DYNAMIC SELF-ORGANIZATION IN CUTTING PROCESS EVOLUTION

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ABSTRACT
This paper differs from the well-known works in the field of cutting process dynamics concerned with stability of the shape-generating tool paths relative to the workpiece, and with attracting sets formed in the vicinity of such paths, in that it contains the results of studies of the variation in the properties of the cutting system in the course of its evolution. In this case, for the purposes of study of the system's modification, the dynamic link parameters are presented as Volterra integral operators of the second kind relative to the phase-plane trajectory of power as a function of work. We offer a method of study of the evolution equations based on digital models, as well as methods of identification of the integral operators' parameters and nuclei. The studies are aimed at prediction of the variation in the dynamic modes and at construction of systems for dynamic diagnostics of the cutting process state.

Keywords: metal cutting, diagnostic, dynamic system, evolution, self-organization.

INTRODUCTION
Formation of the synergetic paradigm of evolution and self-organization has advanced to one of the most important problems of science [1below1-3] and has found its reflection in the study of the cutting process dynamics [4]. In accordance with the synergetic paradigm, study of the machining process dynamics requires the motion equations for two subsystems: those of the tool and of the workpiece being machined, and a mathematical description of the cutting forces in the state coordinates, i.e. a model of the dynamic link uniting the interacting subsystems. The cutting forces are formed as a function of the tool motion paths relative to the workpiece, i.e. the motion paths of the machine-tool executive components, as well as of the paths of elastic deformation displacements of the tool point relative to the workpiece [5 - 14]. The most commonly used are equations of a moving coordinate system, where the motion is determined by the paths of the machine-tool executive components. Then the state coordinates are the elastic deformation displacements of the tool relative to the workpiece, considered in the moving coordinate system and determined by the paths of the executive components. The system equilibrium point in this case is found on the basis of the constant elastic deformation condition. The following classes of models have been identified.

1). Variations of the cutting forces lag behind the deformational displacements, account taken of the coordinate links formed by cutting [5 - 12]. As a rule, account is also taken of the nonlinearity of the forces against the state coordinates [13 - 15].

2). The cutting forces depend on the speed. In such cases the dynamics is practically reduced to the well-known Rayleigh or Van der Pol models. Both deterministic and stochastic models are considered [16 - 18].
3). There is ambiguity in variation of the cutting forces as the tool is moving towards the workpiece being machined or away from it. The related hysteresis dependencies become the source of self-oscillations [19, 20].

4). Mathematical models of the dynamic link have constant and periodically varying parameters. The simplest example for this case is the Mathieu-Hill model [21]. Such models display various parametric phenomena, in particular, parametric self-excitation [22 - 25]. The Floquet theory is commonly used for analysis of the linearized equation in variations [26].

Also used are various combinations of the above models, which just underlines the complexity and diversity of the numerous physical processes that have an effect on the dynamics. The following properties are analyzed: stability of the equilibrium or of the required path, and the attracting sets formed in the vicinity of equilibrium (stable limit cycles, invariant tori and strange attractors) [27, 28]. However, it should be recognized that all models in use take no account of the system's evolutionary change, the external manifestations of which are well known (such as development of the tool wear, changes in the quality parameters of the surface formed by machining, etc.). Thus, the dynamic modification of the system in the course of functioning of the cutting system is ignored. The fundamental difference of this study consists in the development of a mathematical description of the dynamic cutting system as an evolving system. The factor causing the evolutionary changes is the phase-plane trajectory of the power of irreversible transformations in the cutting zone as a function of work done.

2. Mathematical simulation of the evolutionary system.

Methods of analysis

Simulation is based on the following hypothesis: the dynamic link parameters depend on the phase-plane trajectory of the power of irreversible transformations in the cutting zone as a function of work done. Without going to the heart of the physics of irreversible transformations, the dynamic link parameters can be represented as Volterra integral operators of the second kind, while the dynamics equation can be represented as follows, with regard to the evolutionary transformations:

\[
\begin{align*}
\mathbf{m} \frac{d^2 \mathbf{X}}{dt^2} + \mathbf{h} \frac{d \mathbf{X}}{dt} + \mathbf{c} \mathbf{X} &= \mathbf{F}^{[0]}(\mathbf{p}) \mathbf{X}, \\
\mathbf{M} \frac{d^2 \mathbf{Y}}{dt^2} + \mathbf{H} \frac{d \mathbf{Y}}{dt} + \mathbf{C} \mathbf{Y} &= \mathbf{F}^{[0]}(\mathbf{p}) \mathbf{Y}, \\
p^{[0]}(A) &= p_{i,0} + \sum_{\zeta} w_i(A - \zeta) N(\zeta) d \zeta, i = 1, 2, \ldots; \\
A(t) &= \int_0^t N(\tau) d \tau; \quad N(t) = V(t) F(t),
\end{align*}
\]

where \( \mathbf{m}, \mathbf{M}, \mathbf{h}, \mathbf{H}, \mathbf{c}, \mathbf{C} \) are positive definite symmetrical \( 3 \otimes 3 \) matrices of the inertial, dissipative and elastic coefficients relating to the tool and workpiece subsystems; \( \mathbf{F} = [F_1, F_2, F_3]^T \) is the vector function of the cutting forces formed in the system state coordinates, depending on the process modes \( S_p^{[0]}, t_p^{[0]}, V_p^{[0]} \) (longitudinal feed, depth and rate of cutting), which are considered constant for the purposes of this study. It also depends on
the evolving parameters \( p = \{p_1, p_2, ..., p_i\} \in P \) of the dynamic link;  
\[ w_i(A - \varsigma) = \exp[-(A - \varsigma)/T_{i,1}] + \beta_i \exp[(A - \varsigma)/T_{i,2}] \]  
are the nuclei of the integral operators, whereby \( T_{i,1}, T_{i,2} \) are constant works measured in \( kfm \) and reflecting the evolutionary heredity of the parameters as the work progresses; \( \alpha_i \) are the parameters that determine the relation between the power trajectory as a function of work with the parameter under examination; \( A(t), N(t) \) are the work and the power of irreversible transformations.

