \frac{A_c}{A_0}$ and $\eta^*$ are less than $10^{-3}$ for each time step. At the top position ($\theta_0 = 0^\circ$), the cavity planform converged after 9 iterations with both procedures. The differences between the iteratively and the coupled procedures are less than 1% for the perturbation potential when the blade is at the top position. However, larger differences are seen for the pressure coefficient in the cavity closure region. With the iteratively coupled procedure, a relative computational time of 16% is obtained in this case in respect to the complete coupled procedure. The cavity extent and cavity thickness are shown in Fig. 5, where leading-edge partial cavities when the blades are near the top position are obtained. Similar cavity planforms are seen between the present method and the results of Young et al. (2010). The pressure distribution at radial section $r/R = 0.9$ is presented in Fig. 6. Due to the tidal velocity profile, an oscillatory behaviour of the blade pressure distribution is obtained. In this case, the pressure distribution is only presented when the key blade is near the top position, corresponding to the situation where cavitation occurs.
CONCLUDING REMARKS

In this paper an iteratively coupled solution method implemented in a low-order BEM is presented for the calculation of the unsteady potential flow on marine current turbines with sheet cavitation. The iteratively coupled solution method converges to the solution of the usual complete coupled procedure. The maximum differences are in the cavity closure region, which is related to the uncertainty in the cavity extents due to the panel discretisation error. The use of a reduced system for the calculation of the cavity extent and thickness distribution reduces significantly the computational time. This reduction makes the iteratively coupled procedure especially attractive to unsteady computations, where the cavity has to be iterated for each time step.

REFERENCES