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Solution approaches for the soft drink integrated production lot sizing and scheduling problem

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ABSTRACT

In this paper we present a mixed integer programming model that integrates production lot sizing and scheduling decisions of beverage plants with sequence-dependent setup costs and times. The model considers that the industrial process produces soft drink bottles in different flavours and sizes, and it is carried out in two production stages: liquid preparation (stage I) and bottling (stage II). The model also takes into account that the production bottleneck may alternate between stages I and II, and a synchronisation of the production between these stages is required. A relaxation approach and several strategies of the relax-and-fix heuristic are proposed to solve the model. Computational tests with instances generated based on real data from a Brazilian soft drink plant are also presented. The results show that the solution approaches are capable of producing better solutions than those used by the company.

1. Introduction

The soft beverage industry consists of companies which produce, market, package, sell and deliver non-alcoholic beverages, including carbonated soft drinks, to customers (Montag, 2005). While the sector has many small local companies, at least three large producers maintain the highest degree of international brand recognition. The credit quality of a global beverage company is dependent on the relationships between the concentrate producer, which owns the brand, and its bottlers. The concentrate producer owns the trademark and promotes the brand advertising which is aimed at the end-consumers. The bottlers maintain the physical bottling and distribution infrastructure. They also retain and cultivate long-term relationships with the retailers that are not easily replaced. The success of the concentrate producer is largely linked to the presence of bottlers that understand the local market and culture. Given an environment of intense competition, changing consumer demands and rising costs, the soft drink sector is faced with two major challenges: maintaining product relevance, brand strength and market position, as well as balancing pricing strategies with volume growth to achieve sustained growth and stable or improving efficiency and profitability.

Soft beverages are widely popular in many markets and demand is generally stable given that they are low cost, repeated purchase items. They are consumed by a wide range of social and economic groups, although per capita consumption is higher in some markets. Brazil is one of the largest producers of carbonated soft drinks with more than 800 plants, and a consumer market of more than 13 billion l a year, which is the third in the world. However, the national per capita consumption is around 65 l, a modest number when compared to other markets. According to the Brazilian association of soft drink beverages (ABIR, 2007), the stabilization of the currency in the 1990s contributed to a significant increase in the Brazilian production, rising from 5.9 billion l in 1991, to 9.14 billion in 1995, and 11.05 billion in 1999. Over the last years the rate has remained stable producing 13.01 billion in 2006, which is 4.75% higher than in 2005.

The diversity of products offered to consumers, the scale of plants and the complexity of modern filling lines require the adoption of optimization-based programs to generate efficient production plans. The production lot sizing and scheduling problem is present in the Brazilian soft drink production, as well as in other industrial processes such as, e.g., foundry (Araujo et al., 2007), animal nutrition (Toso et al., submitted for publication) and electro fused grains (Luche et al., submitted for publication). In general, in the industry practice, the lot sizing and scheduling problem is solved separately, that is, first the production lot sizes are defined and afterwards the production schedules. However, it is often of interest to integrate the two problems in the decision process. These problems, either integrated or not, have been studied by several authors (e.g., Fleischmann, 1990; Pinedo, 1995; Drexl and Kimms, 1997; Meyr, 2000; Karimi et al., 2003; Toledo and Armentano, 2006). Production lot sizing and scheduling problems can be very difficult depending on the restrictions which have to be met

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and the combinatorial structure (classified in general as NP-hard optimization problems, e.g., Bitran and Yanasse, 1982; Meyr, 2002). The production manager needs to consider product demands, machine capacities, raw material availabilities, setup times, among other aspects.

A few mixed integer programming models for the production planning of beverages have been proposed in the literature. For instance, Clark (2003) studies the production lot sizing problem of a canning line at a drinks manufacturer, considering sequence-independent setup times of the canning line to changeover between canned products if no change of liquid is involved, and extra fixed setup times if a change of liquid is involved. Toledo et al. (submitted for publication, in press) proposes a two-stage multi-machine model that integrates the lot sizing and scheduling decisions, and takes into account sequence-dependent setup times and the synchronisation between the production stages. Solution methods based on heuristics and metaheuristics are also proposed.

In this paper we propose a mixed integer programming model that integrates the lot sizing and scheduling decisions which considers the synchronisation between the stages of the liquid flavour preparation and liquid bottling of a Brazilian soft drink plant. It is simpler than the model presented in Toledo et al. (submitted for publication), as it considers more simplifying assumptions. Both models are based on the GLSP (general lot sizing and scheduling problem) proposed in Fleischmann and Meyr (1997). In GLSP the planning horizon is divided into T macro-periods. It is a big bucket model and to obtain the sequence in which the items will be produced, each macro-period is divided into a number of micro-periods, and only one item can be produced in each micro-period.

The remaining part of this paper is organized as follows. In the next section we briefly describe the soft drink production process. The proposed optimization model is described in Section 3. In Section 4 we present the solution approaches based on a relaxation algorithm and different strategies of the relax-and-fix heuristic. Results of the computational tests are given in Section 5 and final remarks in Section 6.

2. The soft-drink production process

As mentioned, soft drinks of different flavours and bottles types (disposable and recycled) are produced in two main stages: liquid flavour preparation (stage I) and liquid bottling (stage II). In stage I, the liquid flavour (concentrate or syrup plus water) is prepared in tanks of different capacities. Two different liquid flavours cannot be prepared in the same tank at the same time, and for technical reasons the tank must be empty before a new lot of liquid flavour can be prepared in that tank. A minimum quantity of liquid flavour must be prepared in order to assure liquid homogeneity. To properly mix the necessary ingredients, the tank propeller must be completely covered.

In stage II, the liquid flavours are bottled in the filling lines. A filling line is made up of a conveyor belt and machines that wash the bottles, fill them with a combination of liquid flavour and water (carbonated or noncarbonated) and then seal, label and pack them. The production is carried out in the order described above. If for any reason it is necessary to remove a bottle from the conveyor belt, it will be done at the end of the production process, before packaging. There is only one entry point for the bottles in the filling line. For convenience sake, we treat each filling line as a single machine.

Each filling line can receive liquid flavour from only one tank at a time, no matter the number of available tanks. However, a tank can supply a liquid flavour for several filling lines simultaneously if they are bottling soft drinks of the same flavour. In the schematic representation of the production process given in Fig. 1, all the M filling lines receive water from the same source, and liquid flavour from only one tank at a time. Note that, in this example, tank N is assigned to supply liquid flavour for the filling lines k and M simultaneously, but these filling lines only receive liquid flavour from this tank.

The filling lines are initially adjusted to produce soft drinks of a given flavour in a given bottle size. At each changeover of liquid flavour in the tanks and/or soft drink in the filling lines, a sequence-dependent setup time (cleaning and/or machine adjustments) is necessary. For example, if a soft drink of a diet flavour has been prepared and a normal one is to be prepared next, the cleaning time is smaller than the other way round. In the tanks, a changeover time is necessary even if the same liquid flavour is to be prepared next. The production planning thus involves the lot size and production sequence definition at each period of the planning horizon. Although, in general, the production planning is made to meet a pre-specified soft drink demand, it is common for urgent product demands to arrive that should be met at once.