In (1) \( V(t) \) is the speed of motion of the tool point relative to the workpiece, i.e. in the direction of the cutting path. \( F_i(t) \) is the force exerted in the cutting zone and directed against the speed \( V \). If \( \alpha_i = 0 \), then we have the conventional dynamic analysis problem. Evolution exerts two contradicting actions. The first one is connected with the mutual adaptation of the tool and the cutting process. It is simulated by the first integral operator. The second one is determined by degradation of properties. This causes changes in dynamic link parameters. It is simulated by the second integral operator.

Let us make some preliminary notes about the model (1). The system (1) is functional. For the purposes of this study assume that the parameters \( p^{(i)}(A) \) are slowly changing in time. Then we can use the notion of stationary evolutionary trajectory \( X'(A), Y'(A) \), corresponding to the evolving parameters \( p^{(i)}(A) \). Study of the system (1) is performed in two stages. At the first stage the stationary evolutionary trajectory and the evolutionary changes of the parameters are calculated. The parameters and the stationary evolutionary trajectory itself are the solution of the integral equation

\[
\begin{align*}
\mathbf{cX}'(A) &= \mathbf{F}\left[ S_p^{(0)}, t_p^{(0)}, V_p^{(0)}, X', 0, Y', 0, p(A, N) \right]; \\
\mathbf{CY}'(A) &= \mathbf{F}\left[ S_p^{(0)}, t_p^{(0)}, V_p^{(0)}, X', 0, Y', 0, p(A, N) \right]; \\
p^{(i)}(A) &= p_{i,0} + \alpha_i \int_0^A w_i(A - \varsigma) N(\varsigma)d\varsigma, i = 1, 2, ..., s; \\
A(t) &= \int_0^t N(\tau)d\tau; \quad N(t) = V(t)F(t).
\end{align*}
\]

The parameters \( p^{(i)}(A) \) calculated from (2) characterize a certain set corresponding to the set \( (X'(A), Y'(A)) \). These, in turn, correspond to the set of forces \( (F'(A)) \). Since \( p(A, N) \) and \( (X'(A), Y'(A)) \) are slowly changing work functions, then for the fixed section \( \Delta A(i) = A_i - A_{i-1} \) the parameters \( p \) can be considered invariable. Then work is represented by the vector \( \mathbf{A} = \{A_1, A_2, ..., A_i\}^T \), corresponding to the vector of evolutionary parameters \( p^E = \{p(A_1), p(A_2), ..., p(A_i)\}^T \in P \). Thus, analysis (2) allows 1) to determine, in the space \( \mathbf{P} \), the evolutionary trajectory of the parameters \( p^E \), and 2) to calculate the corresponding equilibrium points \( X^{(*)} = \{X^{(*)}(A_1), X^{(*)}(A_2), ..., X^{(*)}(A_i)\}^T \) and \( Y^{(*)} = \{Y^{(*)}(A_1), Y^{(*)}(A_2), ..., Y^{(*)}(A_i)\}^T \), which characterize the stationary evolutionary trajectory.

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At the second stage we analyze the equation in variations relative to the evolutionary stationary trajectory at a fixed value of $A_i$

$$\begin{align*}
\mathbf{m} \frac{d^2 \mathbf{x}}{dt^2} + \mathbf{h} \frac{d \mathbf{x}}{dt} + \mathbf{c} \mathbf{x} &= \Phi \left[ S_p^{(0)}, t_p^{(0)}, V_p^{(0)}, X', Y', \mathbf{x}, \frac{d \mathbf{x}}{dt}, \frac{d \mathbf{y}}{dt}, \mathbf{p}(A) \right]; \\
\mathbf{M} \frac{d^2 \mathbf{y}}{dt^2} + \mathbf{H} \frac{d \mathbf{y}}{dt} + \mathbf{C} \mathbf{y} &= \Phi \left[ S_p^{(0)}, t_p^{(0)}, V_p^{(0)}, X', Y', \mathbf{x}, \frac{d \mathbf{x}}{dt}, \frac{d \mathbf{y}}{dt}, \mathbf{p}(A) \right],
\end{align*}$$

where

$$\Phi \left[ S_p^{(0)}, t_p^{(0)}, V_p^{(0)}, X', Y', \mathbf{x}, \frac{d \mathbf{x}}{dt}, \frac{d \mathbf{y}}{dt}, \mathbf{p}(A) \right] = F \left[ S_p^{(0)}, t_p^{(0)}, V_p^{(0)}, X', Y', \mathbf{x}, \frac{d \mathbf{x}}{dt}, \frac{d \mathbf{y}}{dt}, \mathbf{p}(A) \right] - F \left[ S_p^{(0)}, t_p^{(0)}, V_p^{(0)}, X', Y', \mathbf{x}, 0, y, 0, \mathbf{p}(A) \right]$$

is the new nonlinear function in variations relative to the point $X', Y'$;

while $x$ and $y$ are vectors of variation of the elastic deformation displacements relative to the point $X', Y'$, i.e. $X = X' + x$, $Y = Y' + y$. In (3) for the given value of work, $S_p^{(0)}$, $t_p^{(0)}$, $V_p^{(0)}$, $X'$, $Y'$, $\mathbf{p}(A)$ are the constant parameters. Therefore, analysis of stability of the evolutionary stationary trajectory requires consideration of the linearized system (3) [29]

$$\mathbf{m} \frac{d^2 \mathbf{z}}{dt^2} + \mathbf{h} \frac{d \mathbf{z}}{dt} + \mathbf{c} \mathbf{z} = 0,$n

