A Model Predictive Control Scheme for Autonomous Underwater Vehicle Formation Control

Rui Gomes
Electrical engineering department
SYSTEC, Faculty of Engineering, Porto University
Porto, Portugal
rgomes@fe.up.pt

Fernando Lobo Pereira
Electrical engineering department
SYSTEC, Faculty of Engineering, Porto University
Porto, Portugal
flp@fe.up.pt

***Abstract*—In this article, we focus on the motion control of an AUV formation in order to track a given path along which data will be gathered via a computationally efficient architecture designed based on a special MPC control scheme. This control scheme enables the conciliation of onboard resources optimization with state feedback control - to deal with the typical a priori high uncertainty - while managing the formation with a low computational budget which otherwise might have a very significant impact in power consumption. The key idea is to pre-compute data which is known to be time invariant for a number of likely scenarios and store it on-board in a number of look-up tables. Then, as the mission proceeds, sampled sensory and communicated data is gathered and the proposed MPC scheme allows to determine the best control to be recruited with inexpensive computational operations which include extraction of data from pertinent on-board look-up tables.**

***Index Terms*—Model predictive control, Attainable set, AUV formation control, Obstacle collision avoidance**

I. Introduction

This article concerns the design of a general control scheme for systems in which optimization, and robustness are key requirements, and, at the same time, scarcity of computational resources is a severe constraint. A typical, and increasingly important class of systems are those involving Autonomous Underwater Vehicles (AUVs), possibly networked and articulated with other devices. Here, we present further refinements of the control architecture presented in [1] for the management and control of the formation of AUVs. This development is in the sequel of [2] in which an MPC scheme with very low on-line computational budget obtained by taking advantage of time invariance of the dynamics and some environmental features. This allows to pre-compute a priori objects which are key for the on-line feedback control synthesis. This is the essential feature of the attainable set based MPC scheme that was addressed previously in [1], [3], [4]. In this way, we accommodate in the control scheme the following features which are extremely pertinent for most of the mission scenarios involving this type robotic vehicles: (i) optimization driven, (ii) state feedback, and (iii) computationally parsimonious.

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This innovative approach contrasts strongly with all those so far proposed to AUV systems control as it will be seen in the short state-of-the-art survey in section III. The motivation to investigate sophisticated control schemes relies on the need to acquire data required to understand earth biological, geophysical, climate, as well as for security and surveillance systems increasingly regarded as critical for a sustainable human presence on earth. In [5], [6], some of the large classes of challenges currently being addressed are discussed.

This article is organized as follows: In section II, we state the AUV formation control problem in which the robotic vehicles in a triangle formation to track a planned trajectory along which data is gathered. In section III, we provide a brief overview of the relevant state-of-the-art. In section IV, we present and discuss the computationally efficient MPC scheme for AUV motion control and point out some of its properties. In section V, the most relevant one of this article, we present and discuss the control architecture for control of the AUV formation implemented with the Robust Attainable Set MPC scheme. Some conclusions and an outline of prospective future work are provided in the last section.

II. The AUV Formation Tracking Problem

In this section we describe the AUV formation control problem for a tracking mission. The AUV formation can be regarded as a variable and reconfigurable generalized vehicle and by its control we consider the control of each vehicle and the way the formation evolves to achieve the mission goal: gathering data by sensors carried by three AUVs in triangle formation tracking a given path. Thus, the AUVs motion control should be such that the integral error with respect to a given reference trajectory and the total control effort during a certain period of time is minimized subject to constraints as indicated below.

The general optimal control formulation is

$$
\begin{aligned}
(P)\ \text{Minimize} \quad & g(x(t_0+T)) + \int_{t_0}^{t_0+T} f_0(t,x(t),u(t))dt \\
\text{subject to} \quad & \dot{x}(t) = f(t,x(t),u(t)) \quad \mathcal{L}-a.e. \\
& u(t) \in \Omega \quad \mathcal{L}-a.e., \quad x(t_0+T) \in C_f \\
& h(t,x(t)) \leq 0, \quad g(t,x(t),u(t)) \leq 0
\end{aligned}
$$

where g is the endpoint cost functional, f_0 is the running cost integrand, f, h, and g represent, respectively, the vehicle dynamics, the state constraints, and the mixed constraints, C is a target that may also be specified in order to ensure stability.