Besides the sequence-dependent changeover costs and times, another important factor to be considered in the production planning of soft drink plants is the synchronisation between stages I and II. If the synchronisation is not taken into account, a production schedule may become infeasible in practice. The liquid flavour in a given tank cannot be sent to the filling lines, unless they are ready to initiate the bottling process. In the same way, if the necessary liquid flavour is not ready, the filling line must wait for its preparation. Figs. 2 and 3 show the production planning for three

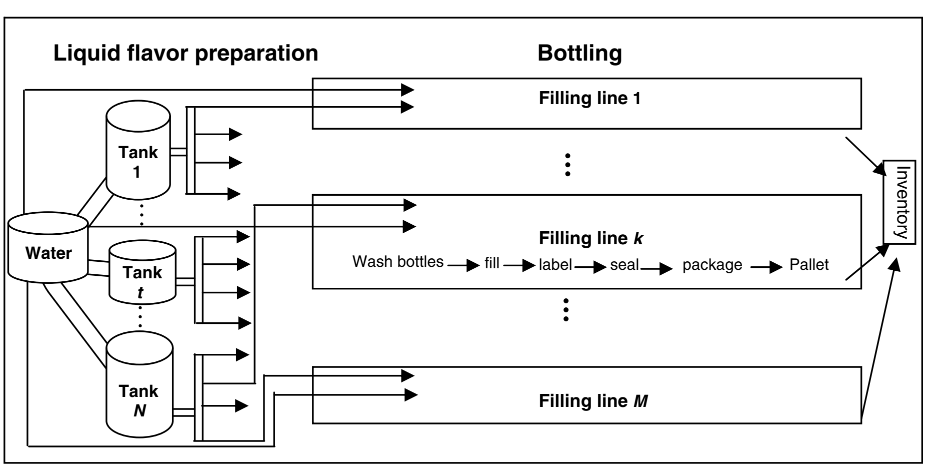

Fig. 1. Soft drink production process.

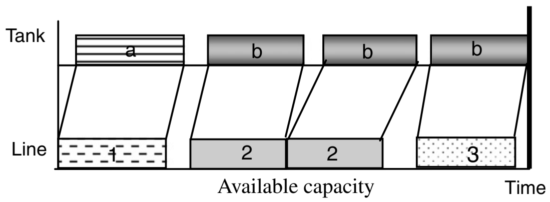

Fig. 2. Non-synchronized schedule.

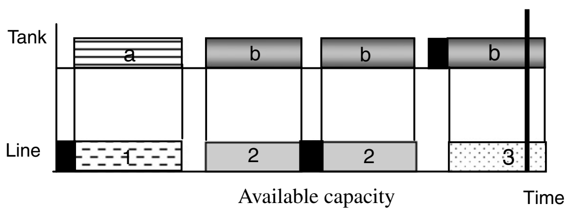

Fig. 3. Synchronized schedule.

types of soft drinks (represented by the numbers 1–3) produced from two types of liquid flavours (represented by the letters a and b). However, in Fig. 2 there is no synchronisation between the two stages. The rectangles represent the lot size and the spaces between them the changeover time from one soft drink to another in the filling line, or the changeover time from one liquid flavour to another in the tank. In Fig. 3, the black rectangles represent the waiting time to synchronize the production of the stages.

Note in Fig. 2 that at the beginning of the planning horizon, the tank needs some time in order to prepare for liquid flavour a. However, the filling line has already begun bottling soft drink 1, which is infeasible. In the first changeover (from liquid flavour a to b and soft drink 1–2), although the changeover time is the same in both stages, the filling line is ahead of the tank since it did not wait for the tank to be ready in the previous period. In the next changeover, the bottling process is initiated before the second lot of liquid flavour b is ready (this type of changeover occurs if more than one full tank capacity of liquid flavour is necessary given the soft drink lot size). The following soft drink produced needs the same liquid flavour as the previous one, but is of a different bottle size. In this case the changeover time in the tank is smaller than the changeover time in the filling line. The tank should have waited for the filling line setup before releasing the liquid flavour. If the waiting times are considered in both stages, the production schedule should be as shown in Fig. 3. Note that, after synchronizing the two stages, the necessary production capacity is higher than the available one and the proposed schedule is not viable. Therefore, the synchronisation between the two stages must be taken into account while the lot size and schedule are being planned.

The production process described above is common for the three soft drink plants visited in this research. The plants differ mainly by the number of final products, the production capacities (tanks and filling lines numbers) and the degree of computer software utilization during the production planning. Plant A (of large scale) produces more than 100 products characterized by the bottle type and flavours (for example, a soft drink of 600 ml and orange flavour is one product, while a soft drink of 2 l and the same flavour is another). It has nine tanks and seven filling lines of different capacities. Plant B (of medium scale) produces 48 products and has seven tanks and three filling lines, all of different capacities. The third one, Plant C (of small scale), has only two filling lines, one to produce soft drinks using glass bottles and the other using plastic bottles. The latter can produce 27 products and has several allocated tanks. The synchronisation between the two stages does not need to be considered in the case of Plant C, since the production bottleneck is always in the bottling stage.

Plant A has some computer software that integrates diverse departments (e.g., sales, production planning and control, logistics). The production planning is done with the help of three pieces of different software. The first one is used to define an initial lot size considering the filling line capacities, the second one is used to adjust the lot sizes considering the tank capacities, and the third one is used to define the production schedule. Various manual adjustments are still necessary when considering other constraints, such as resource availabilities, machine maintenance and urgent product demands. The production planning of Plants B and C are manual and only spreadsheets and databases are used to help the decision process.

The market growth potential together with the increasing numbers of new products and the competition for the market share has posed several challenges to the soft drinks producers. There is a great concern to improve the production and process managing. The optimization model proposed in the next section is useful to develop specific software that helps the decision process and the analysis of different production situations.

3. Model development

The mixed integer optimization model presented in this section considers the synchronisation between the production stages and integrates the lot sizing and scheduling decisions as well as the model in Toledo et al. (submitted for publication). As pointed out in Section 1, the model can be seen as a simplification of Toledo et al.'s model, where each filling line, thereafter called machine, has a dedicated tank and each tank can be filled with all liquid flavours needed by this machine. The model considers that the planning horizon is divided into T periods (macro-periods). It is a big bucket model and to obtain the production sequence, each macro-period is divided into a number of micro-periods, in which only one liquid flavour (or product) can be produced. However, unlike Toledo et al.'s model, the micro-period size is flexible and its length depends on the lot sizes of the liquid flavour (or the product).

The model considers that the production bottleneck may alternate between stages I and II in each macro-period. Therefore, when the bottleneck is in stage I, the size of each micro-period is limited by the minimum and maximum tank capacity, and when it is in stage II, the maximum micro-period size is the maximum machine capacity. The total number of micro-periods in each macro-period is the total number of possible tank setups, and it is the same in both stages. The maximum (and minimum) lot size is related to the tank capacity, as well as to the machine capacity. If, for example, to produce a lot of a given drink it is necessary to produce one and a half tank of liquid flavour (i.e. two tank setups), then this lot will be divided in two micro-periods: the size of the first micro-period will be the full tank capacity and the size of the second will be a half tank. Only one liquid flavour (or product) is produced in each micro-period. The synchronisation between the two stages is taken into account by the model using continuous variables, instead of binary ones as used in Toledo et al.'s model.

The model size is defined by (J,M,F,T,N) representing, respectively, the number of soft-drink products (items), the number of machines (both filling lines and tanks), the number of liquid flavours, the number of periods or macro-periods and the total number of micro-periods (i.e. total number of setups). Let (i,j,m,k,l,t,s) be the index set defined as: $i,j \in \{1,\ldots,J\}$, $m \in \{1,\ldots,M\}$, $k,l \in \{1,\ldots,F\}$, $t \in \{1,\ldots,T\}$ and $s \in \{1,\ldots,N\}$. Consider also, that the following sets and data are known:

Sets

S_t set of micro-periods in period t;
P_t first micro-period of period t;
λ_j set of machines that can produce item j;
α_m set of items that can be produced on machine m;
β_m set of liquid flavours that can be produced on tank m;
γ_{ml} set of items that can be produced on machine m and need liquid flavour l.

