LATERAL TORSIONAL STABILITY OF TIMBER BEAMS

Ivan Baláž(*) 1, Yvona Koleková2

1Department of Metal and Timber Structures, Faculty of Civil Engineering, Slovak University of Technology in Bratislava, Bratislava, Slovak Republic, European Union
2Department of Structural Mechanics, Faculty of Civil Engineering, Slovak University of Technology in Bratislava, Bratislava, Slovak Republic, European Union
(*)Email: ivan.balaz@stuba.sk

ABSTRACT

Lateral torsional stability of timber beams with monosymmetric cross-sections. Proposals are given (Baláž, Koleková, 2000) for approximate formulae enabling to compute the values of elastic critical moments $M_{cr}$ of beams under different loadings and various boundary conditions. These formulae were accepted by prEN 1999-1-1: May 2004 and EN 1999-1-1: May 2007 for design of aluminium structures and are used also for design of steel structures. It is shown that they may be used for design of timber structures too. The aim is to unify different Eurocodes procedures.

Keywords: timber, critical bending moment, lateral torsional stability.

INTRODUCTION

A beam bent about major axis in its stiffer principal plane may buckle out of that plane by deflecting laterally and twisting. Eurocode EN 1995-1-1: 2004 requires to verify the resistance of timber beam as follows

$$\sigma_{m,d} \leq k_{crit} f_{m,d}$$

where

$\sigma_{m,d}$ is the design bending stress,

$f_{m,d}$ is the corresponding design bending strength,

$k_{crit}$ is a factor which takes into account the reduced strength due to lateral buckling due to beam imperfections. For beams with an initial lateral deviation from straightness within the limits defined in EN 1995-1-1: 2004, section 10, $k_{crit}$ may be determined as follows:

$$k_{crit} = 1 \quad \text{for} \quad \lambda_{rel,m} \leq 0.75$$

$$k_{crit} = 1.56 - 0.75 \lambda_{rel,m} \quad \text{for} \quad 0.75 < \lambda_{rel,m} \leq 1.4$$

$$k_{crit} = \frac{1}{\lambda_{rel,m}} \quad \text{for} \quad 1.4 < \lambda_{rel,m}$$

The factor $k_{crit}$ may be put equal to 1.0 for a beam where lateral displacement of the compression side is prevented throughout its length and where rotation is prevented at the supports.

The relative slenderness for bending is defined by
\[ \lambda_{rel,m} = \frac{f_{m,k}}{\sigma_{m,crit}} \]  \hspace{1cm} (2)  

where  
\[ f_{m,k} \] is corresponding characteristic bending strength,  
\[ \sigma_{m,crit} \] is the critical bending stress calculated according to the classical theory of stability, with 5- percentile stiffness values.  

The critical stress \( \sigma_{m,crit} \) at which buckling of an ideal beam takes place is  
\[ \sigma_{m,crit} = \frac{M_{cr}}{W_y} \]  \hspace{1cm} (3)  

The same concept of lateral torsional verification was used in various publications, national standards, prestandard Eurocodes and their National Application Documents or in Eurocodes and their National Annexes. These procedures differ in critical moment \( M_{cr} \) calculations.

**ELASTIC CRITICAL MOMENT \( M_{cr} \)**

Calculation of critical moment \( M_{cr} \) is the classical problem of theory of elasticity. There are many more or less exact approximate formulae for calculation of \( M_{cr} \) value. It was shown by (Hooley and Madsen, 1964), that formulae for elastic critical moment may be used also for timber beams. Today there are also many computer programs which enable to compute \( M_{cr} \) value for any boundary conditions and any loading. Standards contain: a) no formulae for computing of \( M_{cr} \) value (EN 1993-1-1:2005), or b) limited number of approximate formulae valid only for some basic boundary conditions of beams loaded by only basic action types, e.g. EN 1995-1-1:2004, DIN 1052:2004), or c) general formulae for a lot of loading types and many combinations of boundary conditions (e.g. EN 1999-1-1:2007, which is based on works (Baláž, 1999), (Koleková, 1999), (Baláž, Koleková, 1999, 2000a, 2000b, 2002a, 2002b).  

Formulae defining \( M_{cr} \) value are valid for members made from any structural material. It is necessary, of course, to use corresponding material properties \( E, G \) (e.g. for steel, aluminium alloy, concrete or timber). Specialists designing timber beams in bending may use the general formulae for \( M_{cr} \) given in EN 1999-1-1:2007, when they will use material properties of timber \( E = E_{0.05} \) and \( G = G_{0.05} \).

The value of elastic critical moment \( M_{cr} \) of the reference beam, which is characterized by  
- uniform non-warping and doubly-symmetrical cross-section,  
- standard boundary conditions (the beam is simply supported on both ends in all three cases: bending in xz-plane, bending in xy-plane and in torsion about axis x, which may be expressed through the factors \( k_y=1.0, k_z=1.0, k_w=1.0 \), see e.g. (Baláž, Koleková, 2000a, 2000b, 2002a, 2002b) or prEN 1999-1-1: May 2004; EN 1999-1-1: May 2007,  
- and it is loaded by equal end moments \( M \) and \( \psi.M, \psi = 1.0 \), (moment line distribution is uniform),  
may be calculated from simple formula
where: $I_t$ is the torsion constant,
$I_z$ is the second moment of area about the minor axis,
$L$ is the length of the beam between points that have lateral restraint.

All other cases may be transformed in above defined equivalent reference beam by replacing
the length $L$ by the effective length $L_{ef}$ which can take into account the following influences
− mono-symmetry of the cross-section (through mono-symmetry parameter $\zeta_j$)
− warping of the cross-section (through the warping constant $I_{w}$, or $\kappa_{wt}$),
− any combination of boundary conditions,
− any loading type,
− the location of point of load application in cross-section (through $z_g$ or $a_z$).

The general formula may be written in the form

$$M_{cr} = \frac{\pi \sqrt{EI_zGI_t}}{L_{ef}}$$

The effective length $L_{ef}$ can be calculated according to various standards and publications.

