Nonlinear Constant Fatigue Life Diagrams for CFRP Laminates

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Outline

- Background
- Objectives
- Experimental Results
- Modeling & Predictions
- Conclusions
Fatigue Failure Analysis

- Alternating stress
- Mean stress
- Wave sequence
- Frequency

Linear Damage Accumulation Rule

\[
\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \cdots \approx 1
\]

Basic fatigue strength as a function of
Mean Stress Sensitivity to Fatigue

**Unidirectional**
- Salkind (1972)
- Hahn (1979)
- Sims & Brogdon (1977)
- El-Kadi & Ellyin (1994)
- Miyano et al. (1995)
- Philippidis & Vassilopoulos (2002)
- Kawai & Suda (2004)

**Multidirectional**
- Ramani & Williams (1977)
- Bonfield & Ansell (1991)
- Ansell et al. (1993)
- Harris & coworkers:
  - Adam et al. (1989, 1992)
  - Harris et al. (1990, 1997)
  - Beheshty & Harris (1998)
**Constant Fatigue Life (CFL) Diagram**

**Ramani & Williams (1977)**
- Asymmetry about alternating stress axis
- Attributed to strength differential
- Linear CFLD approximation

**Bonfield & Ansell (1991)**
- Asymmetry about alternating stress axis
- Complex shape of CFLD
- Peak at the stress ratio equal to $\chi = \frac{\sigma_C}{\sigma_T}$

**Ansell et al. (1993)**

**Beheshty & Harris (1998)**
- Asymmetry about alternating stress axis
- Complex shape of CFLD
- Bell-shaped CFLD approximation
Objectives

1. Examination of the shape of CFL diagram
2. Development of an efficient fatigue life prediction method

① Fatigue behavior of a [45/90/-45/0]₂s CFRP laminate

② Experimental CFL diagram

③ Fatigue life prediction method based on a theoretical CFLD

④ [0/60/-60]₂s
⑤ [0/90]₃s
<table>
<thead>
<tr>
<th>Kind</th>
<th>Prepreg</th>
<th>Lay-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFRP</td>
<td>T800H/Epoxy #3631</td>
<td>[45/90/-45/0]_{2s}</td>
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<tr>
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<tr>
<td>CFRP</td>
<td>T800H/Epoxy #2500</td>
<td>[0/90]_{2s}</td>
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</tbody>
</table>

**Material Systems**
Specimens

Static tension & T-T fatigue tests:

Based on JIS K7073

Static compression & T-C, C-C fatigue tests:

Based on ASTM D695, Haberle & Matthews (1993)
Stress Ratio

\[ R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \]

Frequency: 10 Hz
Temperature: RT

Fatigue Tests

T-T fatigue

C-C fatigue
A critical stress ratio $\chi$

Frequency: 10 Hz
Temperature: RT

Stress Ratio

$R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}$

$R = -1$

$R = \chi = \frac{\sigma_C}{\sigma_T}$

Tensile failure
Compressive failure
Equal tendency to static failure
T-C fatigue
Experimental Results

T-T

T-C

C-C

Experimental (RT)

T800H/Epoxy#3631

$ [+45/90/-45/0]_{2s} $
Construction of CFLD

\[ \sigma_a = \frac{1}{2} (1 - R) \sigma_{\text{max}} \]

\[ \sigma_m = \frac{1}{2} (1 + R) \sigma_{\text{max}} \]

\( \Delta R = 0.1 \)
\( \square R = 0.5 \)

T800H/Epoxy#3631
Experimental (RT) [+45/90/-45/0]

\( \sigma_{\text{max}} \) for a given life
Experimental CFL Diagram

Fatigue
T800H/Epoxy#3631 RT 10Hz [+45/90/-45/0]_{2s}

\[ \sigma_m = \frac{\sigma_C}{\sigma_T} = -0.681 \]

C-C regime
R = 10
R = 2
R = -1.0

T-T regime
R = 0.1
R = -0.681
R = -0.5

Experimental\n\[ \begin{align*} &\text{Δ } N_f = 10^1 \\
&\text{△ } N_f = 10^2 \\
&\text{+ } N_f = 10^3 \\
&\text{● } N_f = 10^4 \\
&\text{○ } N_f = 10^5 \\
&\text{× } N_f = 10^6 \end{align*} \]

Static failure envelope

[+45/90/-45/0]_{2s}
Linear Approximation of CFL Diagram

\[
\frac{-\sigma_a - \sigma_a^\chi}{\sigma_a^\chi} = \begin{cases} 
\frac{\sigma_m - \sigma_m^\chi}{\sigma_T - \sigma_m^\chi}, & \sigma_T \geq \sigma_m \geq \sigma_m^\chi \\
\frac{\sigma_m^\chi - \sigma_m}{\sigma_C - \sigma_m^\chi}, & \sigma_C \leq \sigma_m < \sigma_m^\chi 
\end{cases}
\]

\[(\sigma_m^\chi, \sigma_a^\chi)\]