To see how this encompasses the AUV formation of 3 vehicles motion problem of tracking of a given reference trajectory, just consider, for the AUV i:

- $x = col(\eta^i, \nu^i)$, and $u = \tau^i$,
- $g(\cdot) = 0$, $f_0(t, x(t), u(t)) = (\eta^i(t) - \eta_r^i(t))^T Q(\eta^i(t) - \eta_r^i(t)) + \tau^{iT}(t) R \tau^i(t)$, where $\eta_r^i(\cdot)$ is the reference trajectory for the i^{th} vehicle.
- By considering, for each vehicle (we drop the index i to lighten the expressions), the state and the controls given by $\eta = [x, y, \psi]^T$ and $\nu = [u, v, r]^T$ and $\tau = [\tau_u, \tau_r]^T$, respectively, the dynamics are those in [7] and [8]. For details, [1].
- Other constraint types include: (i) endpoint state constraints, $\eta^i(t + T) \in C_{t+T}$, (ii) control constraints, $\tau^i(s) \in \mathcal{U}^i$, (iii) state constraints, $(\eta^i(s), \nu^i(s)) \in \mathcal{S}^i$, (iv) communication constraints $g^c_{i,j}(\eta^i(s), \eta^j(s)) \in C^c_{i,j}, \forall j \in \mathcal{G}^c(i)$, and (v) formation constraints $g^f_{i,j}(\eta^i(s), \eta^j(s)) \in C^f_{i,j}, \forall j \in \mathcal{G}^f(i)$.

This problem can be easily scaled for a larger number of vehicles. The implementation is decentralized in the sense that each vehicle has its own controller (RAS MPC scheme in section IV) integrating two components: one underlying its own motion and other activities, and another concerning the cohesion of the specific formation pattern. Each vehicle communicates acoustically with neighbors with full communication graph connectivity. Each vehicle is a node of this graph whose arcs are the bidirectional communication links between two vehicles. The vehicles navigate sufficiently close to each other so that there is no loss of packets. Modes of operation: data gathering, obstacle collision avoidance, communication, and loitering. Each mode of operation requires its own formation pattern. Usually a MPC-like scheme provides a feedback control synthesis enabling to conciliate sub-optimization with feedback control. The MPC scheme consists in computing the control action for the current time subinterval – control horizon – at each sampling time, by solving the on-line optimal control problem (P) over a certain large time horizon – the prediction horizon – with the state variable initialized at the current best estimate updated with the latest sampled value. Once the optimization yields an optimal control sequence, this is applied to the plant during the control horizon. The details of this standard MPC algorithm can be found in [4].

III. Brief State-of-the-Art

Although there is a vast amount of literature, a very good reference in AUV motion control problems tat we single out is [7]. Extended versions of these control systems for very diverse robot craft have been considered for single and multiple vehicles. Non-linear control theory and geometric control provide tools that led to very popular design techniques, [9]–[11]. Early on, it became clear that optimization of resources play a key role in contexts of scarce resources and MPC became a design approach of choice in many application contexts. Moreover, MPC schemes inherit from optimal control a huge flexibility which enables to handle control system with complex dynamics, and subject to very diverse types of constraints, like those arising in the control of formation of vehicles. A wide variety of MPC controllers for formations of autonomous vehicles designed to address issues such as underwater communications failures and delays in continuum and discrete times, centralized and decentralized schemes, linear and nonlinear dynamics, leader-follower and leaderless schemes, collision-free motion, output feedback, cooperative motion, and competitive strategies, single and multiple objectives, as well as a varied range of applications (surveillance, exploration, tracking paths and trajectories), have been considered in a vast literature of which [12]–[22].

In general, these approaches suffer from key pitfalls for AUVs: (i) computationally intensive nature of the usual MPC schemes that involves solving recursively a sequence of highly complex optimal control problems (P) with very limited onboard computation capabilities and energy (as stated it can b inferred from the previous section); and (ii) the MPC schemes are parameterized in order to ensure convergence and stability and, in general, they are not related to onboard sensing capabilities. Motivated by this hard challenge, the approach in this article improves and refines the one proposed in [2] and is an extension of [1] to formation of AUVs.