The data and variables described below with superscript I relate to stage I (tank) and with superscript II relate to stage II (bottling):

Data

d_{jt} demand for item j in period t;
h_j (non-negative) inventory cost for one unit of item j;
g_j (non-negative) backorder cost for one unit of item j;
s^{I}_{kl} changeover cost from liquid flavour k to l;
s^{IIk}_{ij} changeover cost from item i to j;
b^{I}_{kl} changeover time from liquid flavour k to l;
b^{II}_{ij} changeover time from item i to j;
a^{II}_{mj} production time for one unit of item j on machine m;
K^{I}_{m} total capacity of tank m, in litres of liquid;
K^{II}_{mt} total capacity time on machine m in period t;
r_{jl} quantity of liquid flavour l necessary for the production of one unit of item j;
q^{I}_{lm} minimum production of liquid flavour l in tank m (necessary for liquid homogeneity);
I^{+}_{j0} initial inventory for item j;
I^{-}_{j0} initial backorder for item j;
y^{I}_{ml0} 1 if tank m is initially setup for liquid flavour l; 0 otherwise;
y^{II}_{mj0} 1 if machine m is initially setup for item j; 0 otherwise.

Variables

I^{+}_{jt} inventory for item j at the end of period t;
I^{-}_{jt} backorder for item j at the end of period t;
x^{II}_{mjs} production quantity in machine m of item j in micro-period s;
v^{II}_{ms} waiting time of machine m in micro-period s;
y^{I}_{mls} $\{1$ if there is production in tank m of the liquid flavour l in micro-period s; 0 otherwise;
y^{II}_{mjs} $\{1$ if the machine m is setup for item j in micro-period s; 0 otherwise;
z^{I}_{mkls} $\{1$ if there is changeover in tank m from liquid flavour k to l in micro-period s; 0 otherwise;
z^{II}_{mijs} $\{1$ if there is changeover in machine m from item i to j in micro-period s; 0 otherwise.

The two-stage multi-machine lot-scheduling model (P2SMM) is then

$$\text{Min} \quad Z = \sum_{j=1}^{J}\sum_{t=1}^{T}(h_j I^{+}_{jt} + g_j I^{-}_{jt}) + \sum_{s=1}^{N}\sum_{m=1}^{M}\sum_{k\in\beta_m}\sum_{l\in\beta_m} s^{\mathrm{I}}_{kl} z^{\mathrm{I}}_{mkls} + \sum_{s=1}^{N}\sum_{m=1}^{M}\sum_{i\in\alpha_m}\sum_{j\in\alpha_m} s^{\mathrm{II}}_{ij} z^{\mathrm{II}}_{mijs} \tag{1}$$

Subject to:

Stage I (Tank)

$$\sum_{j\in\gamma_{ml}} r_{jl} x^{\mathrm{II}}_{mjs} \leqslant K^{\mathrm{I}}_{m} y^{\mathrm{I}}_{mls}, \quad m = 1,\ldots,M,\ l\in\beta_m,\ s = 1,\ldots,N; \tag{2}$$

$$\sum_{j\in\gamma_{ml}} r_{jl} x^{\mathrm{II}}_{mjs} \geqslant q^{\mathrm{I}}_{lm} y^{\mathrm{I}}_{mls}, \quad m = 1,\ldots,M,\ l\in\beta_m,\ s = 1,\ldots,N; \tag{3}$$

$$\sum_{l\in\beta_m} y^{\mathrm{I}}_{ml(s-1)} \geqslant \sum_{l\in\beta_m} y^{\mathrm{I}}_{mls} \quad m = 1,\ldots,M,\ t = 1,\ldots,T,\ s\in S_t - \{P_t\}; \tag{4}$$

$$z^{\mathrm{I}}_{mkls} \geqslant y^{\mathrm{I}}_{mk(s-1)} + y^{\mathrm{I}}_{mls} - 1 \quad m = 1,\ldots,M,\ k,l\in\beta_m,\ s = 1,\ldots,N; \tag{5}$$

$$z^{\mathrm{I}}_{mkls} \geqslant \sum_{j\in\gamma_{mk}} y^{\mathrm{II}}_{mj(s-1)} + y^{\mathrm{I}}_{mls} - 1 \quad m = 1,\ldots,M,\ k,l\in\beta_m, t = 2,\ldots,T,\ s = P_t; \tag{6}$$

$$\sum_{k\in\beta_m}\sum_{l\in\beta_m} z^{\mathrm{I}}_{mkls} \leqslant 1 \quad m = 1,\ldots,M,\ t = 1,\ldots,T,\ s\in S_t. \tag{7}$$

Stage II (Bottling)

$$I^{+}_{j(t-1)} + I^{-}_{jt} + \sum_{m\in\lambda_j}\sum_{s\in S_t} x^{\mathrm{II}}_{mjs} = I^{+}_{jt} + I^{-}_{j(t-1)} + d_{jt}, \quad j = 1,\ldots,J,\ t = 1,\ldots,T; \tag{8}$$

$$\sum_{j\in\alpha_m}\sum_{s\in S_t} a^{\mathrm{II}}_{mj} x^{\mathrm{II}}_{mjs} + \sum_{i\in\alpha_m}\sum_{j\in\alpha_m}\sum_{s\in S_t} b^{\mathrm{II}}_{ij} z^{\mathrm{II}}_{mijs} + \sum_{s\in S_t} v^{\mathrm{II}}_{ms} \leqslant K^{\mathrm{II}}_{mt}, \quad m = 1,\ldots,M,\ t = 1,\ldots,T; \tag{9}$$

$$v^{\mathrm{II}}_{ms} \geqslant \sum_{k\in\beta_m}\sum_{l\in\beta_m} b^{\mathrm{I}}_{kl} z^{\mathrm{I}}_{mkls} - \sum_{i\in\alpha_m}\sum_{j\in\alpha_m} b^{\mathrm{II}}_{ij} z^{\mathrm{II}}_{mijs}, \quad m = 1,\ldots,M,\ s = 1,\ldots,N; \tag{10}$$

$$x^{\mathrm{II}}_{mjs} \leqslant \frac{K^{\mathrm{II}}_{mt}}{a^{\mathrm{II}}_{j}} y^{\mathrm{II}}_{mjs}, \quad m = 1,\ldots,M,\ j\in\alpha_m,\ t = 1,\ldots,T,\ s\in S_t; \tag{11}$$

$$\sum_{j\in\alpha_m} y^{\mathrm{II}}_{mjs} = 1, \quad m = 1,\ldots,M,\ s = 1,\ldots,N; \tag{12}$$

$$z^{\mathrm{II}}_{mijs} \geqslant y^{\mathrm{II}}_{mi(s-1)} + y^{\mathrm{II}}_{mjs} - 1, \quad m = 1,\ldots,M,\ i,j\in\alpha_m,\ s = 1,\ldots,N; \tag{13}$$

$$\sum_{i\in\alpha_m}\sum_{j\in\alpha_m} z^{\mathrm{II}}_{mijs} \leqslant 1, \quad m = 1,\ldots,M,\ s = 1,\ldots,N; \tag{14}$$

$$I^{+}_{jt}, I^{-}_{jt} \geqslant 0, \quad j = 1,\ldots,J, \quad t = 1,\ldots,T; \quad z^{\mathrm{I}}_{mkls},\ x^{\mathrm{II}}_{mjs},\ v^{\mathrm{II}}_{ms},\ z^{\mathrm{II}}_{mijs} \geqslant 0; \quad y^{\mathrm{II}}_{mjs},\ y^{\mathrm{I}}_{mls} = 0/1, \quad m = 1,\ldots,M,\ k,l\in\beta_m,\ i,j\in\alpha_m,\ t = 1,\ldots,T,\ s\in S_t. \tag{15}$$

The objective function (1) is to minimize the total sum of product inventory, demand backorder, machine changeover and tank changeover costs. In stage I, the demand for liquid flavour l is computed in terms of the production variables of stage II. That is, the demand for liquid flavour l in each tank m in each micro-period s is given by $\sum_{j\in\gamma_{ml}} r_{lj} x^{\mathrm{II}}_{mjs}$. Therefore, the constraints (2) together with constraints (3) guarantee that if there is production in tank m of liquid flavour l in micro-period $s(y^{\mathrm{I}}_{mls} = 1)$, the amount produced will be defined between the minimum quantity necessary for liquid homogeneity and the tank maximum capacity (i.e. the minimum and maximum sizes of micro-period s).