**ACCORDING TO EN 1999-1-1: MAY 2007 or (Baláž, Koleková, 2000a, 2000b, 2002a, 2002b)** it is possible to calculate the effective length may be computed from the formula

$$L_{ef} = L / \mu_{cr}$$

**a) Elastic critical moment $M_{cr}$ of beams with uniform monosymmetric cross-sections**

In the case of a beam of uniform cross-section which is symmetrical about the minor axis, for
bending about the major axis the elastic critical moment for lateral-torsional buckling is given
by the general formula

$$M_{cr} = \mu_{cr} \frac{\pi \sqrt{EI_zGI_t}}{L}$$

where relative non-dimensional critical moment $\mu_{cr}$ is

$$\mu_{cr} = \frac{C_1}{k_z} \left[ 1 + \kappa_{wt}^2 + (C_2 z_g^2 - C_3 \zeta_j) - (C_2 z_g^2 - C_3 \zeta_j) \right]$$

non-dimensional torsion parameter is

$$\kappa_{wt} = \frac{\pi}{k_z L} \frac{\sqrt{EI_w}}{GI_t}$$

relative non-dimensional coordinate of the point of load application related to shear center

$$z_g = \frac{\pi \zeta_g}{k_z L} \frac{\sqrt{EI_z}}{GI_t}$$
relative non-dimensional cross-section mono-symmetry parameter

\[ \zeta_j = \frac{\pi j}{k_z L \sqrt{\frac{E I_z}{G I_t}}} \]  

(11)

where:

- \(C_1, C_2, C_3\) are factors depending mainly on the loading and end restraint conditions (see Tables 1 and 2),
- \(k_z, k_w\) are buckling length factors,
- \(z_g = z_s - z_k\)
- \(z_s\) is the coordinate of the point of load application related to centroid,
- \(z_k\) is the coordinate of the shear center related to centroid,
- \(z_g\) is the coordinate of the point of load application related to shear center.

The sign convention for determining \(z_g\) is for:
- gravity loads \(z_g\) is positive for loads applied above the shear centre,
- general case \(z_g\) is positive for loads acting towards the shear centre from their point of application

\[ z_j = z_s - \frac{0.5}{I_y} \int (y^2 + z^2) z dA \]  

(13)

\[ z_j = 0 \quad (y_j = 0) \] for cross sections with \(y\)-axis (\(z\)-axis) being axis of symmetry.

The following approximation for \(z_j\) can be used:

\[ z_j = 0.45 \psi f h_s \left( 1 + \frac{c}{2h_f} \right) \]  

(14)

where

- \(c\) is the depth of a lip
- \(h_f\) is the distance between centerlines of the flanges.

\[ \psi_f = \frac{I_{fc} - I_{ft}}{I_{fc} + I_{ft}} \]  

(15)

\(I_{fc}\) is the second moment of area of the compression flange about minor axis of the section,
\(I_{ft}\) is the second moment of area of the tension flange about the minor axis of the section,
\(h_s\) is the distance between the shear centre of the upper flange and shear centre of the bottom flange (\(S_u\) and \(S_b\)).

For an I-section with unequal flanges without lips and as an approximation also with lips

\[ I_w = (1 - \psi_f^2 ) I_z \left( \frac{h_s}{2} \right)^2 \]  

(16)

The sign convention for determining \(z\) and \(z_j\) is as follows
- coordinate \(z\) is positive for the compression flange. When determining \(z_j\) from formula in (14), positive coordinate \(z\) goes upwards for beams under gravity loads or for cantilevers.
under uplift loads, and goes downwards for beams under uplift loads or cantilevers under gravity loads

– sign of \( z_j \) is the same as the sign of cross-section mono-symmetry factor \( \psi_f \) in (15). Take the cross section located at the M-side in the case of moment loading, Table 1, and the cross-section located in the middle of the beam span in the case of transverse loading, Table 2.

b) Beam with uniform cross-section symmetrical about major axis, centrally symmetric and doubly symmetric cross-section

![Beams with uniform cross-sections symmetrical about major axis, centrally symmetric and doubly symmetric cross-sections](image)

For these beams loaded perpendicular to the mayor axis in the plane going through the shear centre, \( z_j = 0 \), thus

\[
\mu_{cr} = \frac{C_1}{k_z} \sqrt{1 + \kappa_{wt}^2 + \left(C_2 \psi_f \right)^2 - C_2 \psi_f} 
\]

(17)

\( \mu_{cr} \) values and factors \( C_1 \) and \( C_2 \) may be found in (Baláž, Koleková, 2000b).

For end-moment loading \( C_2 = 0 \) and for transverse loads applied at the shear centre \( z_g = 0 \). For these cases

\[
\mu_{cr} = \frac{C_1}{k_z} \sqrt{1 + \kappa_{wt}^2} 
\]

(18)

If also \( \kappa_{wt} = 0 \)

\[
\mu_{cr} = \frac{C_1}{k_z} 
\]

(19)

Values of \( C_1 \), \( C_2 \) and \( C_3 \) are given in Tables 1 and 2 for various loading cases, as indicated by the shape of the bending moment diagram over the length \( L \) between lateral restraints. The values given in Table 1 correspond to various values of \( k_z \) and in Table 2 correspond to various combinations of values \( k_z \) and \( k_w \).

For cases with \( k_z = 1.0 \) the values of \( C_1 \) for any ratio of end moment loading as indicated in Table 1, are given approximately by formula

\[
C_1 = (0.310 + 0.428\psi_f + 0.262\psi_f^2)^{-0.5} 
\]

(20)
Table 1 - Values of factors $C_1$ and $C_3$ corresponding to various end moment ratios $\psi$, values of buckling length factor $k_2$ and cross-section parameters $\psi_f$ and $\kappa_{wt}$. End moment loading of the simply supported beam with buckling length factors $k_2 = 1.0$ for major axis bending and $k_2 = 1.0$ for torsion.

