Abstract- This paper addresses the dynamic characterization of the autonomous underwater vehicle MARES. The paper presents the main dynamic properties of this underwater robotic platform as well as the procedures employed to obtain the parameters that define the vehicle model. Furthermore, the paper also presents a detailed characterization of the elementary motions that this vehicle is able to perform.

I. INTRODUCTION

The use of autonomous underwater vehicles (AUVs) in different application fields, such as military, homeland defense, underwater surveys, environment monitoring, and oceanographic studies, is becoming more and more common. Nonetheless, there is still a great research effort related to the development of this technology, with a large number of different vehicles being developed either for generic or more specific applications. Regarding small size AUVs, the traditional torpedo shapes with a main propeller and control surfaces are being replaced by vehicles with higher maneuverability that can be advantageously used in more demanding applications.

This paper addresses the dynamic modeling and motion characterization of the MARES AUV, a small size vehicle developed at Porto University and already demonstrated at sea operations in 2007.

The paper starts with a brief description of the MARES AUV and the basic design options. Next, we present the dynamic model that was used to characterize its behavior. Then, in section IV, we present the derivation of the parameters of the model. Section V addresses the characterization of the vehicle behavior in some maneuvers.

II. MARES

MARES, or Modular Autonomous Robot for Environment Sampling (Fig. 1), is a 1.5m long AUV, designed and built by the Ocean Systems Group. The vehicle can be programmed to follow predefined trajectories, while collecting relevant data with the onboard sensors. MARES can dive up to 100m deep, and unlike similar-sized systems, has vertical thrusters to allow for purely vertical motion in the water column. Forward velocity can be independently defined, from 0 to about 1.5 m/s. Major application areas include pollution monitoring, scientific data collection, sonar mapping, underwater video or mine countermeasures.

MARES configuration can change significantly according to the application scenario, so that it is difficult to define what is a standard configuration. In table 1 we summarize the main characteristics of the AUV version that was demonstrated at sea in November 2007.

<table>
<thead>
<tr>
<th>TABLE I MARES CHARACTERISTICS</th>
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<tbody>
<tr>
<td>Length</td>
</tr>
<tr>
<td>Diameter</td>
</tr>
<tr>
<td>Weight in air</td>
</tr>
<tr>
<td>Depth rating</td>
</tr>
<tr>
<td>Propulsion</td>
</tr>
<tr>
<td>Horizontal velocity</td>
</tr>
<tr>
<td>Energy</td>
</tr>
<tr>
<td>Autonomy/Range</td>
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</table>

The vehicle hull is based on a central watertight polyacetal cylinder, where all electronic boards are installed, with the battery packs located at the bottom to lower the center of mass. To simplify the design, this is the only watertight enclosure and therefore all other equipment has to be waterproof. The other polyacetal sections are designed to carry wet sensors and thrusters and they are fully interchangeable. This allows for very easy sensor swapping and/or repositioning, or even to test different configurations of thrusters.

The paper starts with a brief description of the MARES AUV and the basic design options. Next, we present the dynamic model that was used to characterize its behavior. Then, in section IV, we present the derivation of the parameters of the model. Section V addresses the characterization of the vehicle behavior in some maneuvers.
III. DYNAMIC MODEL

The derivation of the dynamic model for the AUV follows the standard approach presented in [1] where two reference frames are considered: an earth fixed frame assumed to have inertial properties and a body frame that moves together with the vehicle. To relate linear and angular velocities as well as forces and torques defined in the two frames it is necessary to define

\[
\eta_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \eta_2 = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}, \quad v_1 = \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad v_2 = \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \quad \tau_1 = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}, \quad \tau_2 = \begin{bmatrix} K \\ M \\ N \end{bmatrix}
\]  

where \( \eta_1 \) and \( \eta_2 \) are the relative position and orientation of the body fixed frame with respect to the inertial frame; \( v_1 \) and \( v_2 \) are the linear and angular velocities expressed in body fixed coordinates; \( \tau_1 \) and \( \tau_2 \) are the forces and torques acting on the vehicle, also expressed in body fixed coordinates. Defining \( \eta = [\eta_1^T \ \eta_2^T]^T, v = [u^T \ v^T]^T \) and \( J = \text{diag}(J_1, J_2) \)

\[
J_1 = \begin{bmatrix}
c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi c\phi c\theta \\
c\psi s\theta + s\psi s\phi & c\psi c\phi & -c\psi s\phi + s\psi s\theta c\phi \\
s\theta & c\phi & -s\theta \\
\end{bmatrix}
\]

\[
J_2 = \begin{bmatrix}
1 & s\phi t\theta & c\phi t\theta \\
0 & c\phi & -s\phi \\
0 & s\phi / c\theta & c\phi / c\theta \\
\end{bmatrix}
\]