Good at a short fatigue life

Fatigue
T800H/Epoxy#3631
RT 10Hz [+45/90/-45/0]_{2s}

Experimental

- ▲ N=10^4
- △ N=10^3
- + N=10^2
- ⃝ N=10^1
- ○ N=10^0
- × N=10^-1
Parabolic Approximation of CFLD
Parabolic Approximation of CFLD

\[
- \frac{\sigma_a - \sigma_a^\chi}{\sigma_a^\chi} = \begin{cases} 
\left( \frac{\sigma_m - \sigma_m^\chi}{\sigma_f - \sigma_m^\chi} \right)^2, & \sigma_f \geq \sigma_m \geq \sigma_m^\chi \\
\left( \frac{\sigma_m - \sigma_m^\chi}{\sigma_C - \sigma_m^\chi} \right)^2, & \sigma_C \leq \sigma_m < \sigma_m^\chi
\end{cases}
\]

\(\sigma_a^\chi, \sigma_a^\chi\)

Good at a longer fatigue life

Fatigue
T800H/Epoxy#3631
RT 10Hz [+45/90/-45/0]_2s

Experimental
\(\blacktriangle N=10^1\)
\(\blacktriangle N=10^2\)
\(\blacktriangle N=10^3\)
\(\blacktriangle N=10^4\)
\(\blacktriangle N=10^5\)
\(\blacktriangle N=10^6\)
Assumptions:

(1) The alternating stress component of the maximum fatigue stress for a given constant value of fatigue life becomes largest at the critical stress ratio equal to the ratio of the compressive strength to tensile one.

(2) A progressive change in shape of CFL curves from a straight line to a parabola occurs with increasing fatigue life.

(3) The CFL diagram is bounded by the static failure envelope made up of two straight lines connecting the peak position of the CFL diagram with the tensile and compressive strengths respectively.
Asymmetric Anisomorphic CFLD

\[
\frac{\sigma_a - \sigma_a^x}{\sigma_a^x} = \begin{cases} 
\left(\frac{\sigma_m - \sigma_m^x}{\sigma_T - \sigma_m^x}\right)^{2-\psi_x}, & \sigma_T \geq \sigma_m \geq \sigma_m^x \\
\left(\frac{\sigma_m - \sigma_m^x}{\sigma_C - \sigma_m^x}\right)^{2-\psi_x}, & \sigma_C \leq \sigma_m < \sigma_m^x
\end{cases}
\]

\[
\psi_x = \frac{\sigma_{\max}^x}{\sigma_B} = f^{-1}(2N_f)
\]

Reference S-N relationship

Fatigue T800H/Epoxy#3631 RT 10Hz [+45/90/-45/0]_2s

Experimental
- ▲ N_f=10^1
- △ N_f=10^2
- + N_f=10^3
- ● N_f=10^4
- ○ N_f=10^5
- × N_f=10^6

\[
R = \chi
\]

R = -0.681

linear
parabolic
Comparison With Experimental Data

\[ \sigma_a - \sigma_a^\chi = \frac{\left( \sigma_m - \sigma_m^\chi \right)^{2-\psi_\chi}}{\sigma_T - \sigma_m^\chi} \quad , \quad \sigma_T \geq \sigma_m \geq \sigma_m^\chi \]

\[ \sigma_a - \sigma_a^\chi = \frac{\left( \sigma_m - \sigma_m^\chi \right)^{2-\psi_\chi}}{\sigma_T - \sigma_m^\chi} \quad , \quad \sigma_C \leq \sigma_m < \sigma_m^\chi \]

where

\[ \psi_\chi = \frac{\sigma_m^\chi}{\sigma_B} = f^{-1}(2N_f) \]

\[ \delta \geq \frac{\sigma_m^\chi}{\sigma_B} \]

\[ \delta \leq \frac{\sigma_m^\chi}{\sigma_B} \]

\[ \sigma \leq \frac{\sigma_m^\chi}{\sigma_B} \]

where

\[ \psi_\chi = \frac{\sigma_m^\chi}{\sigma_B} = f^{-1}(2N_f) \]

Experimental (RT)

T800H/Epoxy#3631

\ [+45/90/-45/0 \]_{2s}

\[ R = -0.681 \]

\[ R = -1.0 \]

\[ R = 10 \]

\[ R = 10 \]

\[ R = 2 \]

\[ R = 0.1 \]

\[ R = 0.5 \]

Good agreements with experiment!
Predicted S-N Relationships

T-T fatigue loading

$\sigma_{\text{max}}$, MPa

$2N_f$

Experimental (RT)