where $\mathbf{z} = \{x_1, x_2, x_3, y_1, y_2, y_3\}^T$; $\mathbf{m} =$

$$\begin{bmatrix}
m & 0 & 0 & 0 & 0 & 0 \\
0 & m & 0 & 0 & 0 & 0 \\
0 & 0 & m & 0 & 0 & 0 \\
0 & 0 & 0 & M & 0 & 0 \\
0 & 0 & 0 & 0 & M & 0 \\
0 & 0 & 0 & 0 & 0 & M
\end{bmatrix};$$

and

$$\mathbf{h} = 
\begin{bmatrix}
h_{1,1} \frac{\partial \Phi}{\partial x_1} & h_{1,2} \frac{\partial \Phi}{\partial x_2} & h_{1,3} \frac{\partial \Phi}{\partial x_3} & -\frac{\partial \Phi}{\partial y_1} & -\frac{\partial \Phi}{\partial y_2} & -\frac{\partial \Phi}{\partial y_3} \\
-\frac{\partial \Phi}{\partial x_1} & h_{2,1} \frac{\partial \Phi}{\partial x_1} & h_{2,2} \frac{\partial \Phi}{\partial x_2} & h_{2,3} \frac{\partial \Phi}{\partial x_3} & -\frac{\partial \Phi}{\partial y_1} & -\frac{\partial \Phi}{\partial y_2} & -\frac{\partial \Phi}{\partial y_3} \\
-\frac{\partial \Phi}{\partial x_1} & -\frac{\partial \Phi}{\partial x_2} & h_{3,1} \frac{\partial \Phi}{\partial x_1} & h_{3,2} \frac{\partial \Phi}{\partial x_2} & h_{3,3} \frac{\partial \Phi}{\partial x_3} & -\frac{\partial \Phi}{\partial y_1} & -\frac{\partial \Phi}{\partial y_2} & -\frac{\partial \Phi}{\partial y_3} \\
-\frac{\partial \Phi}{\partial x_1} & -\frac{\partial \Phi}{\partial x_2} & -\frac{\partial \Phi}{\partial x_3} & H_{1,1} \frac{\partial \Phi}{\partial y_1} & H_{1,2} \frac{\partial \Phi}{\partial y_2} & H_{1,3} \frac{\partial \Phi}{\partial y_3} & -\frac{\partial \Phi}{\partial y_1} & -\frac{\partial \Phi}{\partial y_2} & -\frac{\partial \Phi}{\partial y_3} \\
-\frac{\partial \Phi}{\partial x_1} & -\frac{\partial \Phi}{\partial x_2} & -\frac{\partial \Phi}{\partial x_3} & -\frac{\partial \Phi}{\partial y_1} & H_{2,1} \frac{\partial \Phi}{\partial y_1} & H_{2,2} \frac{\partial \Phi}{\partial y_2} & H_{2,3} \frac{\partial \Phi}{\partial y_3} & -\frac{\partial \Phi}{\partial y_1} & -\frac{\partial \Phi}{\partial y_2} & -\frac{\partial \Phi}{\partial y_3} \\
-\frac{\partial \Phi}{\partial x_1} & -\frac{\partial \Phi}{\partial x_2} & -\frac{\partial \Phi}{\partial x_3} & -\frac{\partial \Phi}{\partial y_1} & -\frac{\partial \Phi}{\partial y_2} & H_{3,1} \frac{\partial \Phi}{\partial y_1} & H_{3,2} \frac{\partial \Phi}{\partial y_2} & H_{3,3} \frac{\partial \Phi}{\partial y_3} & -\frac{\partial \Phi}{\partial y_1} & -\frac{\partial \Phi}{\partial y_2} & -\frac{\partial \Phi}{\partial y_3}
\end{bmatrix};$$

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The system (4) has constant parameters within a small section of work variation. Therefore, for analysis of stability we can use one of the known criteria or analyze the evolutionary diagram of the characteristic polynomial root of the system (4). If, within a certain section of work variation, all characteristic polynomial roots of the system (4) have negative real parts, then the calculated stationary evolutionary trajectory is an attractor, whereby \( z \to 0 \). Otherwise the attracting sets \( \mathbf{z}^*(t) \) are formed in the vicinity of this trajectory and bring distortion into the evolutionary trajectory. This can be corrected by using the method of averaging [17], i.e. by consideration of the system

\[
\begin{bmatrix}
\frac{\partial \Phi_1}{\partial \mathbf{x}_1} & \cdots & \frac{\partial \Phi_1}{\partial \mathbf{x}_{c_1}} & \frac{\partial \Phi_1}{\partial \mathbf{y}_1} & \cdots & \frac{\partial \Phi_1}{\partial \mathbf{y}_{c_1}} \\
\vdots & & \vdots & \vdots & & \vdots \\
\frac{\partial \Phi_{c_1}}{\partial \mathbf{x}_1} & \cdots & \frac{\partial \Phi_{c_1}}{\partial \mathbf{x}_{c_1}} & \frac{\partial \Phi_{c_1}}{\partial \mathbf{y}_1} & \cdots & \frac{\partial \Phi_{c_1}}{\partial \mathbf{y}_{c_1}} \\
\frac{\partial \Phi_1}{\partial \mathbf{x}_2} & \cdots & \frac{\partial \Phi_1}{\partial \mathbf{x}_{c_1}} & \frac{\partial \Phi_1}{\partial \mathbf{y}_2} & \cdots & \frac{\partial \Phi_1}{\partial \mathbf{y}_{c_1}} \\
\vdots & & \vdots & \vdots & & \vdots \\
\frac{\partial \Phi_1}{\partial \mathbf{x}_3} & \cdots & \frac{\partial \Phi_1}{\partial \mathbf{x}_{c_1}} & \frac{\partial \Phi_1}{\partial \mathbf{y}_3} & \cdots & \frac{\partial \Phi_1}{\partial \mathbf{y}_{c_1}} \\
\end{bmatrix} \mathbf{c}_z = 
\begin{bmatrix}
-\frac{\partial \Phi_1}{\partial \mathbf{x}_1} & \cdots & -\frac{\partial \Phi_1}{\partial \mathbf{x}_{c_1}} & -\frac{\partial \Phi_1}{\partial \mathbf{y}_1} & \cdots & -\frac{\partial \Phi_1}{\partial \mathbf{y}_{c_1}} \\
\vdots & & \vdots & \vdots & & \vdots \\
-\frac{\partial \Phi_1}{\partial \mathbf{x}_3} & \cdots & -\frac{\partial \Phi_1}{\partial \mathbf{x}_{c_1}} & -\frac{\partial \Phi_1}{\partial \mathbf{y}_3} & \cdots & -\frac{\partial \Phi_1}{\partial \mathbf{y}_{c_1}} \\
\end{bmatrix}
\]