The relationships between linear and angular velocities in the two frames are

\[
\dot{\eta} = J(\eta_2) v
\]

The dynamics of the vehicle are easily expressed in the body fixed frame by

\[
\tau_{ext} = M_{RB} \ddot{\eta} + C_{RB}(\dot{\eta}) \dot{\eta}
\]

where \( \tau_{ext} \) is the vector composed by the external forces and torques acting on the vehicle, expressed in body frame coordinates, \( M_{RB} \) is the inertial matrix and \( C_{RB}(\dot{\eta}) \) is the Coriolis and centripetal matrix (see [1] for details). The external forces and moments can be decomposed as the sum of the added mass, potential damping, drag, restoring, and propulsion, respectively,

\[
\tau_{ext} = \tau_A + \tau_B + \tau_Y + \tau_C + \tau_{prop}
\]

From (3), (4) and (5), we can conclude that \( \tau_{ext} \in \mathbb{R}^6 \). The three first lines regard to the components of forces, while the last three regard to the components of torques, both in the body-fixed coordinate system that we will define in the next section.

IV. COEFFICIENTS DETERMINATION

In the followings sections and subsections, we will consider that the fixed body coordinate system \( x_B y_B z_B \) coincides with the vehicle center of gravity (CG), as stated in fig.2 and fig.3.

Fig. 2: Vertical projection of MARES

The determination of the vehicle model coefficients is based on theoretical analysis of its shape. Here, we only present the derivation of some added mass and drag coefficients. More details are presented in [2].

The antenna and the handles effects on AUV superior part will be neglected.

A. Added mass

The axial term \( X_a \) is determined using the equation (6) from [1 p.41]. We need to approximate the vehicle by an ellipsoidal with a minor axis \( a = 1/2 \) and a major axis \( b = d/2 \).

\[
X_a = -\frac{a_0}{2-a_0} m
\]

where \( m \) is the total vehicle mass, and \( a_0 \) is given by (1)

\[
a_0 = \frac{2(1-e^2)}{e} \ln \left( \frac{1+e}{1-e} \right) - e
\]

and

\[
e = 1 - \frac{b^2}{a^2}
\]

Based on [3] and [4], we assume that the major contribute to the added mass are the vehicle hull, the horizontal propellers and the sonar transducer. The fluid mass displaced by the AUV, per unit of a transversal “slice” of these three contribution are respectively:

\[
m_a(x) = \rho_f \int_0^{2\pi} \int_{r(x)}^{R(x)} r \ dr \ d\phi = \rho_f \pi R(x)^2
\]

\[
m_{ap}(x) = \rho_f \int_0^{2\pi} \int_{r(\theta)(x)}^{R(\theta)(x)} r \ dr \ d\phi = \rho_f \pi R(\theta)(x)^2
\]

\[
m_{at}(x) = 2\rho_f \int_0^{2\pi} \int_{H} \sqrt{y^2 - (x-x_c)^2} \ dx
\]

where \( R(x) \) is the hull radius, \( R(\theta)(x) \) is the propeller radius, \( H \) is the sonar hull height and \( x_c \) is the sonar hull base center.

The motions in \( y_B y_B \text{ or } y_B w \text{ motion}, \) the fluid present in the holes is transported with the vehicle, but in \( z_B \) or \( pitch \text{ motion}, \) the fluid does not suffer any acceleration. This last case implies
that the fluid passes through the holes does not exercise any force or moment in the vehicle.