T800H/Epoxy#3631

$[+45/90/-45/0]_{2s}$

$R = 0.1$

$R = 0.5$

$\triangle R = 0.1$

$\square R = 0.5$

Predicted
Predicted S-N Relationships

**C-C fatigue loading**

- $R = 2$
- $R = 10$

**T-C fatigue loading**

- $R = 1.0$
- $R = -0.681$

**Graphs**

- T800H/Epoxy#3631
- Experimental (RT): $R = 2$, $R = 10$
- Predicted: $R = 2$, $R = 10$

**Equation**

$$2N_f = \frac{2}{K^*} \left(\frac{1 - \psi_j}{\psi_j}\right)^n$$
Further Verification (1)

Application to a $[0/60/-60]_{3s}$ Laminate
Predicted S-N Relationships

C-C fatigue loading

T-T fatigue loading

σ

R = 0.5
R = 0.1

t

R = 10
R = 2

σ

max, MPa

2Nf

Experimental (RT)

T800H/Epoxy#3631

[0/60/-60]2s

R = 0.1
R = 0.5

Predicted

σ

max, MPa

2Nf

Experimental (RT)

T800H/Epoxy#3631

[0/60/-60]3s

R = 2
R = 10

Predicted
Further Verification (2)

Application to a [0/90]_{3s} Laminate

T800H/Epoxy#2500

[0/90]_{3s}

Experimental (RT)

\( R = 0.5 \)

\( R = 0.1 \)

\( R = -1.0 \)

\( R = 10 \)

\( R = -0.437 \)

Static strength line

Predicted CFD

\( \sigma_a \)

\( \sigma_m \)
Predicted S-N Relationships

T-T fatigue loading

C-C fatigue loading

\[ \sigma_{\text{max}, \text{MPa}} \]

\[ 0 \quad 1000 \quad 2000 \quad 3000 \quad 4000 \quad 5000 \quad 6000 \quad 7000 \quad 8000 \quad 9000 \quad 10000 \]

\[ 10^0 \quad 10^1 \quad 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \quad 10^6 \quad 10^7 \]

\[ 2N_f \]

T800H/Epoxy#2500

Experimental (25°C)

\[ [0/90]_{3s} \]

Predicted

\[ R = 0.1 \]

\[ R = 0.5 \]

\[ R = 2 \]

\[ R = 10 \]
Comparison Between CFLDs for different CFRP laminates
Failure Under T-C & C-C Fatigue

\[ \sigma \]

\[ \text{R} = -0.681 \]

\[ \text{R} = -1.0 \]

\[ \text{R} = 2 \]

\[ \text{R} = 10 \]

T800H/Epoxy#3631

\[ [+45/90/-45/0]_{2s} \]

Experimental (RT)

\[ \text{R} = 10 \]

\[ \text{R} = 2 \]

\[ \text{R} = -1.0 \]

\[ \text{R} = -0.681 \]

Compressive

Tensile
Conclusions

Experimental and theoretical studies to identify a full shape of a CFL diagram were attempted for three kinds of symmetric CFRP laminates of $[45/90/-45/0]_2s$, $[0/60/-60]_2s$ and $[0/90]_3s$ lay-ups.

From experiments:

- It was confirmed that the CFL diagram becomes asymmetric about the alternating stress axis.
- A peak position of the asymmetric CFL diagram corresponds to the fatigue loading near the critical stress ratio equal to the ratio of compressive strength to tensile one.

(These features of the CFL diagram are similar to the findings of Ansell et al. and Harris et al.)
Conclusions

From experiments:

- It was found that the shape of the asymmetric CFL curves progressively changes from a straight line to a nonlinear curve as a given constant value of life increases. The nonlinear shape of the CFL curves in the range of a longer fatigue life can be described approximately by using a parabola.

- Similar features in the shape of CFLD were observed for all the CFRP laminates tested in this study. This fact elucidates that the three types of CFRP laminates have a similar mean stress sensitivity to fatigue.
Conclusions

From modeling:

- A procedure to predict an anisomorphic CFL diagram by using the static strengths in tension and compression and the reference S-N relationship for the critical stress ratio was developed.

- The asymmetric anisomorphic CFL diagrams predicted using the proposed procedure agree well with the experimental CFL diagrams for the types of CFRP laminates.

- The S-N relationships predicted using the theoretical anisomorphic CFL diagram agree well with experimental results for all the stress ratios in the range of fatigue life up to $10^6$ cycles, regardless of the types of CFRP laminates.
It is an advantage of the proposed procedure that only a limited amount of data obtained from relatively simple experiments is required for identification of mean stress sensitivity to fatigue of the material.

Further validation for different types of composite laminates.

The predictive capability demonstrated suggests its application as an essential ingredient to fatigue failure analysis of CFRP structures subjected to complex fatigue loading.
Thank you for kind attention!