THE CUSUM VERSUS MCUSUM MODIFIED CONTROL CHARTS WHEN APPLIED ON DIESEL ENGINES PARAMETERS CONTROL

Suzana Lampreia\(^{(1)}\), Rui Parreira\(^{1,2}\), José Requeijo\(^{3}\), Vitor Lobo\(^{1,2}\)

\(^{1}\)Centro de Investigação Naval (CINAV), Portugal
\(^{2}\)Portuguese Naval Academy, Lisbon, Portugal
\(^{3}\)Faculty of Science and Technology of the Universidade Nova of Lisbon, Portugal
\(^{(1)}\)Email: suzanalampreia@gmail.com

ABSTRACT
The equipment condition monitoring has been developed in order to improve the equipment performance, obtaining more availability with less immobility between operational cycles. The online equipment condition monitoring allied to statistical techniques can widely contribute to a lean maintenance management. Some studies had demonstrate that CUSUM and MCUSUM control charts can be applied on equipment condition monitoring. In this article we will demonstrate the application of both modified charts to functioning parameters of a diesel engine. And will identify the constraints of its application. The basis parameters were obtained from an engine with a good performance, so it will be use to define the chart mean and standard deviation in phase 1. In phase 2 the data will be simulated. Although the application of the CUSUM and MCUSUM chart when in practice it is applied, the implemented statistical system must be flexible and it should be considered in which stage of the cycle the equipment are and the characteristics of the parameters must be well known.

Keywords: Control Charts, Cumulative Sum, Condition Monitoring.

INTRODUCTION
The use of statistical control in equipment’s monitoring may contribute to the implementation of equipment’s lean maintenance management, allowing the advance or delay the decision of a maintenance intervention. The CUSUM charts shows higher sensitivity than the traditional charts (Pereira & Requeijo, 2012).

It will be considered two phases in the implementation of the charts. In phase 1 the traditional charts are used, and in the phase 2 the modified CUSUM and MCUSUM are implement to parameters equipment.

In the phase 2 the data is simulated demonstrating the online parameters monitoring. It will be possible to observe the difference of sensibility between them.

The optimization of the CUSUM control charts should be full considered (Wu et al, 2009) with the equipment cycle evolution.

Phase 1
Because of variables characteristics, in this study only the charts to independent data would be investigated. For more details Pereira & Requeijo (2012) should be consulted.
Univariate Charts – $X$ and $MR$ Charts

In phase 1 the $X$ and $MR$ chart will be used to define the equipment parameters. The charts analysis should reveal equipment’s under statistical control. The charts interpretation is based on eventual random patterns (Norma ISO 8258:1991), considering the data independent and Normal. The under control limit ($UCL$), the lower control limit (LCL) and the central line (CL) are calculated by the equations on Table 1:

Table 1 – Shewhart charts phase 1

<table>
<thead>
<tr>
<th>Carta</th>
<th>LIC</th>
<th>LC</th>
<th>LSC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$ (individual observation)</td>
<td>$X - \frac{3MR}{d_2}$</td>
<td>$X$</td>
<td>$X + \frac{3MR}{d_2}$</td>
</tr>
<tr>
<td>$MR$ (moving range)</td>
<td>$D_3 MR$</td>
<td>$MR$</td>
<td>$D_4 MR$</td>
</tr>
</tbody>
</table>

The $\bar{X}$ and $\bar{MR}$ in table 1 are calculated based on the $m$ (or $m-1$) sample statistics calculated from: $\bar{X} = \frac{1}{m} \sum_{i=1}^{m} X_i$ and $\bar{MR} = \frac{1}{m-1} \sum_{i=1}^{m-1} |MR_i|$, where $MR_i = |X_i - X_{i-1}|$.

If the results shows an under control equipment, the parameters are estimated by $\tilde{\mu} = \bar{X}$ and $\tilde{\sigma} = \bar{MR}/d_2$. The constants $D_3$, $D_4$ and $d_2$ depend exclusively from the sample dimension.