Constraints (4) refer to the idle micro-periods to happen at the end of the associated macro-period (note that $\sum_{l\in\beta_m} y^{\mathrm{I}}_{mls}$ can be equal to 0, meaning that there is no production in tank m of any liquid flavour in micro-period s). Constraints (5) control the liquid flavours changeover. However, if variable $y^{\mathrm{I}}_{mk(s-1)}$ is zero in the last micro-period of a given macro-period $t-1$, $s = P_t$, the changeover between macro-periods is not taken into account by the associated constraint (5). The setup variables in stage II indicate which liquid flavour was prepared in the last non-idle micro-period of each macro-period. Therefore, constraints (6) are needed to consider the changeover between macro-periods. Constraints (7) ensure that there is at most one changeover in each tank m in each micro-period s.

In stage II, constraints (8) represent the inventory balancing constraints for each item in each macro-period. Since the production variable is defined for each micro-period, to obtain the total production of item j in a given macro-period t, it is necessary to calculate the associated production variables over all machines where it can be produced ($m \in \lambda_j$) and micro-periods ($s \in S_t$) of macro-period t. Constraints (9) represent the machine capacity in each macro-period.

As discussed in Section 2, a machine must wait until the liquid flavour is ready in the tank. The set of continuous variables, $v^{\mathrm{II}}_{ms} \geqslant 0$, and the constraints (10) compute this waiting time for each machine m from the beginning of each micro-period s. The waiting time is equal to the difference between the tank changeover time and the machine changeover time. If the machine changeover time from item i to j is greater than the tank changeover time from liquid flavour k to l, the waiting variable is zero and only the machine changeover time is considered in the associated capacity constraint (9). Otherwise, the variables v^{II}_{ms} are positive and the total waiting time of the macro-period is taken into account in constraints (10), thus establishing the synchronisation between the two production stages.

Constraints (11) ensure that there is a production of item j in machine m at micro-period s only if the associated setup variable is set to one. Constraints (12) refer to a single mode of production in each micro-period s and they impose that each machine m is setup for one item at each micro-period s. Note that the production may not occur although the machine is always ready to produce an item. Constraints (13) count the changeover in each machine m in each micro-period s. Constraints (14) ensure that there is at most one changeover in each machine m in each micro-period s.

Finally, constraints (15) define the variables domain. Note that the changeover variables z^{I}_{mkls} and z^{II}_{mijs} are continuous. Constraints (5) and (13) together with the optimization sense (minimization) ensure that these variables will take only 0 or 1 values in an optimal solution.

The above model can be easily adapted to represent the particularities of the different soft drinks companies studied. In the computational tests described in Section 5, the instances were generated based on data collected from Plant A. When defining the soft drinks lot size and schedule, this plant requires that the product inventories in a given macro-period must be sufficient to cover the product demands in the next period. To have a fair comparison between model P2SMM and the company solutions, a new set of constraints should be included in the model

$$I^{+}_{jt} \geqslant d_{j(t+1)}, \qquad j=1,\ldots,J, \quad t=1,\ldots,T, \tag{16}$$

where $d_{j,T+1}$ is the forecasted demand for item j in period $T+1$, i.e. the first period of the next planning horizon.

Plant A has various tanks and filling lines (machines) with different capacities. Some tanks (and machines) are allocated to produce only a given subset of flavours (and items), whereas others can produce any one. There is a single liquid flavour ($l = p$) whose demand is by far superior to the others. While most of the liquid flavours have demands around 20,000 units per period, flavour p has a demand of around 150,000 units. It also has a high setup time and cost. Therefore, in Plant A there is a tank which is completely allocated to continuously produce flavour p. That is, whenever a machine is ready to produce an item that uses this flavour, the tank is also ready to release it. To represent this situation in model P2SMM, the changeover time in stage I from any flavour k to flavour p, and from flavour p to any flavour k, is set to zero ($b^{\mathrm{I}}_{kp} = b^{\mathrm{I}}_{pk} = 0$). Although there is no need to setup the tanks for flavour p, there is still a need to setup the machines when items that need this flavour are produced. However, the tank setup variable, y^{I}_{mps}, cannot be set to zero, since when this is done, constraints (2) together with (3) impose that the production of any item that uses this flavour is zero (if $y^{\mathrm{I}}_{mps} = 0$ then $x^{\mathrm{II}}_{mjs} = 0$, for $j \in \gamma_{mp}$). Therefore, the tank capacity (constraints (2) and (3)) for $l = p$ is dropped.

4. Approaches to solve the model P2SMM

The solution of practical instances of model P2SMM using the exact methods included in standard software such as CPLEX (Ilog, 2006) was not satisfactory (see Section 5). This indicated the need to develop specific solution strategies, which are described next.

4.1. The relaxation approach

In some soft drink plants the liquid preparation in stage I generally does not represent a bottleneck for the production process. That is, the tank capacities are large enough to ensure that, whenever a machine needs a liquid of a given flavour, it will be ready to be released. Therefore, there is no need to control either the changeover in the tanks or the synchronisation between the two production stages. Only the minimum tank capacity constraints to ensure the liquid homogeny are necessary in stage I. This situation was explored as a solution approach to model P2SMM. The relaxation approach (RA) is based on the idea that once the production decision is taken in stage II, the decision for stage I is easily taken.

In the first step, the lot size and schedule of the items in the machines are decided using a one stage model obtained from model P2SMM by dropping the changeover variables of stage I, z^{I}_{mkls}, and the waiting variables of stage II, v^{II}_{ms}. The constraints (9) are replaced by constraints

$$\sum_{j \in \alpha_m} \sum_{s \in S_t} a^{\mathrm{II}}_{mj} x^{\mathrm{II}}_{mjs} + \sum_{i \in \alpha_m} \sum_{j \in \alpha_m} \sum_{s \in S_t} b^{\mathrm{II}}_{ij} z^{\mathrm{II}}_{mijs} \leqslant K^{\mathrm{II}}_{mt}, \quad m=1,\ldots,M, \quad t=1,\ldots,T; \tag{9a}$$

and the objective function is redefined as

$$\mathrm{Min} \quad Z = \sum_{j=1}^{J} \sum_{t=1}^{T} (h_j I^{+}_{jt} + g_j I^{-}_{jt}) + \sum_{s=1}^{N} \sum_{m=1}^{M} \sum_{i \in \alpha_m} \sum_{j \in \alpha_m} s^{\mathrm{II}}_{ij} z^{\mathrm{II}}_{mijs}. \tag{1a}$$

The sets of constraints ((2), (3), (8), (9a), (10)–(16)) and the objective function (1a) define a one-stage multi-machine lot-scheduling model (P1SMM). An adjustment of the solution of this model might be necessary to take into account the synchronisation between the two stages. This can be obtained by model P2SMM with the setup variables of stages I and II fixed according to the solution of model P1SMM. That is, if item j is produced, then the tank and the machine must be setup. This procedure is outlined in Fig. 4, where σ_j is the set of liquid flavours necessary to produce item j. If the models in Algorithm RA are solved by a standard software (e.g. the branch and cut method in CPLEX) and their optimal solutions are not achieved in a pre-defined amount of time, the branch and cut execution is stopped and the best solution is considered.