IV. Attainable Set MPC

In this section we present additional results concerning the Attainable Set MPC (AS-MPC) scheme discussed in [1], [2] whose two underlying key ideas are: (i) Replace the infinite dimensional optimization problem by a sequence of finite dimensional ones; (ii) Take advantage of time-invariant data such as vehicle dynamics, and features of the environment to pre-compute off-line and store on-board a reference short term attainable set, and the value of the value function in an appropriate grid of points to be recruited on-line as a function of a number of real-time parameters.

Let us consider the short term "equivalent" cost functional and the attainable set for the dynamic control system on the time horizon $[0, T]$, with possibly $T = \infty$.

Define $V(t, z) := \min_{u \in \mathcal{U}, \xi \in C_f} \{g(\xi) + \int_t^{T_f} l(\tau, x(\tau), u(\tau)) d\tau\}$ with $x(T_f) = \xi$, $x(t) = z$, $\dot{x}(\tau) = f(\tau, x(\tau), u(\tau))$, $\mathcal{L}$-a.e.. By taking into account the Principle of Optimality (i.e., for $T < T_f$, the solution to (P_{T_f}) restricted to the interval $[t, t + T]$ is also a solution to (P_T)). Thus, on $[t, t + T]$, problems (P_T) and (P_{T_f}) are equivalent, where

$$
\begin{aligned}
(P_T)\ \text{Min} \quad & V(t{+}T, x(t{+}T)) + \int_t^{t+T} l(\tau, x(\tau), u(\tau)) d\tau \\
\text{s. t.} \quad & \dot{x}(\tau) = f(\tau, x(\tau), u(\tau)), \quad \mathcal{L} - a.e. \\
& u \in \mathcal{U}, \text{ and } x(t) \text{ is given, and}
\end{aligned}
$$

$$(P_{T_f}) \text{ Minimize } \quad g(x(T_f)) + \int_0^{T_f} l(t, x(t), u(t))dt$$
$$\text{subject to } \quad \dot{x}(t) = f(t, x(t), u(t)), \ \mathcal{L} - a.e.$$
$$x(T_f) \in C_f, \ x(0) \text{ is given}, \ u \in \mathcal{U},$$

where $\mathcal{U} := \{u : [0, T_f] \to \boldsymbol{R}^m : u(t) \in \Omega\}$, with Ω closed.

Let $t_0 < t$. The Forward Attainable Set (see [23]–[25]) is

$$\mathcal{A}_f(t; t_0, x_0) := \{x(t) : \dot{x} = f(t, x, u), u \in \mathcal{U}, x(t_0) = x_0\}.$$

By a standard change of variable, see [1], and without relabeling, (P_T) can be formulated as follows:

$$(P_T^a) \text{ Minimize } \quad V(t + T, x(t + T))$$
$$\text{subject to } \quad x(t + T) \in \mathcal{A}_f(t + T; t, x(t)).$$

- Remark 1. The computational burden of the Attainable Set leads to consider an efficient approximation, such as polyhedral of either inner or outer type, [23], [26], ellipsoidal, [27], and "cloud of points" as endpoints of trajectory segments generated by constant controls. Complexity analysis led us to opt for the last one.
- Remark 2. For positional systems, [28], the value function may be computed by solving the Hamilton-Jacobi-Bellman equation (HJBE). In general, solving HJBE numerically is extremely computationally intensive. However, there are a number of software packages to solve the HJBE numerically, [29], [30]. In general, we may consider a number of value functions for given typified situations. In real-time "mission" execution, the relevant value function is identified via sensed data and invoked to compute the next optimal control at any (t, x).

Let T, and Δ be, respectively, the optimization, and control horizons, and t the current time. The AS-MPC scheme is follows:

1. Initialization: $t = t_0$, $x(t_0)$
2. Solve (P_T^a) over $[t, t + T]$ to obtain u^*
3. Apply u^* during $[t, t + \Delta]$
4. Sample x at $t + \Delta$ to obtain $\bar{x} = x(t + \Delta)$
5. Slide time, i.e., $t = t + \Delta$, update the Attainable Set with the new $x(t)$ by appropriate translation and rotation, update the value function at the new $t + T$, and goto 2.

It is clear that, in general, the real-time computational burden of this scheme is extremely low as it involves only very simple computational operations.