We approach the holes by cylinders with a diameter \( d_f \) and constant height \( h_f \). We can now define the added mass per unit length, due to holes:

\[
m_{af}(x_B) = \begin{cases} 
\rho_f h_f \sqrt{\frac{d_f^2 \pi^2}{4}} - (x_B - x_f)^2, & x_f + \frac{d_f}{2} \leq x_B \\
\rho_f h_f \sqrt{\frac{d_f^2 \pi^2}{4}} - (x_B - x_f)^2, & x_f + \frac{d_f}{2} \leq x_B \\
0. & \text{other}
\end{cases}
\] (12)

In expression (13), we present the expressions to determine some added mass terms. The other terms can be determined by analogy.

\[
Z_w = - \int_{x_c}^{x_a} m_u(x_B) \, dx_B + 2 \int_{x_c}^{x_a} m_{af}(x_B) \, dx_B + \int_{x_c}^{x_a} m_{af}(x_B) \, dx_B
\]

\[
N_e = - \int_{x_c}^{x_a} x_B^2 m_u(x_B) \, dx_B + 2 \int_{x_c}^{x_a} x_B^2 m_{af}(x_B) \, dx_B + \int_{x_c}^{x_a} x_B^2 m_{af}(x_B) \, dx_B
\] (13)

\[
Y_e = \int_{x_c}^{x_a} x_B m_u(x_B) \, dx_B + 2 \int_{x_c}^{x_a} x_B m_{af}(x_B) \, dx_B + \int_{x_c}^{x_a} x_B m_{af}(x_B) \, dx_B
\]

for the limits of integration, see fig.2 and fig.3.

The most important added mass coefficients are shown in table II.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_d )</td>
<td>(-1.74 \times 10^9)</td>
<td>( N_e )</td>
<td>(-6.32 \times 10^9)</td>
</tr>
<tr>
<td>( Y_s )</td>
<td>(-4.28 \times 10^9)</td>
<td>( Y_e )</td>
<td>(2.89 \times 10^{-2})</td>
</tr>
<tr>
<td>( Z_s )</td>
<td>(-3.88 \times 10^9)</td>
<td>( Z_e )</td>
<td>(-2.10 \times 10^{-1})</td>
</tr>
<tr>
<td>( K_s )</td>
<td>(-2.05 \times 10^{-1})</td>
<td>( M_{af} )</td>
<td>(-2.10 \times 10^{-1})</td>
</tr>
<tr>
<td>( M_{af} )</td>
<td>(-5.59 \times 10^7)</td>
<td>( N_{af} )</td>
<td>(2.89 \times 10^{-2})</td>
</tr>
</tbody>
</table>

**B. Drag**

In this work, we consider only quadratic drag terms. We assume that the linear and angular speeds are sufficiently high to neglect linear terms. The terms greater than second order will be neglected too, assuming that their effects are small comparing to quadratic terms.

From [1] and [5] we have the axial force in the \( x_B \) direction expressed as:

\[
f_{dx}(u_e) = \frac{1}{2} \rho_f C_{D_e} A_{xy} u_e |u_e|
\] (14)

where \( \rho_f = 1.026 \times 10^3 \) kg/m³ is the fluid density, \( C_{D_e} \) is the drag coefficient of an ellipsoidal body depending on the hull form and \( A_{xy} \) is the vehicle projected area in the plane formed by \( y_B \) and \( z_B \) axes.

The determination of drag coefficients can result on wrong estimations [3] because the theory that is behind often based on empirical expressions or experimental results. The best way to estimate them is by experimental tests. However, these estimates are useful as first approach to the characterization of the vehicle motion.

The drag coefficient \( C_{D_e} \) depends on Reynolds Number \( Re \), given by:

\[
Re = \frac{U l}{\nu}
\] (15)

where \( U \) is the axial speed in the \( x_B \)-axis, \( l \) the vehicle length and \( \nu \) the fluid viscosity. We assume \( U = 1 \text{ m/s} \) which is a typical value for the vehicle velocity, \( l = 1.5 \text{ m} \) and \( \nu = 1.005 \times 10^{-6} \text{ m}^2/\text{s} \) at \( T = 20^\circ \text{C} \). These values give \( Re = 1.5 \times 10^6 \). This implies that the vehicle motion is between the laminar and the turbulent flow [5].

Using laminar theory, we have the following drag coefficient:

\[
C_{D_{e,ln}} = 0.44 \frac{d}{l} + 4C_f \frac{l}{d} + 4C_f \left( \frac{1}{T} \right)^{1/2}
\] (16)

\[
C_f = \frac{0.075}{(\log_{10} Re - 2)^2}
\] (17)

Resulting on \( C_{D_{e,ln}} = 0.2 \).