4.2. Relax-and-fix strategies

In the relax-and-fix heuristic, the integer variable set is partitioned into P disjunctive sub-sets, Q_i, $i = 1,\ldots,P$. At each iteration, the variables of only one of these sub-sets are defined as integers, while the variables of the others are relaxed and defined as continuous ones. The resulting sub-model is then solved. The integer

Algorithm RA
Step1 – Solve the P1SMM model.
Step 2 – **If** P1SMM is feasible **then**
for $m = 1,\ldots,M$, $j \in \alpha_m$, $s = 1,\ldots,N$ **do**
If $x^{II}_{mjs} > 0$ **then**
Fix the setup variables of model P2SMM according to:
$y^{II}_{mjs} = 1$ and $y^{I}_{mls} = 1$, $\forall\ l \in \sigma_j$.
endif
endfor
endif
Step 3 – Solve the P2SMM model obtained in Step 2.

Fig. 4. Algorithm RA.

variables of the sub-model are fixed at their current values and the process is repeated for all the sub-sets. Besides the variable set partition, criteria to fix the variables must also be defined before applying the procedure. The main feature of this heuristic is the solution of sub-models that are smaller, and possibly easier to solve, than the original one. The partition of the variable set and the criteria used to fix the variables have a strong connection with the degree of the sub-model difficulty (Wolsey, 1998). The relax-and-fix heuristic can be described as the RF algorithm outlined in Fig. 5.

The relax-and-fix heuristic has been largely used as a method to obtain good primal bounds (feasible solutions) for hard mixed-integer programs either on its own or in hybrid algorithms. In the usual relax-and-fix strategy, the variables are grouped by periods (macro-periods) and only the integer variables are fixed at each iteration – the heuristic iterations number is thus the number of periods (Dillenberger et al., 1994). Various strategies to partition the set of binary variables are explored in Escudero and Salmeron (2005). Toledo (2005) uses a relax-and-fix heuristic to solve the soft drinks lot scheduling problem described in Toledo et al. (submitted for publication). The criterion used is to first fix the binary variables in stage I, then the ones in stage II, in a backward fashion. The relax-and-fix heuristic has also been used in combination with metaheuristics. Pedroso and Kubo (2005) presents a hybrid tabu search procedure in which the relax-and-fix heuristic is used either to initialize a solution or to complete partial solutions. The hybrid approach is applied to solve a big bucket lot sizing problem with setup and backlog costs. At each iteration of the relax-and-fix heuristic, only the variables of a given period that concern a single product are fixed. The main advantage of this strategy is to solve smaller sub-models since some mono-period, mono-machine multi-items lot sizing problems are hard to solve. Other relax-and-fix strategies also fix continuous variables (Federgruen et al., 2007).

The P2SMM model proposed in Section 3 presents various possibilities to build sub-sets Q_i, $i = 1,\ldots,P$. The setup and changeover variables are indexed by stages, machines, items and periods. These indices are explored when defining various strategies of variable set partitions. Different criteria were proposed to fix variables. For example, after solving a sub-model in a given iteration, it is possible to fix only the binary variables associated with non-zero production variables. Table 1 shows 15 relax-and-fix strategies, divided into three groups. Group 1 has five strategies (G1.1–G1.5), Group 2 has seven (G2.1–G2.7), and Group 3 has three. The first column shows the strategy name, the second and third columns show the criteria used for the partition and fixing the variables, respectively. The variables presented in Table 1 are the same ones used to describe the model P2SMM, however, for simplicity sake, some of their indexes were omitted.

Algorithm RF
Initialization – Define a partition of the integer variable set into P sub-sets, Q_k, $k = 1,\ldots,P$ and a criterion to fix variables.
for $k = 1,\ldots,P$ **do**
Step 1 – Relax the integer variables in the sub-set Q_j, $j = k+1,\ldots,P$.
Solve the resulting sub-model.
Step 2 – If the sub-model given in Step 1 is unfeasible, stop. Otherwise, fix the integer variables that satisfy the criteria defined in initialization to their current values.
Endfor

Fig. 5. Algorithm RF.

Table 1
Relax-and-fix strategies

Strategy	Partition	Fixing
G1.1	Macro-period	y^{I}, y^{II}
G1.2	Macro-period	y^{I}, y^{II}, z^{I}, z^{II}
G1.3	Macro-period	y^{I}, z^{I}, y^{II}, z^{II}, x^{II}
G1.4	Macro-period	y^{I}, z^{I}, y^{II}, z^{II} i.th.p.*
G1.5	Macro-period	y^{I}, z^{I}, y^{II}, z^{II} i.th.p. with re-evaluation
G2.1	Machine/macro-period	y^{I}, y^{II}
G2.2	Stage I then Stage II	y^{I}, y^{II}
G2.3	Stage II then Stage I	y^{II}, y^{I}
G2.4	Macro-period/Stage I then Stage II	y^{I}, y^{II}
G2.5	Macro-period/Stage II then Stage I	y^{II}, y^{I}
G2.6	Machine/macro-period/Stage II and machine/macro-period/Stage I	y^{II}, y^{I}
G2.7	Machine/macro-period/Stage II and machine/macro-period/Stage I	y^{II}, z^{II}, y^{I}, z^{I} i.th.p. with re-evaluation
G3.1	Micro-period	y^{I}, y^{II}
G3.2	One micro-period per macro-period	y^{I}, y^{II}
G3.3	First and last micro-period of each macro-period	y^{I}, y^{II}

* i.th.p.: if there is production.

The first five strategies (G1.1–G1.5) use the usual criteria of partitioning the variable set according to macro-periods (using a forward scheduling). They differ from each other by the criteria used to fix the variables in a given iteration. These criteria are based on the idea that the sub-models should have a dimension that favours the decision process. The objective was to evaluate the influence of the variables in the sub-model solution. Take for example strategies G1.1 and G1.2. Note that when the variable y^{II} is fixed to one, it only assures that the machine is prepared, i.e. it does not indicate if the item will be produced or not. Variable y^{I}, besides defining that the tank will be prepared, also states that there will be production of an item between the tank capacities (constraints (2) and (3) in the P2SMM model). Strategy G1.2 is as flexible as G1.1, however, it also fixes the continuous changeover variables (z^{I},z^{II}). This reduces the sub-model size both in terms of variables and constraint numbers. The latter might be reduced by the pre-processing routines usually included in optimization packages.

The lot size is only fixed when variable x^{II} is fixed as in strategy G1.3. Strategy G1.4 is more flexible than strategies G1.1–G1.3. In an iteration k the variables of each micro-period of macro-period k are evaluated and fixed only if there is production ($x^{\mathrm{II}}_{mjs} > 0, \forall s \in S_k$). Otherwise, the binary variables are not fixed and remain binary in the iterations $k+1,\ldots,T$. Besides allowing a redefinition of the lot sizes, this strategy also allows any item to be produced in idle periods at future iterations. An attempt to improve this strategy is done by re-evaluating the idle micro-periods in previous iterations which did not have any variables fixed. So, if in a previously idle micro-period there is production, then the associated variables are fixed. In this strategy, G1.5, at each iteration, the sub-models become less flexible than the ones in strategy G1.4, but might become smaller as more variables are fixed.

Group 2 strategies explore the multi-machine and the two-stage structure of the model to partition the set of variables. The objective was to evaluate the influence of each machine in the solution of the sub-models. Strategy G2.1 is similar to strategy G1.1, but considers both the machine and period index to partition the variable set. In iteration 1, for example, only the variables of machine 1 in macro-period 1 are binary, whereas in iteration 2 only the variables of machine 1 in macro-period 2 are binary, and so on. The setup variables of both stages (y^{I} and y^{II}) are fixed at each iteration. All the other strategies in this group consider one stage at a time. In Strategies G2.2 and G2.3, the partition is done according to the production stage, therefore the order to fix the variables changes accordingly. First, the variables in stage I (or stage II) are fixed and then the ones in stage II (or stage I). Note that this criterion of fixing the variables is different from the strategy used in the RA algorithm. In the RA algorithm all setup binary variables, y^{I} and y^{II}, are kept binary in model P1SMM (Step 1) and fixed in Step 2 only if there is production. In the G2.3 strategy, the binary variables $y^{\mathrm{I}}$are relaxed and consequently constraints (2) and (3) are relaxed too, also the variables are fixed despite the production in the micro-period.