<table>
<thead>
<tr>
<th>Loading and support conditions</th>
<th>Bending moment diagram</th>
<th>Values of factors</th>
<th>$C_1$</th>
<th>$C_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-section symmetry factor $\psi_f$</td>
<td>$M_{pl} ; \psi = +1$</td>
<td>$k_2$</td>
<td>$C_{10}$</td>
<td>$C_{11}$</td>
</tr>
<tr>
<td>$\psi_f \geq 0$</td>
<td>1.0</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.7L</td>
<td>1.016</td>
<td>1.200</td>
<td>1.025</td>
<td>1.000</td>
</tr>
<tr>
<td>0.7R</td>
<td>1.016</td>
<td>1.200</td>
<td>1.025</td>
<td>1.000</td>
</tr>
<tr>
<td>0.5</td>
<td>1.000</td>
<td>1.127</td>
<td>1.019</td>
<td></td>
</tr>
<tr>
<td>0.7L</td>
<td>1.395</td>
<td>1.441</td>
<td>1.150</td>
<td>1.000</td>
</tr>
<tr>
<td>0.7R</td>
<td>1.199</td>
<td>1.321</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.5</td>
<td>1.199</td>
<td>1.285</td>
<td>1.017</td>
<td></td>
</tr>
<tr>
<td>$\psi_f \leq 0$</td>
<td>1.0</td>
<td>1.312</td>
<td>1.320</td>
<td>1.150</td>
</tr>
<tr>
<td>0.7L</td>
<td>1.400</td>
<td>1.416</td>
<td>1.160</td>
<td>1.000</td>
</tr>
<tr>
<td>0.7R</td>
<td>1.400</td>
<td>1.416</td>
<td>1.160</td>
<td>1.000</td>
</tr>
<tr>
<td>0.5</td>
<td>1.400</td>
<td>1.416</td>
<td>1.160</td>
<td>1.000</td>
</tr>
<tr>
<td>$\psi_f = 0$</td>
<td>1.0</td>
<td>1.770</td>
<td>1.887</td>
<td>1.470</td>
</tr>
<tr>
<td>0.7L</td>
<td>2.331</td>
<td>2.663</td>
<td>2.000</td>
<td>1.400</td>
</tr>
<tr>
<td>0.7R</td>
<td>1.453</td>
<td>1.592</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>1.573</td>
<td>2.027</td>
<td>1.200</td>
<td>1.000</td>
</tr>
</tbody>
</table>

1) For beams with rectangular cross-section the values in bold may be used. They may be replaced by approximate values obtained from the formula (17).
2) $C_1 = C_{10} + \left(C_{11} - C_{10}\right)\kappa_{wt} \leq C_{11}$, ($C_1 = C_{10}$ for $\kappa_{wt} = 0$, $C_1 = C_{11}$ for $\kappa_{wt} \geq 1$).
3) 0.7L = left end fixed, 0.7R = right end fixed.
4) In EN 1999-1: May 2007 there are incorrectly:
   - $0.9 - 0.75\psi_f$, instead of correct value $0.9 - 0.77\psi_f$, taken from (Baláž, Koleková,2000b).
   - $0.23 - 0.9\psi_f$, instead of correct value $0.225 - 0.9\psi_f$, taken from (Baláž, Koleková,2000b).
Table 2 - Values of factors $C_1$, $C_2$ and $C_3$ corresponding to various transverse loading cases, values of buckling length factors $k_y$, $k_x$, $k_w$, cross-section monosymmetry factor $\psi_f$ and torsion parameter $\kappa_{wt}$

<table>
<thead>
<tr>
<th>Loading and support conditions</th>
<th>Buckling length factors</th>
<th>Values of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k_y$</td>
<td>$k_x$</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 1 1</td>
<td>1,127</td>
<td>1,132</td>
</tr>
<tr>
<td>1 1 0,5</td>
<td>1,125</td>
<td>1,231</td>
</tr>
<tr>
<td>0,5 1</td>
<td>0,947</td>
<td>0,997</td>
</tr>
<tr>
<td>0,5 0,5</td>
<td>0,947</td>
<td>0,970</td>
</tr>
<tr>
<td>1 1 1</td>
<td>1,346</td>
<td>1,363</td>
</tr>
<tr>
<td>1 1 0,5</td>
<td>1,349</td>
<td>1,452</td>
</tr>
<tr>
<td>0,5 1</td>
<td>1,030</td>
<td>1,087</td>
</tr>
<tr>
<td>0,5 0,5</td>
<td>1,031</td>
<td>1,067</td>
</tr>
<tr>
<td>1 1 1</td>
<td>1,036</td>
<td>1,040</td>
</tr>
<tr>
<td>1 1 0,5</td>
<td>1,039</td>
<td>1,148</td>
</tr>
<tr>
<td>0,5 1</td>
<td>0,922</td>
<td>0,960</td>
</tr>
<tr>
<td>0,5 0,5</td>
<td>0,922</td>
<td>0,945</td>
</tr>
</tbody>
</table>

1) For beams with rectangular cross-section the values in bold may be used.
2) $C_1 = C_{L,0} + (C_{L,1} - C_{L,0}) \kappa_{wt} \leq C_{L,1}$, ($C_1 = C_{L,0}$ for $\kappa_{wt} = 0$, $C_1 = C_{L,1}$ for $\kappa_{wt} \geq 1$).
3) Parameter $\psi_f$ refers to the middle of the span.
4) Values of critical moments $M_{cr}$ refer to the cross-section, where $M_{max}$ is located.
c) Beam with rectangular cross-section

The formulae (17-19) given in previous paragraph are valid also for rectangular cross-section. Parameter $\kappa_w$ in (17, 18) may be calculated with the help of the following formulae.

**Torsion constant**

$$I_t = \frac{1}{3} \left[ 1 - 0.63 \frac{b}{h} + 0.052 \left( \frac{b}{h} \right)^5 \right] b h^3$$  \hfill (21)

**Warping constant**

$$I_{wr} = 0 \text{ m}^6, \quad I_w = I_{wr} + I_{wn} = 0 + \frac{1}{144} \left[ 1 - 4.884 \left( \frac{b}{h} \right)^2 + 4.97 \left( \frac{b}{h} \right)^3 - 1.067 \left( \frac{b}{h} \right)^5 \right] b^3 h^3$$  \hfill (22)

For the timber beams with ratios

$$\frac{L}{h} > 5, \quad \frac{h}{b} < 15, \quad k_w = 1.0, \quad \frac{G}{E} = \frac{G_{0.05}}{E_{0.05}} \approx \frac{1}{16},$$  \hfill (23)

we obtain that $\kappa_w \leq 0.1853$. In many cases the value of $\kappa_w$ may be neglected and formula (17) can be simplified by inserting $\kappa_w = 0$ into the form

$$\mu_w = \frac{C_1}{k_z} \left[ 1 + \left( C_2 \xi_z \right)^2 - C_2 \xi_z \right]$$  \hfill (24)

For end-moment loading $C_2 = 0$ and for transverse loads applied at the shear centre $z_g = 0$ and from formula (24) we obtain simple formula (19).