The simplicity of the optimization problem is apparent due to the complexity of the computation of the attainable set. However, the invariance of the dynamics allows the off-line pre-computation of an approximation of $\mathcal{A}_f(t_0 + T; t_0, x_0)$. In the figure 1, it is shown: (i) the forward attainable set for the unicycle, and (ii) the value function in the absence of obstacles. The controls to be applied to the vehicle are found by searching for the minimum value within the vehicle's forward attainable set.

In [1], [4], the following properties of the AS-MPC scheme are stated in detail and proved. Denote by $(x^*_{T,\Delta}, u^*_{T,\Delta})$ the associated MPC optimal control process. Let $J(x, u)$ be the value of the cost functional associated with the (x, u) over $[0, \infty)$, by $J(x, u)_{|[\alpha,\beta]}$ be its restriction to the interval $[\alpha, \beta]$, and by $J_k(x, u)$ its restriction to the interval $[k\Delta, (k + 1)\Delta]$.

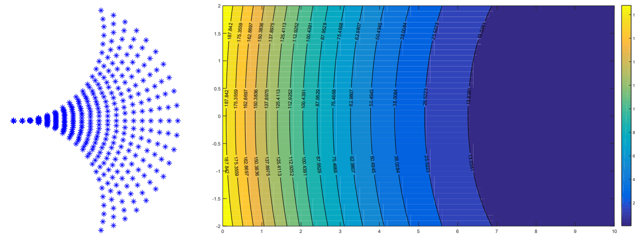

Fig. 1. i) AUV forward attainable set and ii) The control value function

Proposition 1. *Let $T_f = \infty$ and let (x^*, u^*) be an optimal control process s.t. $\lim_{t\to\infty} x^*(t) = \xi^*$, with ξ^* as an equilibrium point in C_∞. Then,*

(i) $\lim_{\Delta \downarrow 0, T \uparrow \infty} \sum_{k=1}^{\infty} J_k(x^*_{T,\Delta}, u^*_{T,\Delta}) = J(x^*, u^*)$

(ii) $\lim_{k\to\infty} \left| J_k(x^*_{T,\Delta}, u^*_{T,\Delta}) - J(x^*, u^*)_{|[k\Delta,(k+1)\Delta]} \right| = 0.$

Since we are using the cloud of points as approximation to the Attainable Set, a good estimate of the Hausdorff distance[1] between these sets to determine the worst case of sub-optimality.

Let Ω_ε denote the set $\{u_i \in \Omega : i = 1, \ldots, N_\varepsilon\}$ satisfying the following properties: (i) $\Omega \subset \bigcup_{i=1}^{N_\varepsilon} (u_i + \varepsilon B)$, and (ii) $\forall i \, \exists j$ s.t. $\|f(t, x, u_i) - f(t, x, u_j)\| < \varepsilon$. Denote by $\mathcal{A}_f(t_1; t_0, x)$ and $\mathcal{A}_f^\varepsilon(t_1; t_0, x)$ the points attainable at $t_1 > t_0$ from x at t_0, by the dynamic system with controls, respectively, in L^∞ with values in Ω, and piecewise constant with values in Ω_ε.

Proposition 2. *Let Δ be a positive number. Under mild assumptions on the dynamics, we have, for any $(t, x) \in \boldsymbol{R} \times \boldsymbol{R}^n$,*

$$d_H\left(\mathcal{A}_f(t + \Delta; t, x), \mathcal{A}_f^\varepsilon(t + \Delta; t, x)\right) \leq \Delta(\varepsilon + \Delta K_f K).$$

Another key issue concerns the fact the point $\bar{x} \in \boldsymbol{R}^n$ to which the system is steered at a given time is very likely not listed in the stored value function look-up table.

Proposition 3. *Assume that V at $\bar{x}$ is not known, and that there is a grid of points G_δ in $\boldsymbol{R}^n$ such that the maximum distance between neighboring points in G_δ is less than $\delta > 0$. Then, there is a simplex $S_{\bar{x}} = \{x_i : i = 1, \ldots n + 1\}$ which are the closest to $\bar{x}$ s.t. the estimate $\tilde{V}$ of V at $\bar{x}$ is given by*

$$\tilde{V}(\bar{x}) = \frac{\sum_{i=1}^{n+1} V_i \|\bar{x} - x_i\|^{-1}}{\sum_{i=1}^{n+1} \|\bar{x} - x_i\|^{-1}}$$