The system (4) has constant parameters within a small section of work variation. Therefore, for analysis of stability we can use one of the known criteria or analyze the evolutionary diagram of the characteristic polynomial root of the system (4). If, within a certain section of work variation, all characteristic polynomial roots of the system (4) have negative real parts, then the calculated stationary evolutionary trajectory is an attractor, whereby \( z \to 0 \). Otherwise the attracting sets \( \mathbf{z}^*(t) \) are formed in the vicinity of this trajectory and bring distortion into the evolutionary trajectory. This can be corrected by using the method of averaging [17], i.e. by consideration of the system

\[
\begin{bmatrix}
\mathbf{m} \frac{d^2 \mathbf{x}}{dt^2} + \mathbf{h} \frac{d \mathbf{x}}{dt} + \mathbf{c} \mathbf{x} = \dot{\mathbf{\Phi}} [S_p(t), \mathbf{t}_p(t), V_p(t), \mathbf{X}, \mathbf{Y}, \mathbf{x}^*(t), \frac{dx^*(t)}{dt}, \frac{dy^*(t)}{dt}, \mathbf{p}(A)]; \\
\mathbf{M} \frac{d^2 \mathbf{y}}{dt^2} + \mathbf{H} \frac{d \mathbf{y}}{dt} + \mathbf{C} \mathbf{y} = \dot{\mathbf{\Phi}} [S_p(t), \mathbf{t}_p(t), V_p(t), \mathbf{X}, \mathbf{Y}, \mathbf{x}^*(t), \frac{dx^*(t)}{dt}, \frac{dy^*(t)}{dt}, \mathbf{p}(A)].
\end{bmatrix}
\]

(5)

where \( \dot{\mathbf{\Phi}}[A, x^*(t), y^*(t)] = \frac{1}{T_0 - i_0} \int_{i_0}^{i} \dot{\mathbf{\Phi}} [S_p(t), \mathbf{t}_p(t), V_p(t), \mathbf{X}, \mathbf{Y}, \mathbf{x}^*(t), \frac{dx^*(t)}{dt}, \frac{dy^*(t)}{dt}, \mathbf{p}(A)] dt \).

Without going into details, consider an example of analysis of the evolutionary modification of the system. Note that examples are often just as useful as the general formulation of the problem.

EXAMPLE OF ANALYSIS OF EVOLUTIONARY TRANSFORMATIONS IN THE CUTTING SYSTEM

Let us dwell on the dynamic model of orthogonal cutting (Fig. 1), which includes the following properties [4 - 8]: the workpiece is rigid and rotating at a constant speed \( \Omega = 1/T = \text{const} \) \( T \) is the time of one revolution; the deformations \( \mathbf{X} = \{X_1, X_2,\}^T \) are considered in the plane normal to the cutting plane; the elastic and dissipative properties of the tool subsystem are linear. Consequently, there are ellipses of rigidity and dissipation with only one orientation in the plane given by the angle \( \alpha \) (Fig. 1). The forces \( \mathbf{F} = \{F_1, F_2,\}^T \) have an invariable orientation in the plane \( X_1, X_2,\) and their module changes with variation of the cut layer area. The equation determining the system dynamics has the following form

\[
\begin{bmatrix}
\mathbf{m} \frac{d^2 \mathbf{x}}{dt^2} + \mathbf{h} \frac{d \mathbf{x}}{dt} + \mathbf{c} \mathbf{x} = \dot{\mathbf{\Phi}} [
\begin{bmatrix}
S_p(t), \mathbf{t}_p(t), V_p(t), \mathbf{X}, \mathbf{Y}, \mathbf{x}^*(t), \frac{dx^*(t)}{dt}, \frac{dy^*(t)}{dt}, \mathbf{p}(A)
\end{bmatrix}; \\
\mathbf{M} \frac{d^2 \mathbf{y}}{dt^2} + \mathbf{H} \frac{d \mathbf{y}}{dt} + \mathbf{C} \mathbf{y} = \dot{\mathbf{\Phi}} [
\begin{bmatrix}
S_p(t), \mathbf{t}_p(t), V_p(t), \mathbf{X}, \mathbf{Y}, \mathbf{x}^*(t), \frac{dx^*(t)}{dt}, \frac{dy^*(t)}{dt}, \mathbf{p}(A)
\end{bmatrix].
\end{bmatrix}
\]

(5)

where \( \dot{\mathbf{\Phi}}[A, x^*(t), y^*(t)] = \frac{1}{T_0 - i_0} \int_{i_0}^{i} \dot{\mathbf{\Phi}} [S_p(t), \mathbf{t}_p(t), V_p(t), \mathbf{X}, \mathbf{Y}, \mathbf{x}^*(t), \frac{dx^*(t)}{dt}, \frac{dy^*(t)}{dt}, \mathbf{p}(A)] dt \).