In strategies G2.4 and G2.5, a reduction in the sub-model size of strategies G2.2 and G2.3 is obtained by including the variable macro-period in the partition criteria. A further reduction in the sub-model size is obtained in strategy G2.6 by, besides considering the variable period and stage, also considering the machines in the partition. The criterion used to fix the variables in strategies G2.1–G2.6 is the same as the one used in strategy G1.1, i.e. only the binary variables (y^{I}, y^{II}) are fixed at each iteration. Strategy G2.7 combines the sub-models' size of strategy G2.6 with the flexibility to fix the variables of strategy G1.5. Besides the binary variables, the continuous variables (z^{I}, z^{II}) are also considered to be fixed when there is production ($x^{\mathrm{II}} > 0$). In this strategy, the idle micro-periods in previous iterations which did not have any fixed variables are also re-evaluated for further variable fixing. Note that only strategies G2.1, G2.6 and G2.7 consider the machine index in the partition criteria.

The criterion used to partition the variable set in the Group 3 strategies moves forward in micro-periods. This criterion implies a higher number of iterations, but smaller sub-models. For example, if strategy G1.1 is applied to model P2SMM, there are T iterations and sub-models with $|\, S_t \,| \sum_m (|\, \alpha_m \,| + |\, \beta_m \,|)$ binary variables each, and if strategy G3.1 is applied, there are N iterations and sub-models with $\sum_m (|\, \alpha_m \,| + |\, \beta_m \,|)$ binary variables each. However, fixing a variable of a given micro-period also reduces the heuristic flexibility. The results obtained testing strategies G3.1–G3.3 were not good compared to the ones in Groups 1 and 2. For some instances, they even failed to produce feasible solutions. Therefore, these results were not included in the computational tests described in the next section.

5. Computational tests

In this section we present and analyze the computational results of the solution approaches presented in Section 4, applied to solve real instances of the production planning problem of Plant A. The P2SMM model (Section 3) and the RA and RF algorithms (Section 4) were coded in the AMPL modelling language (Fourer et al., 1993). The P2SMM model and the sub-models necessary in the RA and RF algorithms were solved using the optimization system CPLEX version 10.0 (Ilog, 2006). At each iteration of algorithms RA and RF, a mixed integer optimization model needs to be solved. Although generally simpler than model P2SMM, these models are still difficult to solve. Therefore, if they were not solved to optimality in a pre-specified amount of time, their execution was stopped and the best solution was analysed. The runs were executed using an Intel Pentium 4, 1.0 GB de RAM, 3.2 GHz.

5.1. The test bed

Several visits to Plant A were made in order to understand its production processes and to collect the necessary data to simulate its lot sizing and scheduling problem. Data such as product demands, changeover times in both stages, tank and machine capacities, among others, were collected during the visits. Several interviews with workers from different departments of the plant were conducted in order to obtain the appropriate information. For reasons of confidentiality, some values were modified to protect interests of the company. The data was used to generate 15 instances. Each period of the planning horizon refers to one production week.

The first instance (P1) was generated based on data related to two machines that can produce items in common. The first one (machine 1) can produce 23 items and the second one (machine 2) only 10 out of these. That is, there are 13 items that can only be produced on machine 1. Eighteen different flavours are necessary to produce this set of items. Machine 1 was available for four working days per week (total of 5760 minutes per week) and machine 2 for six working days (total of 8640 minutes per week). It was estimated that the tank could have up to five changeovers per day. Taking the average working days (5 days) of the machine, it is possible to have up to 25 changeovers per week. The product demand data for instance P1 is associated with three weeks. Therefore, this instance has three macro-periods (3 weeks) with a total of 75 micro-periods (25 per macro-period). The production scheduling for this instance was provided by Plant A, making the comparison with the solution approaches tested possible.

Four other instances (P2–P5) were generated by modifying part of the data used in instance P1. In instance P2 the inventory costs

Table 2
Instances dimensions P1–P15

Model	Variables	Binary variables	Constrains
P2SMM	86,359	4,575	86,140
P1SMM	54,559	4,575	49,544

of P1 were doubled, in instance P3 the backorder costs of P1 were doubled, in instance P4 the total demand of each item in the planning horizon of P1 was randomly redistributed among the three periods, and in instance P5 the machine capacities were reduced by 25%.

The remaining ten instances (P6–P15) are based on product demand data related to a period of 30 weeks. Each one of these instances is associated with three consecutive weeks. Instances P6, P7, P14 and P15 are associated with periods of higher demands when compared to P8–P13. Except for the demands, all the other parameters used in these instances are the same ones used to generate instance P1. More details on the test bed (e.g. AMPL codes, data, etc.) are given in Ferreira (2006). All the 15 instances of models P2SMM and P1SMM (used in Step 1 of algorithm RA) have the same dimensions, as shown in Table 2. Note that the number of variables and constraints of the models are relatively large.

5.2. Results

The computational experiments were divided into three parts. First, all the solution approaches were applied to solve instance P1 and compared to the company solution. Instances P1–P5 were used to evaluate the solution strategies (algorithms RA and RF). The best solution strategies were then used to solve instances P6–P15. To simulate Plant A practice, the maximal execution time for solving each instance of the P2SMM model by CPLEX, algorithm RA and algorithm RF, was set to 4 hours. In general, the soft drink lot sizing and scheduling are prepared three to four days in advance and, according to the production manager it takes around 4 hours to be completed. Therefore, this time limit was used only as a time reference and it is considered acceptable for the decisions involved. In practice the algorithm could also run automatically overnight for more than 4 hours.

All the relax-and-fix strategies presented in Table 1 were applied to solve the models P2SMM and P1SMM (Step 1 of algorithm RA). To have a fair comparison, the execution time was divided as follows. The maximum execution time for algorithm RF when applied to solve the P2SMM model was set to 3 hours. The relax-and-fix solution was then used as a first integer solution in CPLEX, which was executed for another hour. In the RA algorithm, algorithm RF was executed for at most 3 hours and only to solve the model in Step 1. The model in Step 3 was then solved by CPLEX with a maximum of 1-hour execution time (maximum of 4 hours to execute algorithm RA).

5.3. Results for instance P1

Six CPLEX strategies were used to solve the P1 instance of model P2SMM and the P1 instances of the sub-models in Steps 1 and 3 of algorithm RA. The CPLEX system allows for the adjustment of various parameters of its branch and cut algorithm. In this test we compared the performance of the subroutines to find primal bounds (heuristics), the pre-processing subroutines, and the subroutines to obtain dual limits (cutting planes generation). The system default parameters automatically decide when to apply these subroutines to solve a problem. The symbol "×" in Table 3 indicates which one of the above subroutines was activated or deactivated. For example, the use of the CPLEX strategy run 5 means that the instance was solved by the CPLEX default parameters, except for the intensive use of the CPLEX heuristic and the deactivation of the cutting plane generation subroutines.

