APPLICATION OF BAYESIAN UPDATING AND STOCHASTIC FINITE ELEMENT METHOD TO THE LOAD AND RESISTANCE FACTOR DESIGN (LRFD) OF A ROUND BAR HAVING RANDOM GEOMETRY

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**ABSTRACT**

This study has applied the Bayesian updating scheme and the stochastic finite element method (SFEM) to the Load and Resistance Factor Design (LRFD) method of a round bar having random geometry. The LRFD method is able to take into account separately the stochastic variability in material strength, in geometrical randomness of structural components and in loads applied to the components. Therefore, this method makes it possible to keep up the reliability of the components during the period of use. The LRFD method, however, has a problem that it requires vast amount of data for estimation. Addressing this problem, in this study, the Bayesian updating scheme, which requires only small amount of data, has been applied to the LRFD method. The SFEM and the Markov chain Monte Carlo (MCMC) method have been adopted in this study to analyse the stochastic variability in component geometry as well as in loads and to perform the Bayesian updating, respectively in the LRFD.

**Keywords:** Bayesian updating, LRFD method, Markov chain Monte Carlo, Stochastic FEM.

**INTRODUCTION**

Unlike the conventional allowable stress design (ASD) method, the Load and Resistance Factor Design (LRFD) method uses separate factors for material resistance and geometrical randomness of structural components, and applied loads of different types. The LRFD method can therefore reflect the degree of uncertainty of material strength, loads and their combinations. As a result, more uniform reliability can be achieved (Hoshiya, 1986, Chen, 1997). The LRFD method, however, requires a vast amount of statistical data for material strength and loadings considered (Okamura, 1979). This method was adopted in construction standards for reinforced concrete structures and bridges as early as in 1970’s in European countries and in US (Hoshiya, 1986). In Japan, however, the method has drawn attention recently. Especially, architectural engineers showed interests in the method in Japan and published recommendations for the method for steel structures. The Japan Society of Mechanical Engineers (JSME) recently has started a research committee to pursue the establishment of a structural design standard by the LRFD method. However, the method requires a vast amount of data for estimation. Addressing this problem, in this study, the Bayesian updating scheme (Nakasone, 2013 and 2014), which requires only small amount of data, has been applied to the LRFD method.

**RESULTS AND CONCLUSIONS**

This paper considers a problem depicted in Fig. 1 where a round bar having a strength of $x_1$ and a diameter of $x_3$ is subjected to longitudinal load of a magnitude of $x_2$; i.e., the limit state...
function of the present problem is \( Z = z(x_1, x_2, x_3) = x_1 - 4\pi x_2 / x_3^2 \). For \( Z \leq 0 \), the bar is fractured. Table 1 shows the values of statistical parameters of each variable used in this study.

The SFEM can directly obtain stochastic variability in stress caused by variations in component geometry and in applied loads, so that the limit state function is simply expressed as \( Z' = z'(x_1, x_2') = x_1 - x_2' \) where \( x_2' \) is a random variable representing stress variation caused by the random geometry of a structural component and by the random applied load.

Figures 2(a) through (c) show the variations of the reliability index value \( \beta \) with the cumulative number of data used for updating the \( \beta \) values, \( \beta_L \), \( \beta_0 \), and \( \beta_m \) obtained by the lower bound, the mean and the upper bound, respectively, of the 90% highest region of the mean and the variance of each random variable estimated by the Bayesian updating method. The solid lines in the figures indicate the target reliability index value of \( \beta_0 = 3.7 \) which corresponds to a fracture probability of 0.01%. The open and solid circular symbols in the figures indicate the results obtained by ordinary LRFD using the limit state function \( Z \) and those by LRFD/SFEM using the function \( Z' \). These figures show that the results obtained by the two approaches agree with each other and that, for the number of data larger than 50, the estimates \( \beta_L \) and \( \beta_m \) give reasonable reliability index values nearly equal to \( \beta_0 \) whereas \( \beta_m+ \) does not.

### Table 1 - Values of statistical parameters of random variables in the present LRFD simulations.

<table>
<thead>
<tr>
<th></th>
<th>( X_1 ), MPa</th>
<th>( X_2 ), kN</th>
<th>( X_3 ), mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>600</td>
<td>5</td>
<td>20.0</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>60</td>
<td>1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Fig. 1 - Present round bar model.

Fig. 2 - Variations of the value of the reliability index value \( \beta \) with the cumulative number of data used for updating the \( \beta \) values, (a) \( \beta_L \), (b) \( \beta_m \) and (c) \( \beta_m+ \).

### REFERENCES