For beams supported on both ends ($k_y = 1$, $k_z = 1$, $0.5 \leq k_w \leq 1$) or for beam segments laterally restrained on both ends, which are under any loading (e.g. different end moments combined with any transverse loading), the following value of factor $C_1$ may be used in the formula (18) to obtain approximate value of the critical moment

$$C_1 = \frac{1.7 \left| M_{\text{max}} \right|}{\sqrt{M^2_{0.25} + M^2_{0.5} + M^2_{0.75}}} \leq 2.5$$  \hfill (25)

where

- $M_{\text{max}}$ is maximum design bending moment,
- $M_{0.25}$, $M_{0.75}$ are design bending moments at the quarter points and
- $M_{0.5}$ is design bending moment at the midpoint of the beam or beam segment with length equal to the distance between adjacent cross-sections which are laterally restrained.

Factor $C_1$ defined by (25) may be used also in formula (17), but only in combination with relevant value of factor $C_2$ valid for given loading and boundary conditions. This means that for the six cases in Table 2 with boundary condition $k_y = 1$, $k_z = 1$, $0.5 \leq k_w \leq 1$, as defined above, the value $C_2 = 0.5$ may be used in (17) together with factor $C_1$ calculated from (25) as an approximation. Similar approximation is used in German DIN 18 800: 1990, Part 2.
d) Beam with rectangular cross-section under uniform bending moment

For all above given $M_{cr}$ formulae the influence of curvature due to major axis bending was ignored. It has been shown in (Broude, 1953, p.85), see also (Baláž, Koleková, 1999, p.61), that the influence of pre-buckling displacements is positive one. For special case of doubly symmetric cross-section of the beam under uniform moment with $k_w = 1.0$, $k_z = 1.0$ it leads to the increase of the critical moment described by the factor $\xi \geq 1.0$

$$
\mu_{cr,ξ} = \xi \mu_{cr} = \frac{1}{\sqrt{1 - \frac{I_z}{I_y}} \sqrt{1 - \frac{I_z}{I_y} \frac{GI_z}{EI_z} (1 + K_{wz})^2}}
$$

Formula (26) is valid only for above mentioned special case. It is often used as an evidence that no lateral torsional buckling occurs if $I_y \leq I_z$. Such rules may be found in many codes including German ones. It is necessary to stress, that generally it is not true. No lateral torsional buckling will occur if $I_y \leq I_z$ only if point of load application in to cross-section is not too high above the centre of shear.

Examples of factor $\xi$ calculation

(i) IPE 360: $L=4.5m$, $I_z/I_y=0.064$, $K=1.029$, $\sqrt{GI_z/EI_z}=0.118$, $\xi=1.035$.

(ii) HD 360x162: $L=4.5m$ $I_z/I_y=0.360$ $K=1.526$, $\sqrt{GI_z/EI_z}=0.078$, $\xi=1.255$.

The factor $\xi$ increasing the critical moment due to pre-buckling displacements is for doubly symmetric beam under uniform bending with $k_z=1.0$, $k_w=1.0$ given in Table 3.

<table>
<thead>
<tr>
<th>$\frac{I_z}{I_y}$</th>
<th>$K = \frac{\pi}{L} \sqrt{\frac{EI_{cr}}{G I_z}}$</th>
<th>$\sqrt{\frac{GI_z}{EI_z}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 0</td>
<td>1.054</td>
<td>1.055 1.057 1.068</td>
</tr>
<tr>
<td>0.1 1</td>
<td>1.056</td>
<td>1.061 1.081</td>
</tr>
<tr>
<td>0.1 3</td>
<td>1.062</td>
<td>1.089 1.217</td>
</tr>
<tr>
<td>0.3 0</td>
<td>1.195</td>
<td>1.198 1.207 1.243</td>
</tr>
<tr>
<td>0.3 1</td>
<td>1.201</td>
<td>1.218 1.296</td>
</tr>
<tr>
<td>0.3 3</td>
<td>1.224</td>
<td>1.326 2.390</td>
</tr>
<tr>
<td>0.6 0</td>
<td>1.581</td>
<td>1.589 1.612 1.715</td>
</tr>
<tr>
<td>0.6 1</td>
<td>1.596</td>
<td>1.644 1.899</td>
</tr>
<tr>
<td>0.6 3</td>
<td>1.661</td>
<td>2.000 -</td>
</tr>
</tbody>
</table>

In the national and international standards coefficient $\xi = 1$ being on the safe side.

e) Cantilever with uniform cross-sections symmetrical about the minor axis

For bending about the major axis the elastic critical moment for lateral-torsional buckling is given by the formula (7), where relative non-dimensional critical moment $\mu_{cr}$ is given in Table 4 and 5. In Table 4 and 5 non-linear interpolation shall be used (see Fig. 3 and 4).