Without going into details, consider an example of analysis of the evolutionary modification of the system. Note that examples are often just as useful as the general formulation of the problem.
\[ \mathbf{m} \frac{d^2 \mathbf{X}}{dt^2} + \mathbf{h} \frac{d \mathbf{X}}{dt} + \mathbf{c} \mathbf{X} = \mathbf{F}, \]  

(6)

where  
\[
\mathbf{m} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}; \quad \mathbf{h} = \begin{bmatrix} h_{0,1} \cos^2 \alpha + h_{0,2} \sin^2 \alpha & \frac{1}{2} (h_{0,2} - h_{0,1}) \sin 2\alpha \\ \frac{1}{2} (h_{0,2} - h_{0,1}) \sin 2\alpha & h_{0,1} \sin^2 \alpha + h_{0,2} \cos^2 \alpha \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{2,1} \\ h_{1,2} & h_{2,2} \end{bmatrix};
\]

\[
\mathbf{c} = \begin{bmatrix} c_{0,1} \cos^2 \alpha + c_{0,2} \sin^2 \alpha & \frac{1}{2} (c_{0,2} - c_{0,1}) \sin 2\alpha \\ \frac{1}{2} (c_{0,2} - c_{0,1}) \sin 2\alpha & c_{0,1} \sin^2 \alpha + c_{0,2} \cos^2 \alpha \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{2,1} \\ c_{1,2} & c_{2,2} \end{bmatrix}.
\]

The matrices \( \mathbf{m} \), \( \mathbf{h} \) and \( \mathbf{c} \) are symmetrical and positive determined, and the system (1) has, without going into the force formation mechanism, the only asymptotically stable path determined by \( \mathbf{F}(t) \). When \( \mathbf{F} = \text{const} \), we have an asymptotically stable equilibrium point.

In Fig. 1 the transition of the tool point to that point is shown by transformation of the point "0" to the point "\( X_1, X_2 \)". The situation changes fundamentally, if it is remembered that the forces depend on "\( X_1, X_2 \)". This view is based on the synergetic concept including a description of the system – environment interaction [2, 4, 5, 30]. Let us first analyze the system without evolution. The principal assumptions of simulation of forces, as detailed below, consist in the following.

1. The forces depend on the cut layer area [4 – 9, 23]. In the considered system the cut layer thickness is \( b = \text{const} \). The area varies with the thickness \( a \). Dependence of the forces on the depth is usually determined experimentally for the steady state conditions. Strictly speaking, this dependence is nonlinear, but monotone. At sufficiently large thickness variations, it can be represented in the linearized form:
\[ F_1 = \chi_1 r p \alpha b ; \quad F_2 = \chi_2 r p \alpha b \]  

where \( a, b \) are the thickness and the width of the cut layer in \([\text{mm}]\); \( \chi_1 = \text{const}, \chi_2 = \text{const} \) are angle coefficients; \( \rho \) is the pressure of the chips on the tool face in \([\text{kg/mm}^2]\). This is one of the evolving parameters.

The thickness is determined by the integral operator

\[ a(t) = \int_{t-T}^{t} \{ V^{(II)}(t) - V_{x_1}(t) \} dt , \]

where \( V^{(II)}(t) \) is the speed of the toolpost longitudinal travel; \( V_{x_1}(t) = dX_1 / dt \) is the speed of elastic deformation displacement of the tool in the same direction. In the present study \( V^{(II)} = \text{const} \). Therefore, the thickness

\[ a(t) = \int_{t-T}^{t} \{ V^{(II)}(t) - V_{x_1}(t) \} dt = S_p - [X_1(t) - X_1(t-T)]. \]

Here \( S_p = V^{(II)} T \) is the traditionally considered feed rate. Equilibrium of the system is achieved when \( V_{x_1}(t) = 0 \), that is \( X_1(t) \equiv X_1(t-T) \). Then \( a(t) = S_p = \text{const} \). Otherwise we observe redistribution of forces and deformational displacements depending on the cutting forces. This redistribution characterizes one of the reasons for lag in the variation of the forces caused by variation of the elastic displacements. The main properties of the lag can be accounted for by introduction of an aperiodic link.

2. The chip pressure \( \rho \) depends on the speed. When the speed is increased in the range (40.0-160) m/min, the \( \rho \) is monotonically decreased. In [15] this dependence is presented in the following form (Fig. 2)

\[ \rho(V) = \rho_0 \left(1 + \mu e^{-\alpha(V^{(II)} - dX_1 / dt)}\right), \]

where \( \rho_0 \) is the chip pressure in the high-speed region; \( \mu, \alpha \) are the parameters. In Fig.2 the triangles show the experimental points. Here, as the speed of the tool's deformational displacements \( dX_1 / dt \) grows, the chip pressure and, consequently, the forces, increase. Thus, a positive feedback is formed, which contributes to the loss of equilibrium stability.

3. If the system's equilibrium point becomes unstable and develops periodic motions, then, depending on the speed of the tool's motion towards the workpiece \((-dD_1 / dt)\), the tool's back edge approaches the machined part of the workpiece. The magnitude of the tool's sum back angle \( \theta_{\Sigma} \) changes (see bottom illustrations in Fig. 1). The back angle of the cutting tool may even assume negative magnitudes \( \theta_{\Sigma} = \theta_0 - \Delta \theta \). Here \( \theta_0 \) is the magnitude of the back angle under static conditions, while \( \Delta \theta = \arctg \left(V^{(II)} / V \right) \). As a result, a force is formed, which depends on the speed \( V^{(II)} - dX_1 / dt \) and is conveniently approximated by the exponential function. The force acting on the back edge depends on the oscillation speed, therefore it is characterized in [25] as nonlinear damping.