The turbulence theory, gives \( C_{D_{e,tu}} = 0.08 \). The experimental results obtained by (3) to determine the drag coefficient point to a value close to \( C_{D_{e,tu}} \). Taking into account that the dimensions and the speed of operation are similar, we consider:

\[
C_{D_e} = C_{D_{e,ln}}
\] (18)

We are now in condition to determine the axial term in \( x_B \) direction:

\[
f_{dx}(u_e) = \frac{1}{2} \rho_f C_{D_e} A_{xy} u_e |u_e|
\]

(19)

for the limits of integration, see fig.2 and fig.3.

The motion in the \( y_B \) and \( z_B \) leads a flow around the vehicle that we approximate by a cylinder. According to [5] and [6], for a cylinder with a ratio \( l/d \approx 7.5 \), the drag coefficient is:

\[
C_{D_c} = 0.8
\] (20)

The sonar hull under the vehicle is also cylindrical with a ratio \( l_g/d_g \approx 1 \). From [5] and [6], we obtain a drag coefficient equal to:

\[
C_{D_{og}} = 0.68
\] (21)
The drag coefficient terms are determined as shown below:

\[ y_{\text{hl}} = -\frac{1}{2} \rho \int_{x_l}^{x_h} R_x(x) \, dx + 2 \rho \int_{x_l}^{x_h} R_y(x) \, dx \]

\[ m_{\text{hl}} = -\frac{1}{2} \rho \int_{x_l}^{x_h} \left( x^2 \, R_y(x) \right) \, dx + 2 \rho \int_{x_l}^{x_h} x \, R_y(x) \, dx \]

\[ z_{\text{hl}} = \frac{1}{2} \rho \int_{x_l}^{x_h} x \, x \, R_y(x) \, dx \]

\[ \text{for the limits of integration, see fig.2 and fig.3} \]

The remaining drag terms can be determined by analogy. The most important drag coefficients are shown in Table III.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_{\text{hl}} )</td>
<td>(-4.05 \times 10^7)</td>
</tr>
<tr>
<td>( Y_{\text{hl}} )</td>
<td>(-1.15 \times 10^4)</td>
</tr>
<tr>
<td>( Z_{\text{hl}} )</td>
<td>(-2.29 \times 10^{-2})</td>
</tr>
<tr>
<td>( M_{\text{hl}} )</td>
<td>(-4.30 \times 10^9)</td>
</tr>
</tbody>
</table>

### V. LIMITS ANALYSIS

The MARES propellers have a finite actuation force. This fact implies that they reach saturation in some maneuvers. Obviously, these limits depend on vehicle angles, speed and acceleration. In this section, we will study the limits of actuation due to propellers limited force. We will consider particular cases where linear and angular accelerations are null, on maneuver steady-state. The following results will be important whereas we will know the limit relation between the maneuver and speed.

#### A. Motion at constant depth and pitch

We consider that the linear and angular acceleration are null, i.e. \( \dot{u}_x = 0 \). The speeds \( u = u_{\text{ref}} \) and \( w = w_{\text{ref}} \) are constant. The remaining linear and angular speeds are null.

To maintain a constant depth, it is required that the speed component of \( u \) in \( z \)-axis is compensated by the \( w \) component in the same axis. So, we can conclude:

\[ w = u \tan \theta \]  

(23)

where \( \theta \) is the pitch angle as we can see in fig.4.

Fig. 4: AUV moving with constant depth and pitch angle

In this analysis, we consider that the body-fixed coordinate system coincides with the center of gravity.

1. **Common mode**

Selecting the third line from (5), we easily get:

\[ f_{p3} + f_{p4} = -2 \rho \int \nu \tan \theta + (B-W) \cos \theta \]  

(24)

In steady-state the acceleration in \( z_B \)-axis depends on the drag, propellers and restoring forces.