The sign convention for determining $z_j$ is below formula (16) and $z_g$ is below formula (12).
### Table 4 - Relative non-dimensional critical moment $\mu_{cr}$ for cantilever ($k_y = k_z = k_w = 2$) loaded by concentrated end load $F$

<table>
<thead>
<tr>
<th>Loading and support conditions</th>
<th>$\pi \frac{EI_w}{L \sqrt{G_h}}$</th>
<th>$\pi \frac{Ez}{L \sqrt{G_h}}$</th>
<th>$\frac{\pi z_j}{L \sqrt{G_h}} \frac{EI_z}{z_j}$</th>
<th>$\mu_{cr} = z_j = \zeta_j$</th>
<th>$\zeta_j &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$k_w \kappa_{wt} = \kappa_{wt0}$</td>
<td>$k_w \kappa_{wt} = \kappa_{wt0}$</td>
<td>$= \kappa_{wt0}$</td>
<td>$\kappa_{wt0}$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>4</td>
<td>0.107, 0.156, 0.194, 0.245, 0.316, 0.416, 0.759</td>
<td>0.107, 0.156, 0.194, 0.245, 0.316, 0.416, 0.759</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.123, 0.211, 0.302, 0.463, 0.759, 1.312, 4.024</td>
<td>0.123, 0.211, 0.302, 0.463, 0.759, 1.312, 4.024</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.128, 0.254, 0.478, 1.280, 3.178, 5.590, 10.730</td>
<td>0.128, 0.254, 0.478, 1.280, 3.178, 5.590, 10.730</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.129, 0.258, 0.508, 1.619, 3.894, 6.500, 11.860</td>
<td>0.129, 0.258, 0.508, 1.619, 3.894, 6.500, 11.860</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.129, 0.258, 0.511, 1.686, 4.055, 6.740, 12.240</td>
<td>0.129, 0.258, 0.511, 1.686, 4.055, 6.740, 12.240</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1) For beams with rectangular cross-section the values in bold may be used. See Fig. 3.

2) For $z_j = 0$, $z_g = 0$ and $\kappa_{wt0} \leq 8$: $\mu_{cr} = 1.27 + 1.14 \kappa_{wt0} + 0.017 \kappa_{wt0}^2$.

For beams with rectangular cross-section $\mu_{cr} = 1.27$.

3) For $z_j = 0$, $-4 \leq \zeta_g \leq 4$ and $\kappa_{wt} \leq 4$, $\mu_{cr}$ may be calculated also from formulae (17-19), where the following approximate values of the factors $C_1, C_2$ should be used for the cantilever under tip load $F$:

$$C_1 = 2.56 + 4.675 \kappa_{wt} - 2.62 \kappa_{wt}^2 + 0.5 \kappa_{wt}^3, \text{ if } \kappa_{wt} \leq 2 \text{ or } C_1 = 5.55, \text{ if } \kappa_{wt} > 2.$$  

$$C_2 = 1.255 + 1.566 \kappa_{wt} - 0.931 \kappa_{wt}^2 + 0.245 \kappa_{wt}^3 - 0.024 \kappa_{wt}^4, \text{ if } \zeta_g \geq 0 \text{ or } C_2 = 0.192 + 0.585 \kappa_{wt} - 0.054 \kappa_{wt}^2 - (0.032 + 0.102 \kappa_{wt} - 0.013 \kappa_{wt}^2) \zeta_g, \text{ if } \zeta_g < 0.$$  

For beams with rectangular cross-section $C_1 = 2.56$ and $C_2 = 1.255$, if $\zeta_g \geq 0$ or $C_2 = 0.192 - 0.032 \zeta_g$, if $\zeta_g < 0$. 

---

-2362-
Table 5 - Relative non-dimensional critical moment $\mu_{cr}$ for cantilever ($k_y = k_z = k_w = 2$) loaded by uniformly distributed load $q$

<table>
<thead>
<tr>
<th>Loading and support conditions</th>
<th>$\frac{\pi z}{L} \sqrt{\frac{EI}{GJ_t}} = k_y \zeta_g$</th>
<th>$z_j &lt; 0$</th>
<th>$z_j &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{\pi z}{L} \sqrt{\frac{EI}{GJ_t}} = k_z \zeta_g$</td>
<td>$\frac{(C)}{(T)}$</td>
<td>$\frac{(C)}{(T)}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{\pi z}{L} \sqrt{\frac{EI}{GJ_t}} = k_w \zeta_g$</td>
<td>$\zeta_g &gt; 0$</td>
<td>$\zeta_g &lt; 0$</td>
</tr>
<tr>
<td>-4</td>
<td>0.0113</td>
<td>0.1730</td>
<td>0.2250</td>
</tr>
<tr>
<td>-2</td>
<td>0.0126</td>
<td>0.2225</td>
<td>0.3400</td>
</tr>
<tr>
<td>0</td>
<td>0.0132</td>
<td>0.2630</td>
<td>0.5160</td>
</tr>
<tr>
<td>-2</td>
<td>0.0134</td>
<td>0.2680</td>
<td>0.5370</td>
</tr>
<tr>
<td>-4</td>
<td>0.0134</td>
<td>0.2700</td>
<td>0.5410</td>
</tr>
</tbody>
</table>

1) For beams with rectangular cross-section the values in bold may be used. See Fig. 4.

2) For $z_j = 0$, $z_g = 0$ and $\kappa_{w0} \leq 8$: $\mu_{cr} = 2.04 + 2.68 \kappa_{w0} + 0.021 \kappa_{w0}^2$.

For beams with rectangular cross-section $\mu_{cr} = 2.04$.

3) For $z_j = 0$, $-4 \leq \zeta_g \leq 4$ and $\kappa_{w} \leq 4$, $\mu_{cr}$ may be calculated also from formulae (14-16), where the following approximate values of the factors $C_1, C_2$ should be used for the cantilever under uniform load $q$:

$$C_1 = 4.11 + 11.2 \kappa_{w0} - 5.65 \kappa_{w0}^2 + 0.975 \kappa_{w0}^3,$$

if $\kappa_{w0} \leq 2$, or $C_1 = 12$, if $\kappa_{w0} > 2$.

$$C_2 = 1.661 + 1.068 \kappa_{w0} - 0.609 \kappa_{w0}^2 + 0.153 \kappa_{w0}^3 - 0.014 \kappa_{w0}^4,$$

if $\zeta_g \geq 0$ or

$$C_2 = 0.535 + 0.426 \kappa_{w0} - 0.029 \kappa_{w0}^2 - (0.061 + 0.074 \kappa_{w0} - 0.0085 \kappa_{w0}^2) \zeta_g,$$

if $\zeta_g < 0$.