where, for $i = 1, \ldots, n + 1$, $V_i = V(x_i) + \nabla V(x_i) \cdot \bar{v}_i$, with $\bar{v}_i = \bar{x} - x_i$ and the $n \times (n + 1)$ unknowns of the vectors $\nabla V(x_i)$, $i = 1, \ldots, n + 1$ are given as a solution of the set of $n + 1$ set of equations $\nabla V(x_i) \cdot (\bar{v}_i - \bar{v}_k) = \frac{V(x_k) - V(x_i)}{\|x_i - x_k\|}$. Moreover, we have that, for some $c > 0$, $\|V(\bar{x}) - \tilde{V}(\bar{x})\| \leq \max_{x_i, x_j \in S_{\bar{x}}} \{|V(x_i) - V(x_j)|\} + c\delta$.

[1]The Hausdorff distance between sets A and B, $d_H(A, B)$, is defined by $d_H(A, B) := \max \left\{ \sup_{x\in A} \{d_B(x)\}, \sup_{y\in B} \{d_A(y)\} \right\}$, where $d_A(a)$ is the Euclidian distance between the point a and the set A.

The Robust Attainable Set MPC (RAS-MPC) scheme is a variant of the AS-MPC scheme detailed in [1], [4] where the optimization in each step is relaxed and the loop is closed within the control horizon Δ with feasible controls. This is to prevent difficulties due to persistent drifting perturbations during the control horizon.

V. The Control Architecture

The role of the control architecture consists in organizing the overall motion control problem of the AUV formation into simpler problems. This involves the: (i) Management of the overall formation that includes the maintenance of each of the pre-defined formation patterns, as well as their adaptation to circumstances specified in requirements of the listed missions; (ii) Switching between formation patterns whenever expected or unexpected events occurs to ensure the success of the mission; (iii) Managing the interaction among the AUVs in order to ensure communications and the control actions to sustain the cohesion of the vehicles; (iv) Controlling each one of AUVs in the order to carry out either pre-planned, or, if needed given the high environment variability, replanned, tasks in which the overall mission is organized.

The variability of the environment due to expected or unexpected events, e.g., the emergence of unmapped obstacles, unforeseen changes in the phenomenon of interests, equipment failures, etc., requires different modes in the motion control of the AUV formation. This amounts to regard the overall model the formation as a hybrid dynamic control system, i.e., a collection of dynamic control systems - one per mode of operation -, and a set of either controlled or uncontrolled discrete events associated which one of them. The occurrence of an event causes a switch from one mode of operation to another one, entailing a change in the behavior of the overall dynamic control system. Thus, the implementation of the RAS-MPC controller described in section IV in a context where the dynamics are of hybrid nature amounts, with additional controlled discrete events to ensure liveness and nonblocking properties, to the embedding of the RAS-MPC controller in a control architecture. Figure 2 shows the overall system automata representing the highest layer of the control architecture.

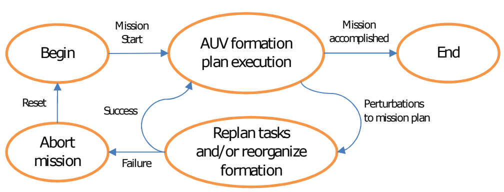

Fig. 2. Main System Automaton

In order to facilitate the explanation of the main idea, we focus solely in the motion control of a formation of three AUVs in a plane carrying the required navigation and payload sensors whose mission consists in gathering data along a given path. The specific spatial distribution of the sensors is imposed by the observation requirements. This simple scenario is easily scalable.

A realistic scenario includes several tasks of which we focus on the following ones: (i) Gathering data along a given path. The AUVs keep the triangle formation and the decentralized controller as described in section II ensures the simultaneous path tracking and formation maintenance; (ii) Avoiding collision with obstacles. This task, involves obstacle detection, and characterization, path replanning, and, possibly, reconfiguration of the formation; (iii) Communicating with the external systems. This task is required to either transmit gathered data to enable the mission follow-up and to receive commands to change the mission if necessary.