4. The transient processes in the area of contact between the tool's back face and the workpiece are disregarded. Besides, it is taken into account that \( \theta_{\Sigma} \) is a small value. Therefore, the projection of the force formed in the area of the tool's back face to the direction
$X_2$ differs from the force acting, in the push-away direction, on the friction coefficient $k_{tp}$.

There are two main sources of self-excitation in the model: lag of the cutting forces relative to the deformational displacements, and their reduction with the speed increase.

Besides, allowance is made for the nonlinear damping of oscillations in the direction.

Fig. 2 - Speed characteristic of the variation of the chip pressure on the tool's front face (turning of Steel 45 with a tool made of T15K6 w/o coolant)

\[
\frac{d^2X_1}{dt^2} + h_{1,1} \frac{dX_1}{dt} + h_{2,1} \frac{dX_1}{dt} + c_{1,1}X_1 + c_{2,1}X_2 = \rho_0X_1 \left(1 + \mu e^{-\alpha_1(V_d - dX_1/dt)}\right)(S_p - Y) + F_{(3)}(e^{\alpha_2(V_d - dX_1/dt)} - 1);
\]

\[
\frac{d^2X_2}{dt^2} + h_{1,2} \frac{dX_1}{dt} + h_{2,2} \frac{dX_1}{dt} + c_{1,2}X_1 + c_{2,2}X_2 = \rho_0X_2 \left(1 + \mu e^{-\alpha_1(V_d - dX_1/dt)}\right)(S_p - Y) + F_{(3)}k_{tp}(e^{\alpha_2(V_d - dX_1/dt)} - 1); T_p \frac{dY}{dt} + Y = X_1,
\]

where $T_p$ is the time constant; $F_{(3)}$ is the force acting on the back edge of the tool; $k_{tp}$ is the friction coefficient in the area of contact between the tool's back edge and the workpiece; $\alpha_2$ is the rate of rise of the force acting on the back edge of the tool.

**Property of equilibrium**

From (10) we obtain the equation for calculation of the equilibrium $X^* = \{X_1^*, X_2^*\}^T$

\[
e_{x}X^* = F_x \left(S_p, V, V^{(n)}\right),
\]

where

\[
F_x = \left[\rho_0X_1 \left(1 + \mu e^{-\alpha_1V}\right)S_p + F_{(3)}(e^{\alpha_2V} - 1)\right] \left[\rho_0X_2 \left(1 + \mu e^{-\alpha_1V}\right)S_p + F_{(3)}k_{tp}(e^{\alpha_2V} - 1)\right]^T
\]
The evolving parameters in (11) are \( \rho_0, F^{(3)}_1, \alpha_1 \) and \( \alpha_2 \). Their variation cause displacement of the steady-state magnitude of the elastic deformatonal displacements, i.e. the current magnitude of the workpiece diameter. However, in the present work we focus on the system's dynamic modification causing the changes in the topology of the phase space of the system. Therefore, we shall, above all, dwell on the particular changes in the stability of the slowly shifting equilibrium. Analysis of its stability requires consideration of the linearized equation in variations relative to \( (X_1^*, X_2^*) \) [29]. After substitution of \( X_1 = X_1^* + x_1(t) \) and \( X_2 = X_2^* + x_2(t) \) from (10) we obtain

\[
\frac{d^2 x}{d t^2} + \mathbf{h}_x \frac{d x}{d t} + \mathbf{c}_x x = 0,
\]

where \( \mathbf{m} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \); \( \mathbf{c}_x = \begin{bmatrix} c_{1,1} + \rho_0 \chi_1 \left( 1 + \mu \exp(-\alpha_1 V) \right) & c_{2,1} \\ c_{1,2} + \rho_0 \chi_2 \left( 1 + \mu \exp(-\alpha_1 V) \right) & c_{2,2} \end{bmatrix} \);

\[
\mathbf{h}_x = \begin{bmatrix} h_{1,1} + F^{(3)}_1 \alpha_1 e^{\alpha_1 V} & h_{2,1} - \rho_0 \chi_1 \mu \alpha \alpha_1 e^{-\alpha_1 V} \\ h_{1,2} + F^{(3)}_2 \alpha_2 e^{\alpha_2 V} & h_{2,2} - \rho_0 \chi_2 \mu \alpha \alpha_2 e^{-\alpha_2 V} \end{bmatrix}.
\]

Stability will then be determined by distribution of the roots of the characteristic equation in the complex plane [29]

\[
\Delta(p) = \left( h_{1,1} + F^{(3)}_1 \alpha_1 e^{\alpha_1 V} - \rho_0 \chi_1 \left( 1 + \mu \exp(-\alpha_1 V) \right) T_p \right) p + c_{1,1} + \rho_0 \chi_1 \left( 1 + \mu \exp(-\alpha_1 V) \right) \quad \Leftrightarrow \quad \left( h_{1,2} + F^{(3)}_2 \kappa_\alpha \alpha_2 e^{\alpha_2 V} - \rho_0 \chi_2 \left( 1 + \mu \exp(-\alpha_2 V) \right) T_p \right) p + c_{1,2} + \rho_0 \chi_2 \left( 1 + \mu \exp(-\alpha_2 V) \right) \quad \Leftrightarrow \quad \left( h_{2,1} - \rho_0 \chi_1 \mu \alpha \alpha_1 e^{-\alpha_1 V} (S_p - X_1^*) \right) p + c_{2,1} \quad \Leftrightarrow \quad mp^2 + \frac{c_{2,2}}{p} \quad \Rightarrow \quad mp^2 + \left( h_{2,2} - \rho_0 \chi_1 \mu \alpha \alpha_1 e^{-\alpha_1 V} (S_p - X_1^*) \right) p + c_{2,2} = 0.
\]

(13)

In particular, if variable parameters are specified, then we can use the methods of selection of stability regions in the space of such parameters. However, the stability loss mechanisms common for the dynamic system can be identified by analysis of the structure of the matrices \( \mathbf{h}_x \) and \( \mathbf{c}_x \) in (12).