Multivariate Chart - $T^2$ Traditional Control Charts

$T^2$ charts are applied when the number of variables is greater than one. If the observations of $p$ variables in control are independent, we have, $X_{ij} = \mu_j + \epsilon_{ij}$ where $X_{ij}$ is the observation $i$ for variable $j$, $\mu_j$ is the process mean for the variable $j$, and $\epsilon_{ij}$ are iid normal random variables with mean zero and standard deviation $\sigma_\epsilon$ (white noise). The mean vector ($\bar{X}$) and the covariance matrix ($S$) are calculated using data from phase 1, and are respectively given by:

$$\bar{X} = (\bar{X}_1, \bar{X}_2, ..., \bar{X}_p)^T$$

and

$$S = \begin{bmatrix}
S_{11} & S_{12} & S_{13} & \cdots & S_{1p} \\
S_{21} & S_{22} & S_{23} & \cdots & S_{2p} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
S_{p1} & S_{p2} & S_{p3} & \cdots & S_{pp}
\end{bmatrix}$$ (1)

The $T^2$ control charts for each $k$, are based on the statistic:

$$\left(T^2\right)_k = (X_i - \bar{X})^T S^{-1}(X_i - \bar{X})$$ (2)
The $LCL$ and the $UCL$ to phase 1 are constant in Table 2.

<table>
<thead>
<tr>
<th>Chart</th>
<th>$LCL$</th>
<th>$UCL$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase I</td>
<td>0</td>
<td>$(m-1)^2/m\beta_{n,p-2,(m-1)/2}$</td>
</tr>
</tbody>
</table>

**Phase 2 - Univariate Chart - Modified CUSUM Chart**

The modified CUSUM (M-CUSUM) chart is based on a statistic with memory, because the present statistic is based on the last observation. (Pereira & Requeijo (2012)

The modified CUSUM chart is built based on the cumulative sum, $C$, and it is defined by:

\[
C_t = \max\{0, C_{t-1} + (Z_t - k)\} \quad C_0 = 0
\]

\[
T_t = \min\{0, T_{t-1} + (Z_t + k)\} \quad T_0 = 0
\]

where $Z_t = \left(\hat{X}_t - T_L\right)/\sigma_{\hat{X}}$, $\sigma_{\hat{X}} = \sigma/\sqrt{n}$, $\Delta = \delta \sigma_{\hat{X}}$, $k = \delta/2$, $T_L = (T_L)_{\text{Norma}} - \Delta_S$ and $\Delta_S = \delta_1 \sigma$, where $\delta_1$ is a constant. In this equation $\hat{X}_t$ is the mean of the $t$ sample, $T_L$ is the maximum value defined by the equipment fabricant, $\sigma$ is the standard deviation from the equipment parameters, $n$ the sample dimension, $Z_t$ the Normal reduced variable referee to $\hat{X}_t$, $k$ the reference value and $\Delta_S$ the security factor. For the design of the modified CUSUM chart, the alert level ($AL$) and the upper control level ($UCL$) limits are considered. The limits are calculated based on Gan (1991), and are function of the $ARL$ ($Average\ Run\ Length$) value and $k$.

**Multivariate Chart - Modified MCUSUM Chart**

When a selected equipment is under study the modified cumulative multivariate chart should be considered for some specific variables. It will be more sensitive to little changes and accurate on the observed equipment parameters.

The modified MCUSUM (M-MCUSUM) Chart to control the mean vector is represented by (Villalobos, 2005):
\[ Y_t^* = \left( C_t' \sum C_t \right)^{1/2}; C_t = 0 \text{ if } C_t^* \leq k \]  
(5)

For independent data:

\[ C_t = \left( C_t' + X_t - T_k \right) \left( 1 - \frac{k}{C_t^*} \right); C_t^* > k \]  
(6)

\[ C_t^* = \left( \left( C_t' + X_t - T_k \right) \sum C_t' \right)^{1/2}; C_0 = 0 \text{ and } k > 0 \]  
(7)

The modified MCUSUM control charts depend either on the ARL_{inControl} value, if the values \( Y_t^* > h \), where \( h \) is the control limit, the situation is out of control. (Pereira & Requeijo, 2012).