For beams with rectangular cross-section $C_1 = 4.11$ and

$C_2 = 1.661$, if $\zeta_g \geq 0$ or $C_2 = 0.535 - 0.061 \zeta_g$, if $\zeta_g < 0$. 

-2363-
**ACCORDING TO NATIONAL APPLICATION DOCUMENTS TO STANDARDS ČSN P ENV 1995-1-1: 1996; STN P ENV 1995-1-1: 2002**

The effective length is defined as follows. The \( m \) values are given in Table 6.

\[
L_{ef} = mL
\]  \( \text{(27)} \)

**ACCORDING TO EN 1995-1-1: 2004**

\( L_{ef} = l_{ef} \) may be calculated using values from Table 6, see also Table 7.

Table 6 - Effective length as a ratio of the span (EN 1995-1-1: 2004)

<table>
<thead>
<tr>
<th>Beam type</th>
<th>Loading type</th>
<th>( l_{ef}/l^a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simply supported</td>
<td>Constant moment</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Uniformly distributed load</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>Concentrated force at the middle of the span</td>
<td>0.8</td>
</tr>
<tr>
<td>Cantilever</td>
<td>Uniformly distributed load</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Concentrated force at the free end</td>
<td>0.8</td>
</tr>
</tbody>
</table>

\( a \) The ratio between the effective length \( l_{ef} \) and the span \( l \) is valid for a beam with torsionally restrained supports and loaded at the centre of gravity. If the load is applied at the compression edge of the beam, \( l_{ef} \) should be increased by \( 2h \) and may be decreased by \( 0.5h \) for a load at the tension edge of the beam.

Table 7 - Effective length \( l_{ef} \) (Colling, 2014)

<table>
<thead>
<tr>
<th></th>
<th>( l_{ef} )</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td><img src="Diagram1.png" alt="Diagram" /></td>
<td>1.0 ( l )</td>
</tr>
<tr>
<td>1b</td>
<td><img src="Diagram2.png" alt="Diagram" /></td>
<td>1.0 ( l ) + 2 ( h ) - ( h/2 )</td>
</tr>
<tr>
<td>2</td>
<td><img src="Diagram3.png" alt="Diagram" /></td>
<td>0.9 ( l ) + 2 ( h ) - ( h/2 )</td>
</tr>
<tr>
<td>3</td>
<td><img src="Diagram4.png" alt="Diagram" /></td>
<td>0.8 ( l ) + 2 ( h ) - ( h/2 )</td>
</tr>
<tr>
<td>4</td>
<td><img src="Diagram5.png" alt="Diagram" /></td>
<td>0.5 ( l ) - ( h/2 ) + 2 ( h )</td>
</tr>
<tr>
<td>5</td>
<td><img src="Diagram6.png" alt="Diagram" /></td>
<td>0.8 ( l ) - ( h/2 ) + 2 ( h )</td>
</tr>
</tbody>
</table>
ACCORDING TO DIN 1052: 2004, ANNEX E.3

The effective length is given by

\[ L_{ef} = k_{1,ef} L = \frac{L}{a_1 \left( 1 - a_2 \frac{a_z}{L} \frac{EI_z}{GI_l} \right)} \]  

(28)

where

\[ a_z = z_g \]  

(29)

is the coordinate of the point of load application related to shear center.

<table>
<thead>
<tr>
<th>System</th>
<th>Moment</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Moment Diagram 1]( Moment Diagram 1 )</td>
<td>![Moment Diagram 2]( Moment Diagram 2 )</td>
<td>1.77</td>
<td>0</td>
</tr>
<tr>
<td>![Moment Diagram 3]( Moment Diagram 3 )</td>
<td>![Moment Diagram 4]( Moment Diagram 4 )</td>
<td>1.35</td>
<td>1.74</td>
</tr>
<tr>
<td>![Moment Diagram 5]( Moment Diagram 5 )</td>
<td>![Moment Diagram 6]( Moment Diagram 6 )</td>
<td>1.13</td>
<td>1.44</td>
</tr>
<tr>
<td>![Moment Diagram 7]( Moment Diagram 7 )</td>
<td>![Moment Diagram 8]( Moment Diagram 8 )</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

COMPARISONS OF CORRESPONDENT VALUES \( L_{ef}/L \approx 1/\mu_{cr} \), \( l_{ef}/L \), \( k_{l,ef} \), \( m \)

The comparisons of corresponding values of \( L_{ef}/L \approx 1/\mu_{cr} \), \( l_{ef}/L \), \( k_{l,ef} \), \( m \) of the beams under various loading types and boundary conditions taken from various publications are given in Table 9. From Table 9 it is clear that the procedure proposed in (Baláž, Koleková, 2002b), which was accepted by prEN 1999-1-1: May 2004 and EN 1999-1-1: May 2007, is the most general and the most exact one.
Table 9 Comparisons of \( \frac{I_{ed}}{L} \) values for elastic critical moment \( M_{cr} \), see formula (5), defined by various publications. Evaluation of rules for \( \alpha_a = 0 \) m.