The fundamental difference of \( \mathbf{h}_x \) and \( \mathbf{c}_x \) from the matrices \( \mathbf{h} \) and \( \mathbf{c} \) in (7) is their asymmetry. We shall start by examining the matrix \( \mathbf{h}_x \) and representing it as its symmetric and antisymmetric components.
The asymmetric component in (14) determines the gyroscopic forces [31, 32]. It is well-known that if the symmetric component (14) is positive definite, it is the dissipation matrix and, subject to positive definiteness of the symmetric part of the elasticity matrix, the system has an asymptotically stable equilibrium point [31, 32]. The gyroscopic forces enhance the stability of the system. If, however, in the same conditions, the symmetric component of the matrix (14) is negative definite, it generates accelerating forces. Then the equilibrium point becomes unstable and the gyroscopic forces cannot stabilize the equilibrium. Thus, the first stability loss mechanism is determined by transformation of the symmetric part of the matrix of speed coefficients from positive definite to negative definite. We shall also represent the matrix \( \Sigma \) in the symmetric and the antisymmetric forms

\[
\begin{bmatrix}
0 & 0,5 & 1 \exp \alpha \rho \\
0,5 & 1 \exp \alpha \rho & 0 \\
1,1 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

The symmetric part in (15), subject to its positive definiteness, refers to the potential forces, while the antisymmetric part refers to circulatory forces orthogonal to the direction of the deformational displacements. They cause formation of circular paths relative to the equilibrium. The published experimental studies of tool oscillations show that when stability is lost, their paths are close to an ellipse. If the symmetric part becomes negative definite, then the equilibrium is unstable and the system develops periodic precession oscillations.

The above considerations supplement and extend the well-known studies of the cutting process stability [4 - 8]. First, stability depends on the control parameters \( S_p \) and \( V \), which preset the equilibrium point. Second, the fundamental effect on stability is exerted by the lag argument \( T_p \) and the rise rate of the coefficient \( \rho_0 \) as the cutting speed grows. Third, all main features of the dynamic system can be analyzed only through the use of the vector (spatial) model. Note that the circulatory and gyroscopic forces do not exist, in concept, in scalar dynamic systems.

**3.2. Evolutionary modification of the dynamic system**

The system (10) simulates two interrelated oscillatory circuits, which have two sources of self-excitation. The first one derives from the forces lagging relative to the deformations,
while the second one is conditioned by the area where the speed rise is matched by the reduction of forces. Hence the three possible stationary states in the system: the asymptotically stable equilibrium; the stable limit cycle; and the two-dimensional invariant torus. Let us consider the system, for which the parameters of the tool dynamic model are shown in Table 1. The considered case is machining of Steel 45 with a tool made of Steel T15K6. The method of parameter identification is described in detail in earlier works [4]. The evolving parameters are $T_p$, $\rho_0$, $\alpha_1$, $\alpha_2$ and $\mu$. The studies show that $T_p$, $\rho_0$ are the most sensitive to evolution. Therefore, in this paragraph the parameters $\alpha_1$, $\alpha_2$ and $\mu$ are regarded as fixed. These are shown in Table 2.

![Table 1 - The parameters of the tool dynamic model](image)

![Table 2 - The parameters $\alpha_1$, $\alpha_2$ and $\mu$](image)

Consider the bifurcation diagram in the plane $T_p$, $\rho_0$ (Fig.2). The shaded part is the asymptotic stability region. The hatched part is the region of the two-dimensional invariant torus. The white areas coincide with generation of self-oscillations. Fig.2 also shows the D-partitioning areas. These divide the space in the B-O-C, C-O-F, and F-O-A regions, and the shaded region. They differ in the number of roots of the characteristic polynomial in the right complex half-plane. The regions B-O-C and F-O-A have a pair of complex conjugate roots with positive real roots, while the region C-O-F has two pairs of such roots. However, the boundaries in the bifurcation diagram (Fig.2), where the two-dimensional invariant torus is formed, are displaced. The line C-O is transformed into the line D-O, while the line D-O becomes the line E-O. For explanation, the phase paths corresponding to the point "1" can be examined. The time diagrams for this case are shown in Fig. 3. It can be seen that the two-frequency process is transformed, with time, into a single-frequency process, i.e. there is pulling of the high-frequency oscillations to the low-frequency range. A similar situation occurs in the (E-O-F) region.

The evolution changes the parameters of the dynamic link formed by the cutting process. The parameters of the tool subsystem remain unchanged. Consider an example of evolutionary modification of the system, the parameters of which are shown in Tables 1 and 2. The evolving parameters are $p_1 = \rho_0$, $p_2 = T_p$. The parameters of the nuclei of the integral operators for the cutting speed of 0.5 m/s are shown in Table 3. The point locus corresponding to the set of evolutionary parameters is shown in Fig. 2 as the thick line on which the dark circles denote the values of the irreversible transformations work, for which Fig. 4 shows examples of trajectories in different sections of the phase space. It must be emphasized that as the work progresses, one can see the areas where the topology of the phase space remains unchanged, and the bifurcation points at which the topology of the phase space changes. Let us discuss the results of the study in terms of their scientific and practical value.
Fig. 3 - Bifurcation diagram in plane \((T_p, \rho_0)\)

Fig. 4 - Example of transformation of a two-frequency process into a single-frequency one, i.e. into an orbitally asymptotically stable limit cycle