The set of discrete modes associated with these tasks and the events causing the transition between modes are represented by the automaton diagram below. Once the mission starts, the

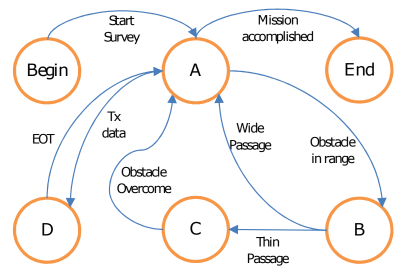

Fig. 3. Formation Pattern Automaton

AUVs enters in the nominal mode **A** of data gathering while tracking the given path on a triangle formation. The *Mission Accomplished* event prompts the AUVs to the recovery operation. The follow-up of the mission requires the monitoring of the data being gathered. This is done in mode **D** and it means that, from time to time, one of the AUV surfaces, transmits the gathered data as well as the health status of the vehicles, which after a scrutiny, might entail a change in the mission. The occurrence instants can be pre-planned or the result of either controlled or uncontrolled events. Once the exchange of information is complete, the surfacing vehicle joins the other two AUVs that, in the meantime, had been waiting loitering, in order to pursue the operation mode **A**. If an *Obstacle in range* event is detected by any of the AUVs, then the system moves to mode **B**. In this mode, the obstacle is characterized and a collision avoidance path is computed. Then, two events might occur: either a *Wide passage* is available and the formation is kept unchanged and the systems moves to the nominal mode **A** tracking the original path, or a *Thin passage* is available and the system transits to mode **C** where the formation is reconfigured to overcome de obstacle. Once this action is completed the systems pursues its operation in the nominal mode **A**.

The complexity of the collision avoidance and the fact that it illustrates well the point concerning the interaction between mission planning and control, and the onboard resources are the reasons why it deserves a special attention. We impose the following assumptions: (i) Data for obstacle detection and characterization is obtained by using a rotating pointed range finder; (ii) The unmapped obstacles in the environment are relatively sparse. This does not exclude the possibility of some obstacles being very close to one another; (iii) Obstacles are locally modelled by circles, being this simplification justified by the low computational complexity in estimating the circle with smallest radius that includes all the points detected by the range finder; (iv) The range finder sensor reaches a distance significantly larger than that transversed by the vehicle within the time interval of length Δ (defined in section IV and whose direction can vary between two limiting angles δ_{max} and $-\delta_{max}$ (see figure 5).

The automaton diagram 4 shows the various modes and associated transition events. Once an obstacle is detected, it

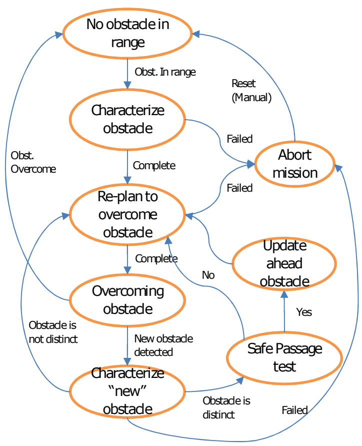

Fig. 4. Obstacle Collision Avoidance Automaton

has to be characterized in order to compute the best strategy to avoid collision and remain as close as possible to the path to be tracked. Besides the range finder of each AUV rotation, the formation pattern can be deformed in order to characterize the obstacle, location and both outer limits of the obstacle. This, together with the closest path to the mission path, enables to optimally replan the path of each AUV, possibly including changes in the formation pattern, how to overcome the obstacle. When an AUV is close to an obstacle, then its RAS-MPC is modified by adding penalization function guaranteeing a safety distance d_s to the obstacle. If, while circumventing an obstacle, a new obstacle is detected, then the pertinent AUVs range finders proceeds with the characterization of the detected obstacle. If it is a new obstacle and the path between both obstacles is optimal, then it is necessary to decide whether the passage is safe, even if the AUVs have to navigate in a "line formation".

Figure 5, helps to understand how the criterion for this decision is defined. The passage is safe if $H_1 + H_2 - R_1 - R_2 - 2d_s > 0$ where d_s is given, R_1, R_2, C_1, C_2, and P_L are estimated with the range finder, $H_1 = \sqrt{R_1^2 + L_1^2}$, $H_2 = \sqrt{R_2^2 + L_2^2}$, $L_2 = |P_L - A|$, $L_1 = |A - P_V| - |\sqrt{(R_1 + d_s)^2 - R_1^2}|$, P_V is the position of the AUV, and the point A is the intersection of the segments $\overline{C_1, C_2}$ and $\overline{P_V, P_L}$.