<table>
<thead>
<tr>
<th>Boundary conditions of the beam</th>
<th>Type of loading</th>
<th>Kirby, P.A. – Nethercot, D.A. 1979; Slovak NAD to ČSN P ENV 1995-1-1: 1996; (( \frac{I_{ed}}{L} = m )) ( \alpha_a = 9 ) m.</th>
<th>Czech NAD to ČSN P ENV 1995-1-1: 2002 (( \frac{I_{ed}}{L} = m )) ( \alpha_a = 9 ) m.</th>
<th>Eurocode EN 1995-1-1: 2003; (( \frac{I_{ed}}{L} = \frac{E}{G} )) ( \alpha_a = 9 ) m.</th>
<th>German code DIN 1052: 2004; (( \frac{I_{ed}}{L} = \frac{E}{G} )) ( \alpha_a = 9 ) m.</th>
<th>Balaž, I. – Kolekář, J. 2002b; Eurocode EN 1999-1-1: 2007; (( \frac{I_{ed}}{L} = \frac{E}{G} )) ( \alpha_a = 9 ) m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>beam simply supported on both ends ( k_y = 1.0 ) ( k_z = 1.0 ) ( k_u = 1.0 )</td>
<td>end moments ( M ) and ( M ), ( W = 1 )</td>
<td>( \frac{I_{ed}}{L} ) according to different codes. Quantity ( \alpha_a ) denotes distance of point of load application from shear center ( S ).</td>
<td>( \frac{I_{ed}}{L} )</td>
<td>( \frac{I_{ed}}{L} )</td>
<td>( \frac{I_{ed}}{L} )</td>
<td>( \frac{I_{ed}}{L} )</td>
</tr>
<tr>
<td>( \alpha_a = + h/2 ), if point of load application is above ( S )</td>
<td>( \alpha_a = 0 ) m, if point of load application is in ( S )</td>
<td>( \alpha_a = - h/2 ), if point of load application is below ( S )</td>
<td>( \alpha_a = 0 ) m, if point of load application is in ( S )</td>
<td>( \alpha_a = 0 ) m, if point of load application is in ( S )</td>
<td>( \alpha_a = 0 ) m, if point of load application is in ( S )</td>
<td>( \alpha_a = 0 ) m, if point of load application is in ( S )</td>
</tr>
<tr>
<td>end moments ( M ) and ( M ), ( W = 0 )</td>
<td>0.57</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>end moments ( M ) and ( M ), ( W = -1 )</td>
<td>0.43</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>point load ( F ) at midspan</td>
<td>0.74</td>
<td>0.8</td>
<td>0.75</td>
<td>0.7</td>
<td>0.8</td>
<td>1 / 1.35 = 0.741</td>
</tr>
<tr>
<td>point load ( F ) at quarter-span</td>
<td>0.69</td>
<td>0.74</td>
<td>0.69</td>
<td>0.64</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>point load ( F ) at ( x )-section, ( \xi = x/L ) ( \alpha = 1.125 - 1.42(1 - \xi) )</td>
<td>---</td>
<td>0.8/( \alpha )</td>
<td>0.75/( \alpha )</td>
<td>0.7/( \alpha )</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>two point loads ( F ) and ( F ) at quarters-span</td>
<td>0.95</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>uniformly dist. load ( q )</td>
<td>0.88</td>
<td>0.95</td>
<td>0.9</td>
<td>0.85</td>
<td>0.9</td>
<td>1 / 1.13 = 0.885</td>
</tr>
<tr>
<td>beam fixed on both ends ( k_y = 0.5 ) ( k_z = 0.5 ) ( k_u = 1.0 )</td>
<td>point load in mid. ( F )</td>
<td>0.39</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>1 / 6.81 = 0.147</td>
</tr>
<tr>
<td>uniformly dist. load ( q )</td>
<td>0.59</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>1 / 5.12 = 0.192</td>
</tr>
<tr>
<td>interior span of continuous beam ( k_y = 0.5 ) ( k_z = 1.0 ) ( k_u = 1.0 )</td>
<td>point load in mid. ( F )</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>1 / 1.7 = 0.588</td>
</tr>
<tr>
<td>uniformly dist. load ( q )</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>1 / 1.3 = 0.769</td>
<td></td>
</tr>
<tr>
<td>cantilever ( k_y = k_z = k_u = 2 )</td>
<td>end moment ( M )</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>1 / 2.576 = 0.388</td>
</tr>
<tr>
<td>point load ( F ) at free-end</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>1 / 1.28 = 0.781</td>
</tr>
<tr>
<td>uniformly distributed load ( q )</td>
<td>---</td>
<td>---</td>
<td>1.2</td>
<td>0.5</td>
<td>---</td>
<td>1 / 2.054 = 0.487</td>
</tr>
</tbody>
</table>

The values in bold and boxes are incorrect. The value 0.769 is valid for section at midspan and not at the support. The correct value at the support is 0.769 / 2 = 0.385.
The procedure given in EN 1995-1-1: 2004 is limited only to few cases and gives very rough values of \( L_{ef}/L \approx l_{ef}/L \). The procedure from DIN 1052: 2004, which is offered to designers also in German National Annex to DIN EN 1995-1-1: December 2005 is less limited but contains incorrect value.

Authors recommend not to use the procedure given in prestandard Eurocodes ENV 1993-1-1:1992 Design of steel structures and ENV 1999-1-1:1998 Design of aluminium structures which contain a lot of problems and incorrect values \( C_1, C_2, C_3 \). Authors criticized the draft of prEN 1993 which originally contained the same tables as above prestandards ENV. As a consequence of the criticism these tables were removed from the final version of EN 1993-1-1: March 2005. Eurocode for design of steel structures EN 1993 therefore does not contain any procedure for calculation of critical moment \( M_{cr} \). Designers should avoid to use that part of publications containing ENV procedure for \( M_{cr} \) calculation, e.g. very good books (Hirt, Bez, 1998) and (Hirt, Bez, Nussbaumer, 2007).

Eurocodes prEN 1999-1-1: May 2004 and EN 1999-1-1: May 2007 accepted without change the procedure proposed by (Baláž, Koleková, 2000b). It may be found in Annex I. Prof. Sedlacek accepted this procedure in draft of DAS Richlinie 023, Stabilität von Tragwerken aus der Haupttragebene Biegeknicken und Biegedrillknicken.

The tables created in (Baláž, Koleková, 2000b) are used after modification in handbook (ECCS Technical Committee 8 – Stability Rules for Member Stability EN 1993-1-1, No. 119, 2006) without any reference to the original source. But the ECCS handbook No. 119 contains incorrect rules and some incorrect \( C_i \) values relating to \( M_{cr} \) calculation. Details see in (Baláž, Koleková, 2014a).

Authors of another ECCS publication (Dubina, Ungureanu, Landolfo, 2012) during copying the tables from handbook No.119 produced so many print errors in \( C_i \) values that using of these tables is dangerous.

Consequence of the facts that: (i) Eurocode EN 1993-1-1: March 2005 for design of steel structures contains no rules for \( M_{cr} \) calculation and (ii) Eurocode EN 1999-1-1: May 2007 for design of aluminium structures is not known to many designers is, that incorrect procedures given in prestandard ENV 1993-1-1:1992 or in two above ECCS publications are used in many books, National Annexes and in design practice.