**METHODOLOGY**

a. In phase 1 the data is collected considering the good functioning of the engine.

b. Considered the variables independent, normal, and checking the equipment stability, the \( X \) and \( MR \) or \( T^2 \) charts should be built and the mean/covariance and the mean vector/covariance matrix are respectively estimated.

c. In phase 2 built the modified CUSUM or the modified \( T^2 \) to monitor the equipment performance, and watch each sensitivity:

   - Estimate the Upper Control Limit (UCL) and the Alert Value (AL) basis on the limits from fabricant.
   - Establish rules for action on the system. The next are suggested:

      - Execute an intervention to detect any anomalous situation when 4 consecutive points above the AL are observed.
      - Proceed to a maintenance intervention when 2 consecutive points above UCL are observed.

**RESULTS**

**Phase 1 – Univariate Charts**

In phase 1 for univariate study the traditional charts will be used (Figure 1).
Because the charts represents an under control equipment the parameters are calculated and $X=5.76$ and $\sigma=0.16$ are obtained.

**Phase 1 - Multivariate Chart**

The application of modified $T^2$ chart shows equipment under control so the $X$ vector, the $S$ and $S^{-1}$ matrix can be calculated:

$$X = \begin{bmatrix} 5.7599 \\ 2.0108 \\ 564.14 \\ 560.18 \end{bmatrix}$$

$$S = \begin{bmatrix} 0.02396 & 0.00138 & 0.02143 & 0.16616 \\ 0.00138 & 0.25060 & -0.28737 & -0.17897 \\ 0.02143 & -0.28737 & 59.4610 & -3.39725 \\ -0.16616 & -0.17897 & -3.39725 & 56.02524 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} -42.6 & -0.156 & -0.009 & 0.125 \\ -0.1559 & 4.0 & 0.0203 & 0.0136 \\ -0.0090 & 0.0203 & 0.02 & 0.001 \\ 0.125 & 0.01362 & 0.001 & 0.02 \end{bmatrix}$$

The $T^2$ chart shows an under control equipment so the parameters can be defined.
Phase 2 - Univariate Chart

For the modified CUSUM charts to define the control limits $\alpha = 1\% \ (ARL = 100)$ for the $AL$ and $\alpha = 0,2\% \ (ARL = 500)$ for the $UCL$ are considered.

Considering the modified CUSUM for equipment monitoring with $\Delta = 0,5\sigma$ it represents lower observations values than for $\Delta = 1\sigma$. For lower $\Delta$ the sensitivity is higher (Fig. 3 and Fig. 4).

![Fig. 3 - Phase 2 Modified CUSUM Chart - 2nd Progression ($\Delta = 0,5\sigma$)](image)

![Fig. 4 - Phase 2 Modified CUSUM Chart - 2nd Progression ($\Delta = 1,5\sigma$)](image)

Multivariate Chart - Modified MCUSUM Chart

From zero to the second progression non observation are registered. Since the M-MCUSUM represents the results of the combination of various variables and on the previous observation values, although the application of the modified MCUSUM Chart, the variables should be study individual.
For the third progression great sensitivity has been shown.

**CONCLUSIONS**

It is possible to implement the modified CUSUM and MCUSUM control charts in condition monitoring.

The parameters choice to be used with the M-MCUSUM charts must be based in its relation.

To choose the variables for multivariate charts this study had considered the relation between them.

The M-CUSUM charts are more sensitivity than the Shewhart charts.

If the M-MCUSUM chart is applied, in spite of its sensitivity, the results congregate various variables, so the analysis should be made with caution, and individual variables study should be considered.

It should be considered the adjustment of the Gan abacus for the M-MCUSUM charts.

The M-CUSUM and M-MCUSUM charts can probably be use in equipment condition monitoring (Requeijo et al, 2012), although the implemented statistical systems must be flexible and be adapted according the equipment and it’s cycle.

**ACKNOWLEDGMENTS**

Portuguese Naval School and CINAV are kindly acknowledged for the use of diesel engine database and also for the fruitful collaboration of the Faculty of Science and Technology from the Universidade Nova of Lisbon.
REFERENCES


[3]-Requeijo J, Lampreia S, Barbosa P, Dias J, Controlo de Condição de equipamentos mecânicos por análise de vibrações com dados autocorrelacionados (Control provided by mechanical vibration analysis with autocorrelated data), Riscos, Segurança e Fiabilidade (Risks, Safety and Reliability), 2012, Salamandra, Lisboa, 1, p. 483-497.