Table 3 - The parameters of the nuclei of the integral operators

<table>
<thead>
<tr>
<th>(p_i)</th>
<th>(p_{i,0})</th>
<th>(\alpha_i)</th>
<th>(\beta_i)</th>
<th>(T_{i,1})</th>
<th>(T_{i,2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p_i = p_o)</td>
<td>500 (\frac{kg}{mm^2})</td>
<td>(2 \times 10^{-7} \frac{s}{kg(mm)^4})</td>
<td>0.7</td>
<td>2.99 (10^3) kgm</td>
<td>5.54 (10^3) kgm</td>
</tr>
<tr>
<td>(p_i = T_p)</td>
<td>(1.1 \times 10^{-3})</td>
<td>(1.2 \times 10^{11} \frac{s^2}{(kg)^2(mm)^2})</td>
<td>1.5</td>
<td>4.46 (10^3) kgm</td>
<td>9.66 (10^3) kgm</td>
</tr>
</tbody>
</table>
DISCUSSION OF THE RESULTS
The cutting process is a typical evolving dynamic system, in which, as the work is done by the cutting forces, there emerge, as a rule, bifurcation points where the topology of the phase space changes. Between the bifurcation points a monotone variation of the attracting sets is observed. Even in the case where the equilibrium point in the system remains asymptotically stable, the system's evolution is manifested by displacement of the characteristic polynomial roots. Since the system is perturbed by the stationary wideband force noise, the root displacement induces changes in the frequencies of the main oscillators, the set of which corresponds to the easily observed signal of the vibroacoustic emission of the cutting process. Therefore, the use of autoregressive spectral analysis allows us to create a new data base, which enables us to evaluate the cutting process and the current quality parameters of the workpieces directly in the machining process. In addition to our earlier studies [33, 34 and the well-known research in the field of vibroacoustic diagnostics of the cutting process, the above mathematical models and the method of their study make it possible to identify the significant frequencies, which can be used for construction of the vibroacoustic signal model. One of the most familiar manifestations of the evolutionary changes in cutting is the development of the tool wear and the magnitude of its wear rate. Thus, the combination of simulation of the evolutionary modification of the system and construction of a model of vibroacoustic emission opens a new direction for diagnostics of the tool state during machining by cutting. The concrete results of construction of such diagnostics systems will be presented in our next publications.
The above data show that in the course of evolutionary transformations there occur displacements of the system equilibrium point, i.e. variations of the current magnitudes of the workpiece diameter. Moreover, the vibration sequences of the tool motion relative to the workpiece also change. Hence, the simulation model of dynamics provides for calculation of the transformation of the measurable vibration sequences in the series of the deformational displacements of the tool point relative to the workpiece. This opens the way for creation of systems for dynamic monitoring of the variation of the quality parameters in the manufacture of parts directly in the course of machining.

Analysis of the results of digital simulation of the evolutionary system shows that the bifurcation points and the system's properties depend not only on the parameters of the equation of connection between the interacting subsystems, but also on the parameters of such systems. In particular, if the matrices $c$, $h$ in (6) have equal diagonal elements, then for each orthogonal coordinate system the spatial model of the interacting subsystems is transformed into a system of independent scalar equations, i.e. $c$, $h$ become diagonal. In such a system, identification of the matrices $c_\xi$, $h_\xi$ allows us to immediately identify the elastic and the speed reactions of the cutting process and their variation throughout the system's functioning, including in the course of evolution. In the general case, the system's properties and all stages of its evolution depend mainly on the matrices $c$, $h$, $m$. In particular, at high magnitudes of the tool compliance in the cutting direction, there may emerge chaotic attractors formed via the cascade of period-doubled oscillations, i.e. according to the Feigenbaum scenario.

The formulated new general postulates concerning the mechanisms of equilibrium loss in a system integrate essentially all known stability loss mechanisms, excluding the parametric ones. This makes it possible not only to extend the known postulates, but also to direct the way to assurance of a stable cutting process with regard to the evolutionary variation of parameters.

**CONCLUSION**

1. The dynamic cutting system is determined as a result of interaction of mechanical subsystems through a dynamic link formed in the machining zone. The dynamic link representing the model of forces in the state coordinates is nonlinear relative to the elastic deformation displacements and their rates. Therefore the dynamic cutting system can serve as an example of various effects in nonlinear dynamics.

2. In the linearized equation in variations relative to the point of equilibrium of the elastic deformation displacements, the matrices of speed coefficients and elasticity become asymmetric due to the cutting process reaction. Therefore, they are the natural source of the following force structures:

- dissipative (satisfying the condition of positive definiteness of the symmetric part of the matrix of speed coefficients);
- accelerating (satisfying the condition of negative definiteness of the symmetric part of the matrix of speed coefficients);
- gyroscopic (corresponding to the antisymmetric component of the matrix of speed coefficients);
- potential (corresponding to the symmetric component of the matrix of elasticity);
- circulatory (non-potential) forces formed by the antisymmetric component of the matrix of elasticity.
The main mechanisms of equilibrium stability loss are connected with transformation of the symmetric part of the matrix of speed coefficients from positive definite into negative definite, and also with a significant increase of the circulatory forces relative to their potential components.

3. In between the bifurcation points the topology of the phase space remains unchanged. However, the system's properties, e.g. the self-oscillation frequencies vary monotonically. This opens up possibilities for construction of systems for diagnostic of the cutting process state based on vibration characteristics, e.g. on the basis of models of autoregressive spectral analysis.

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REFERENCES


[19]-Drozdov N.A. On the question of the vibration machine for turning. Machines and tools, 1937, №22. (in russian)


[33]-Zakovorotny V.L., Bordachev E.V. Information support of a system of dynamic diagnostics of the tool wearout on the example of turning. Problems of mechanical engineering and reliability of the machines. 1995.№ 3. a. 95 - 101. (in russian)