P15NC - TRIANGULAR FINITE ELEMENT WITH CURVED EDGES FOR PLATE BENDING

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ABSTRACT
A triangular finite element with curved hedges for plate bending is presented. This element is based on the Kirchhoff’s theory for plate bending and has 12 nodes and 15 degrees of freedom (dof). The nodal configuration, Fig. 1, is as follows:

Corner nodes (1,2,3) at which the displacement normal to the plane of the element (w) is taken as parameter. Midside nodes (1b, 2b, 3b), with two dof each: displacement (w) normal to the element plane and rotation (∂w/∂n) normal to the element side. Loof nodes along the sides (1a, 1c, 2a, 2c, 3a, 3c) at which the rotation normal to the element side is taken as parameter.

INTRODUCTION
The finite element (P15NC) presented here constitutes a generalization, to curved edges, of the finite element with straight edges (P15N) presented in (Martins, 1997). The consideration of curved edges implies however a considerably more complex formulation of the directional derivatives along the sides (Silva, 2002). For this element (with curved edges) the vector \( \{ Y \} \) and corresponding unit vector \( \{ \hat{Y} \} \) tangent to the line \( \xi=\text{constant} \), are given respectively by:

\[
\{ Y \} = \frac{d\{ X \}}{d\xi} \quad \text{and} \quad \{ \hat{Y} \} = \frac{\{ Y \}}{||\{ Y \}||}
\]

Once obtained this unit vector, a direction normal to this one in the plane of the plate, is defined imposing that the inner product of the corresponding vectors is equal to zero:
\[ \{ X \} \{ Y \} = 0 \quad \text{and} \quad \{ n \} = \frac{\{ X \}}{\| X \|} \]

Considering shape functions \( M_i \) \((i=1,6)\) for corner and midside nodes the vectors tangent to each side are given by:

Side 1: \( Y_1^i = \sum_{i=1}^{6} \frac{\partial M_i}{\partial r} x_i ; \quad Y_2^i = \sum_{i=1}^{6} \frac{\partial M_i}{\partial r} y_i \) (similar expressions in \( t \) and \( s \) for sides 2 and 3)

Making \((r=\xi, \ s=\eta, \ t=1-\xi-\eta)\) it is possible to calculate the vectors normal to each of the nodes where there is a directional derivative as dof.

The derivative of the displacement \( w \) in order to the normal at each node with rotation as dof is calculated as:

\[ \{ n \} = \begin{bmatrix} \frac{\partial x}{\partial n} \\ \frac{\partial y}{\partial n} \end{bmatrix}^T \quad \frac{\partial w}{\partial n} = \begin{bmatrix} \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} \end{bmatrix} \{ n \} \]

RESULTS AND CONCLUSIONS

Several meshes were analyzed in terms of computation of normal vectors at Loof nodes and nodal parameters. Table 1 presents the results obtained for a triangular plate with two edges simply supported and one edge free. The results obtained with this element are compared with the ones obtained with the element with straight edges. The results differ only at the 5th decimal place.

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<th>Curved edges element</th>
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REFERENCES