Table 3
Runs with different CPLEX parameters

Name	Default	Intensive use of CPLEX heuristics – on	Cutting planes generation – off	Pre-processing – off
Run 1	×			
Run 2	×		×	
Run 3	×			×
Run 4	×	×		
Run 5	×	×	×	
Run 6	×	×		×

The total cost values (in thousands of monetary units) and the corresponding integer gap for each one of the six CPLEX strategies applied to solve model P2SMM and the models associated with algorithm RA are shown in Table 4. The gaps were computed by formula: $100(Z - Z_{lb})/Z$, where Z and Z_{lb} are the values of the best integer solution and the best lower bound obtained by CPLEX. As discussed in Section 5.1, instance P1 is the only one of the problem sets for which we are aware of the corresponding production schedule used by Plant A. This company solution meets all product demands with no delays, yielding a total cost of 422.7 in thousands of monetary units. Comparing this solution with the ones given in Table 4, we note that all the six CPLEX strategies supplied solution values worse than the company one. The solution values given by algorithm RA are better than model P2SMM for all but one CPLEX strategy (run 2). However, the associated gap values are too high ($\geqslant$98.0% for P2SMM and $\geqslant$97.6% for RA), which indicates that the linear relaxations of the models are very weak and the total execution time was not enough for CPLEX to find good primal bounds. The best three strategies for P2SMM were run 2, run 6 and run 1, and for RA they were run 3, run 4 and run 6. Among the best strategies, only two of them (runs 4 and 6) make intensive use of the CPLEX heuristics. Moreover, the variation of the gap value among the first three strategies (run 1–run 3) was between 97.6% and 98.5%, while among the last three the gap value was between 97.7% and 98.6% for both P2SMM and RA ($\leqslant$0.9%). These results indicated the need to further explore the model structure to develop specific solution strategies and justify the need for algorithms RA and RF.

The solutions obtained with the intensive use of the CPLEX heuristic (run 4–run 6) were not strongly better than the ones given by the other three strategies. Therefore, in the remaining part of the tests, CPLEX was used with the parameters defined in run 1–run 3. All the relax-and-fix strategies given in Table 1 were combined with the CPLEX strategies and applied to solve the P1 instance of model P2SMM and the P1 instance of the sub-model in Step 1 of the RA algorithm. Due to space limitation, only the solution values that are better than the company one are presented (all the other results can be found in Ferreira (2006)). The P2SMM allows for backorders and since the solution value given by Plant A does not include backorder costs, a version of the problem without backorders was also solved. Table 5 summarizes all the runs executed presenting the best solution values (total cost in thousands of monetary units) with and without backorders (Z and Z_{nb}, respec-

Table 4
Results for different CPLEX strategies – instance P1

Strategy	P2SMM (Z)	GAP (%)	RA (Z)	GAP (%)
Run 1	631.507	98.4	524.784	98.0
Run 2	523.850	98.0	544.068	98.0
Run 3	652.568	98.5	429.542	97.6
Run 4	640.902	98.4	461.918	97.7
Run 5	724.378	98.6	473.698	97.8
Run 6	564.283	98.2	450.970	97.7

Table 5
Best solution values – instance P1

Approach	Strategy	Z	Percentage (%)	Z_{nb}	Percentage
P2SMM	Run 1_G2.7	384.081	9.1	529.866	−25.3
	Run 3_G2.1	413.415	2.2	511.965	−21.1
RA	Run 1_G1.1	339.793	19.6	453.901	−7.4
	Run 1_G1.3	346.671	18.0	630.318	−49.1
	Run 1_G1.5	380.754	9.9	483.760	−14.4
	Run 1_G2.1	327.795	22.5	**311.469**	**26.3**
	Run 1_G2.8	334.264	20.9	367.402	13.1
	Run 2_G1.2	403.162	4.6	418.593	1.0
	Run 2_G1.5	323.086	23.6	482.115	−14.1
	Run 2_G2.1	**306.834**	**27.4**	342.556	19.0
	Run 2_G2.8	311.553	26.3	344.949	18.4
	Run 3_G2.1	379.642	10.2	363.221	14.1
	Run 3_G2.8	324.859	23.1	346.717	18.0

tively). The first column in Table 5 indicates the solution approach, the second one shows the CPLEX and the relax-and-fix combination (CPLEX_RF), the third column the Z value, the fourth and sixth column the percentage by which the given solution is better (or worse) than the company's one. The fifth column presents the Z_{nb} value. The best solution values are highlighted in bold.

Observe in Table 5 that the RA algorithm combined with run 2_G2.1 presented the best solution value when backorders are allowed. When backorders are not allowed, the RA algorithm is still better than the direct solution of P2SMM. However, it is 26.3% better than the company's one when combined with run 1_G2.1. It is interesting to note that the inventory and changeover costs in this solution are both smaller than the ones in the solution from the company. In general, the RA algorithm provided more competitive solutions, even when backorders are allowed. All together, there were 20 solutions that were better than the company's one, seven without backorders. Therefore, the proposed model and solution approaches are competitive with the industrial practice. Nevertheless, the associated gap is still high, above 96%.

5.4. Results for instances P1–P5

All the solution strategies used to solve instance P1 were also applied to solve instances P2–P5. However, only the best results are presented in Table 6. Below each solution value (Z, in thousands of monetary units), the used relax-and-fix strategy is shown. Note that since only the tank capacity constraints are included in stage I of P1SMM model (used in Step 1 of the RA algorithm), the relax-and-fix strategy G2.7 had to be adapted and renamed as G2.8 in order to be used in RA. The best solution value for each instance is highlighted in bold. Among the 15 relax-and-fix strategies proposed in Section 4.2, four strategies provided the best solution values. For P2SMM the best relax-and-fix strategies were G2.1, G2.6 and G2.7, and for RA the best ones were G2.1 and G2.8. In general, for P2SMM, the relax-and-fix strategies provided best results when combined with run 1, while for RA the best relax-and-fix results were obtained with run 2 (three out of five instances for both approaches). Although the results are non-uniform, it is possible to point out the best combination CPLEX_RF for both P2SMM and RA, which are, respectively, run 1_G2.7 and run 2_G2.1.

5.5. Results for instances P6–P15

Considering the results in Table 6, the P6–P15 instances were solved using only the combination run 1_G2.7 for P2SMM and run 2_G2.1 for RA. Table 7 presents, for each instance, the solution value (Z, in thousands of monetary units) and the percentage (pw) by which the solution is considered worse when compared to the one given by the other approach. The best solution for each instance is considered to be 0% worse.

Table 6
Best solutions values for instances P1–P5

	Instances	Run 1	Run 2	Run 3
P2SMM	P1	**384.081**	467.257	413.415
		G2.7	G2.7	G2.1
	P2	490.303	**472.918**	556.104
		G2.7	**G2.1**	G2.7
	P3	445.063	467.173	**415.605**
		G2.7	G2.1	**G2.1**
	P4	**402.984**	470.560	496.267
		G2.6	G2.1	G2.6
	P5	**603.706**	738.698	762.777
		G2.7	G2.7	G2.6
	Average	**465.227**	523.321	528.834
RA	P1	327.796	**306.834**	324.860
		G2.1	**G2.1**	G2.8
	P2	298.972	**276.185**	351.690
		G2.1	**G2.8**	G2.1
	P3	332.035	**290.841**	366.926
		G2.8	**G2.1**	G2.1
	P4	**314.402**	317.599	351.833
		G2.1	G2.1	G2.1
	P5	366.486	379.529	**348.554**
		G2.1	G2.1	**G2.1**
	Average	327.938	**314.197**	348.773

Table 7
Solution values for instances P6–P15 – Z (pw%)

Nome	P2SMM run 1_G2.7	RA run 2_G2.1
P6	663.939 (20.7%)	**526.473 (0.0%)**
P7	591.466 (13.9%)	**509.464 (0.0%)**
P8	569.571 (10.5%)	**509.668 (0.0%)**
P9	685.703 (39.0%)	**412.237 (0.0%)**
P10	474.031 (9.3%)	**429.868 (0.0%)**
P11	579.246 (50.1%)	**289.170 (0.0%)**
P12	**436.811 (0.0%)**	491.725 (12.6%)
P13	621.971 (40.6%)	**369.540 (0.0%)**
P14	588.450 (23.6%)	**449.511 (0.0%)**
P15	671.317 (33.5%)	**446.194 (0.0%)**
Average	588.250	**443.385**

The RA algorithm gave the best solution value for nine out of the ten instances solved, which are up to 50.1% better than the ones given by P2SMM. The average solution values are also the smaller ones. The results confirm the superiority of the RA algorithm when compared to the direct solution of the P2SMM model instances. However, it is worth mentioning that, for all instances P1–P15, the gap between the best feasible solutions of model P2SMM (found within the time limit) and its linear relaxation solution is over 90%. This fact suggests that there is scope for further research exploring specific solution approaches. Plant A did not provide the corresponding production lot sizes and schedules to these instances. Therefore, it was not possible to compare the solution values obtained by the proposed approaches used to solve instances P6–P15 to the company's ones.