In (Živner, 2010) there is a large collection of many procedures for \( M_{cr} \) calculation. The procedure (Baláž, Koleková, 2000b), which is in EN 1999-1-1: May 2007, was found as the best one. This was confirmed also in the Brazilian PhD thesis (Fruchtengarten, 2010). See also (Kulmann et al., 2014, p. III-144 and III-146).

The formulae taken from the DIN 1052: 2004 seems to be only simplifications of the more exact (Baláž, Koleková, 2000b) formula because \( a_1 = C_1/k_z, a_2 = C_2/k_z, a_3 = z_k \). The error as a consequence of such simplification is described by the ratio of the less exact DIN 1052: 2004 formula to more exact (Baláž, Koleková, 2000b) and EN 1999-1-1:2007 formula

\[
\frac{M_{cr,DIN1052}}{M_{cr,Balaz,Kolekova,EN1999-1-1}} = \frac{\left( \frac{L}{l_{ef}} \right)_{DIN1052}}{\mu_{cr}} = \frac{1}{1-x} = \frac{\sqrt{1-x^2} - x}{1-x} \tag{30}
\]

The graphical interpretation of (30) is shown in Fig. 2. Fig. 2 shows that the less exact DIN 1052: 2004 formula gives smaller \( M_{cr} \) values comparing with the more exact (Baláž,
Koleková, 2000b) and EN 1999-1-1:2007 formula. The error on the safe side (Fig. 2) is less than 7% for the values

\[ x = a_2 - \frac{a_z}{L} \sqrt{\frac{EI}{GL}} \leq 0.3 \]  

(31)

NOTE: it is necessary to stress that in Table 3 and 4 non-linear interpolation shall be applied. For beams with rectangular cross-section diagrams in Fig. 3 and 4 may be used.

Fig. 2 - Graphical interpretation of the ratio \( M_{cr,\text{DIN 1052}} / M_{cr,\text{Baláž, Koleková, EN 1999-1-1}} \)

Fig. 3 - Interpolation in Table 4 for \( z_j = 0, \kappa_{\text{wr}} = 0 \)

Fig. 4 - Interpolation in Table 5 for \( z_j = 0, \kappa_{\text{wr}} = 0 \)
SOLUTIONS BASED ON THE BESSEL FUNCTIONS

They may found in (Baláž, Koleková, 2000c) and in the very good book (Voľmir, 1967). For the force $F$ acting on the cantilever end ($k_z = 2$) in the shear center, we can obtain the value $\mu_{cr}$ by applying Bessel functions $J_{\nu}(x)$ and gamma function $\Gamma(\nu + 1)$ for solution of differential equation (Kamke, 1965) of lateral torsional buckling (Voľmir, 1967) as follows

$$\frac{\zeta_g}{4} \mu_{cr} \Gamma(0.25) \sum_{k=0}^{n} \frac{(-1)^k}{k! \Gamma(0.25 + k + 1)} \left( \frac{\pi}{8 \mu_{cr}} \right)^{2k} = -\Gamma(0.75) \sum_{k=0}^{n} \frac{(-1)^k}{k! \Gamma(-0.25 + k + 1)} \left( \frac{\pi}{8 \mu_{cr}} \right)^{2k}$$

(32)

For the smaller values of parameter $\zeta_g$ the following approximate formula may be used

$$\mu_{cr} = 4.013 \frac{2}{\pi} \left(1 - \frac{2}{\pi} \zeta_g\right)$$

(33)

The error in approximate formula (33) is lesser than 9.8% for $|\zeta_g| \leq 0.4$. The comparable value with the value $\pi / 4.013 = 0.783$ taken from the Table 10 is the value $\pi / 4.013 = 0.783$.

CONCLUSION

This study shows that for verification of lateral torsional stability of timber beams it is possible to use procedure proposed in (Baláž, Koleková, 2000b), which is used also in the Annex I of Eurocode EN 1999-1-1: May 2007 for design of aluminium structures. It is shown that this procedure is convenient after considerable simplification also for timber beams with rectangular cross-section or cross-sections with other shape.

The comparisons in Table 10 illustrate that this procedure gives the best results and show disadvantages of all other investigated procedures including ones used in Eurocode EN 1995-1-1: 2004, German standard DIN 1052: 2004 and German National Annex to DIN EN 1995-1-1: December 2005.

ACKNOWLEDGMENTS

Project No. 1/0748/13 was supported by the Slovak Grant Agency VEGA.

REFERENCES


[19]-Broude BM. Ultimate Limit State of Metal Beams. (In Russian). GILSA (Publisher),
Moscow, Leningrad. 1953.


Vieweg 2004b.

Springer Vieweg. 2014.

Rules for Buildings. ČSNI Praha.

[24]-ECCS Technical Committee 8 – Stability (2006) Rules for Member Stability EN 1993-1-

[25]-DASt Richlinie 023. Stabilität von Tragwerken aus der Haupttragebene Biegeknicken
und Biegedrillknicken (Sedlacek). Anlage 1 zur DAS-St-Richtlinie 023 Bemessungshilfe zur
Bestimmung des idealen Biegedrillknickmomentes Mcr (Baláž, Koleková). (Out-of-plane
stability of plane structural frames. Annex 1. Design aid to determining the elastic critical

Bemessungsregeln und Bemessungsregeln für den Hochbau. Ausgabe: August 2004

Holzbauten; Allgemeines; Allgemeine Regeln und Regeln für den Hochbau.


[29]-Dubina D, Ungureanu V, Landolfo R. Design of Cold-formed Steel Structures. Eurocode
3: Design of Steel Structures. Part 1-3: Design of Cold-formed Steel Structures. ECCS


and Rules for Buildings. CEN Brussels.


[35]-Fruchtengarten J. Sobre o estudo da flambagem lateral de vigas de aço por meio da
utilizaçao de uma teoria não-linear geometricamente exata, Dissertação (Mestrado),
Departamento de Engenhari de Estruturas e Fundações, Escola Politécnica da Universidade de

[36]-Hirt MA, Bez R. Stahlbau. Grundbegriffe und Bemessungsverfahren, 1st German