Simulation results obtained with the proposed control structure are shown in figure 6. The mission consists in gathering data while tracking a path defined by the line segment joining points A and B in a given triangle formation.

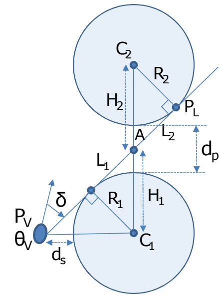

Fig. 5. Safe passage detection

At time t_1, obstacle O_1 is detected in the vehicle's path. The value function is locally altered around O_1's area by increasing significantly it's cost to keep the vehicle out of it. This forces the vehicle to overcome the obstacle by the right. Since at time t_2 obstacles O_1 and O_2 are in range, and O_1 is the closest obstacle, the value function alteration around O_1 is kept while the system decides if there is a safe passage. At time t_3, a safe passage between O_1 and O_2 is detected and the value function is now locally altered around O_1 and O_2 to prevent collisions against each obstacle. The path is now chosen by the left of O_2 as it minimizes the value function. The same happens at time t_4. A safe passage is detected and the path to B is straightforward. Had the distance between O_2 and O_3 been such that the passage was unsafe, a not-so-optimal solution would have been obtained as the traveled distance by the left of O_3 would be longer than that by the right of O_2.

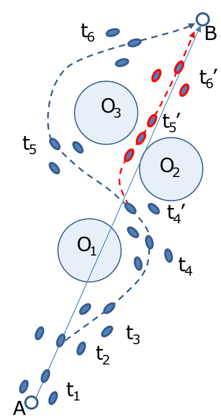

Fig. 6. Obstacle avoidance simulations results

This scheme can only deliver a sub-optimal solution since the decision-making is determined by the range sensor range locally based decisions does not guarantee the overall optimality. However, the sparser the unmapped obstacles are, the better approximation to optimality is achievable.

After deployment, the AUVs are loitering in the triangle formation for the survey around the departure point A. Once the survey starts at time t_1, mode **A** is activated and the triangle formation tracks the given path to the final destination B. At time t_2, the leading vehicle detects obstacle O_1. Then, the formation switches to mode **B** to characterize the obstacle. In order to do this efficiently the AUVs change to a transversal line formation and, once this is done, a path to overcome it is defined by the RAS-MPC by computing minimum of a mapping obtained by adding a penalization to the value function. Since there is plenty of space, the AUVs return to the triangle at some time t_3 while circumventing the obstacle and trying to reach the mission path.

At time t_4, obstacle O_2 is detected on the right. RAS-MPC determines that the best path is between O_1 and O_2. Moreover, the safety criteria above determines that the passage between O_1 and O_2 is safe for the whole formation, and the triangle formation is kept. At time t_4', a third obstacle O_3 is detected and its characterization in mode $\mathbf{B}$ together with the safety criteria determines two different scenarios to pursue the mission:(i) Follow through a thin passage between O_2 and O_3 safe for a single vehicle. Choosing it means changing to longitudinal line formation that results in being closer to the mission path but with loss of quality of the gathered data due to the adopted formation; (ii) Circumvent O_3 by the left what would entail a longer route far away from the mission path, but would allow to preserve the triangle formation with a higher quality of the data gathered.

A simple onboard optimization procedure determines that the first option is the best one. Once the obstacles are overcome at time t_6', the formation resumes to the normal triangle until it reaches the final destination B where the mission mode changes to $\mathbf{D}$ to proceed with data transmission. In this state all the vehicles surface, transmit data, and remain loitering around the final destination B.

VI. Conclusions

In this article, we extended the RAS-MPC scheme for a single AUV presented in [1] to the path tracking control of a formation of AUVs. The key drivers of the approach concern the mitigation of the real-time computational burden and the ability of adapting to unmapped obstacle avoidance. While the former is motivated by limited onboard energy and computational power, in a context of strict real-time constraints, it shows the flexibility of the RAS-MPC scheme with a control architecture to handle unmapped obstacle as well as the various tasks of the mission and the management of the formation pattern in order to conciliate the optimization of onboard resources with feedback control to handle uncertainty and variability. The mathematical details have been omitted due to the lack of space. The obtained simulation results are encouraging and point to the next step: migrate the developments to a multiple AUV based system for field testing.

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