5.6. Overall results

Forty five combinations of CPLEX and relax-and-fix strategies were proposed and tested in two approaches (direct solution of model P2SMM and the RA algorithm) to solve actual instances of the lot scheduling problem of a soft drink plant. For the first five instances tested (P1–P5), the RA algorithm provided the best results with different CPLEX and relax-and-fix strategies. The best solution found for the practical instance, P1, was found by the RA algorithm with the combination run 2_G2.1 and it is 27.4% better than the solution used in the soft drink plant.

From the results for instances P1–P5 two strategies were selected to solve the remaining 10 instances, direct solution of model P2SMM with the combination run 1_G2.7 and the RA algorithm with run 2_G2.1. The latter approach provided the best solutions for 9 instances. These results showed that the RA algorithm is better than the direct solution of model P2SMM. They also showed that solution strategies that explore the problem structure can improve the performance of general-purpose optimization software.

6. Final remarks

In this paper a two stage multi-machine lot-scheduling model (P2SMM) is presented to solve the production-planning problem of soft drink plants. The mixed integer programming model proposed considers that the production bottleneck alternates between two stages: liquid flavour preparation and bottling. Besides the bottleneck variation, the model P2SMM also considers the synchronisation between these two stages and integrates the lot sizing and scheduling decisions. The proposed model is useful to represent the problem and showed the limitations of the available general-purpose optimization software.

A relaxation algorithm (RA) and various relax-and-fix strategies that explore the model structure were proposed and used to solve real instances of the problem. The procedures were capable of providing solutions that are up to 27.4% better than the solution used in the Brazilian soft drink plant studied. However, the integrality gap of the solution values obtained by all the solution approaches tested shows scope for further research.

The production schedule in the P2SMM model is obtained by dividing a given period into micro-periods. Another interesting strategy is to use the asymmetric travelling salesman problem to obtain the items scheduling in each period of the lot sizing problem. Toso et al. (submitted for publication) explore this idea in the solution of an integrated lot sizing and scheduling problem in the animal nutrition industry with encouraging results. Pochet and Wolsey (2006) present various solution methods for production planning problems based on mixed integer models. A classification of lot sizing problems and a subroutine library for problem reformulations, LS-LIB, are also presented. An interesting topic for future research is to use the LS-LIB to reformulate some of the P2SMM substructures. Other heuristic methods based on local branching and/or variable neighbourhood search can also contribute to improve the proposed solution approaches.

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References

ABIR., Associação Brasileira das Indústrias de Refrigerantes e de Bebidas Não Alcoólicas, Dados de Mercado, available at: http://www.abir.gov.br (last visit 01/02/2007).

Araujo, S.A., Arenales, M.N., Clark, A.R., 2007. Joint rolling-horizon scheduling of materials processing and lot sizing with sequence-dependent setups. Journal of Heuristics 13, 337–358.

Bitran, G.R., Yanasse, H.H., 1982. Computational complexity of the lot size problem. Management Science 28 (10), 1174–1186.

Clark, A.R., 2003. Hybrid heuristics for planning lot setups and sizes. Computers & Industrial Engineering 45, 545–562.

Dillenberger, C., Escudero, L.F., Wu Zhang, A.W., 1994. On practical resource allocation for production planning and scheduling with period overlapping setups. European Journal of Operational Research 75, 275–286.

Drexl, A., Kimms, A., 1997. Lot sizing and scheduling – survey and extensions. European Journal of Operational Research 99, 221–235.

Escudero, L.F., Salmeron, J., 2005. On a fix-and-relax framework for a class of project scheduling problems. Annals of Operations Research 140, 163–188.

Federgruen, A., Meissner, J., Tzur, M., 2007. Progressive interval heuristics for multi-item capacitated lot sizing problems. Operations Research 55 (3), 490–502.

Ferreira, D., 2006. Abordagens para o problema integrado de dimensionamento e sequenciamento de lotes da produção de bebidas, Ph.D. Thesis, Universidade Federal de São Carlos, Departamento de Engenharia de Produção, Brazil.

Fleischmann, B., Meyr, H., 1997. The general lot sizing and scheduling problem. OR Spektrum 19, 11–21.

Fleischmann, B., 1990. The discrete lot sizing and scheduling problem. European Journal of Operational Research 44, 337–348.

Fourer, R., Gay, M.D., Kernighan, B.W., 1993. AMPL – A Modeling Language for Mathematical Programming. The Scientific Press, Danvers, Massachusetts.

ILOG – ILOG CPLEX, Mathematical Programming Optimizers, version 10.0, 2006.

Karimi, B., Ghomi, S.M.T.F., Wilson, J.M., 2003. The capacitated lot sizing problem: A review of models and algorithms. Omega International Journal of Management Science 31 (5), 365–378.

Luche, J.R., Morabito, R., Pureza, V., submitted for publication. Combining process selection and lot sizing models for the production scheduling of electrofused grains.

Meyr, H., 2000. Simultaneous lot sizing and scheduling by combining local search with dual reoptimization. European Journal of Operational Research 120, 311–326.

Meyr, H., 2002. Simultaneous lot sizing and scheduling on parallel production lines. European Journal of Operational Research 39, 277–292.

Montag, L., 2005. Global soft beverage industry, Moody's Investors Service, Inc., August 2005. available at: http://www.moodys.com.br/brasil/pdf/soft%20beverage.pdf (last visit: June, 2007).

Pedroso, J.P., Kubo, M., 2005. Hybrid tabu search for lot sizing problems. In: Blesa, M., Blum, C., Roli, A., Sampels, M. (Eds.), Lecture Notes in Computer Science, vol. 3636. Springer, Berlin/Heidelberg, pp. 66–77.

Pinedo, M., 1995. Scheduling – Theory, Algorithms and Systems. Prentice Hall.

Pochet, Y., Wolsey, L.A., 2006. Production Planning by Mixed Integer Programming. Springer.

Toledo, C.F.M., 2005. Problema conjunto de dimensionamento de lotes e programação da produção, Ph.D. Thesis, Universidade Estadual de Campinas, Faculdade de Engenharia Elétrica e Computação, Brazil.

Toledo, F.M.B., Armentano, V.A., 2006. A Lagrangian-based heuristic for the capacitated lot sizing problem in parallel machines. European Journal of Operational Research 175, 1070–1083.

Toledo, C.F.M., França, P.M., Kimms, A., Morabito, R., submitted for publication. A mathematical model for the synchronized and integrated two-level lot sizing and scheduling problem.

Toledo, C.F.M., França, P.M., Morabito, R., Kimms, A., in press. A multi-population genetic algorithm to solve the synchronized and integrated two-level lot sizing and scheduling problem. International Journal of Production Research.

Toso, E.A.V., Morabito, R., Clark, A.R., submitted for publication. Production setup-sequencing and lot sizing through ATSP subtour elimination and patching.

Wolsey, L.A., 1998. Integer Programming. John Wiley & Sons.