2. **Differential mode**

As in common mode determination, we select the fifth line from and we obtain the following expression:

\[ x_p f_{p3} + x_p f_{p4} = -M_{\text{efp}} \nu \tan \theta + (Z_u - X_u) u^2 \tan \theta - z_B B \sin \theta \]  

(25)

Assuming that \( -x_p f_{p3} \approx x_p \), we easily get:

\[ f_{p4} - f_{p3} = \frac{-M_{\text{efp}} \nu \tan \theta + (Z_u - X_u) u^2 \tan \theta - z_B B \sin \theta}{x_p} \]  

(26)

3. **Propeller forces**

Subtracting and adding (26) to (24), we can obtain, respectively:

\[ f_{p3} = \frac{1}{2} \left[ -Z_{\text{efp}} \nu \tan \theta \nu \tan \theta + (B-W) \cos \theta \right] \]  

(27)

\[ f_{p4} = \frac{1}{2} \left[ -Z_{\text{efp}} \nu \tan \theta \nu \tan \theta + (B-W) \cos \theta \right] \]  

(28)

Assuming that the propellers have a maximum actuation force \( f_{p\text{max}} \), it is possible to determine the maximum angle as a function of \( u \). To do this, we solve numerically (27) with \( f_{p3} = f_{p\text{max}} \), taking into account that

Fig. 5: Force versus pitch angle for a constant speed \( u = 1m/s \)
p3 is the propeller that reach $|f_{p_{\text{max}}}|$ first. The next figure shows the maximum pitch as function of $u$. In this case, we consider only $\theta > 0$.

![Image](image.png)

**Fig. 6:** Maximum pitch angle versus speed

### B. Circumference following at constant depth and zero pitch

In this case, we assume that there is no vertical motion and pitch angle is zero, i.e. $z = c_z$ and $\theta = 0$. The vehicle will have constant surge, sway and yaw speed components in steady-state. Remaining speeds component will be zero. Given that, we can conclude that the linear and angular accelerations in body fixed coordinates will be null, in steady-state. So, the yaw speed is given by:

$$ r = \pm \frac{\sqrt{u^2 + v^2}}{R} $$

(29)

where $R$ is the circumference radius described by the vehicle motion. We consider $r > 0$. The following results would be symmetric if $r < 0$.

Selecting the second line of hydrodynamic equation, we can write:

$$ |v| = -\frac{(X_u u - mu + Y_r r)r}{V_{brv}} $$

(30)

Substituting $r$ into (30) by (29), we get:

$$ v^2 = \begin{cases} 
\left( X_u u - mu + Y_r r, \frac{\sqrt{u^2 + v^2}}{R} \right) \frac{\sqrt{u^2 + v^2}}{R}, & v < 0 \\
\left( X_u u - mu + Y_r r, \frac{\sqrt{u^2 + v^2}}{R} \right) \frac{\sqrt{u^2 + v^2}}{R}, & v \geq 0
\end{cases} $$

(31)

We will consider only $R$ in the range of 1 to 50 meters. The expression (31) has no solution for $v \geq 0$ for $R \in [1; 50]$. Thus, in the range considered:

$$ \text{sign}(v) = -\text{sign}(r) $$

(32)

Defining $\delta$ as the difference between yaw and the circumference tangent angles, as shown in fig. 7, we get:

$$ \delta = \text{atan}\left( \frac{v}{u} \right) $$

(33)

![Image](image.png)

**Fig. 7:** AUV describing a circumference ($v < 0, r > 0$)

**Fig. 8:** Difference between yaw and the circumference tangent angles in function of radius

1. **Common mode**

Selecting the first line of hydrodynamic equation (5), we easily obtain:

$$ f_{p1} + f_{p2} = (Y_r v + Y_r r)r - mrv - X_{brv} |v|u $$

(34)

2. **Differential mode**

Manipulating the sixth line from (5), the propellers differential mode results:

$$ f_{p2} = f_{p1} = -\frac{\left( Y_r v + Y_r r \right) r - mrv - X_{brv} |v| |v|u - N_{p_{\text{brv}}} \right)}{Y_p} $$

(35)

where $y_p$ is the distance in $y_p$-axis between the propellers center and the center of gravity.

3. **Propeller forces**

By subtracting and adding (35) to (34), we easily obtain, respectively:

$$ f_{p1} = \frac{1}{2} \left[ \left( Y_r v + Y_r r \right) r - mrv - X_{brv} |v|u \right] $$

$$ f_{p2} = \frac{1}{2} \left[ \left( Y_r v + Y_r r \right) r - mrv - X_{brv} |v|u \right] $$

(36)

In the next figure, we show the force applied by each propeller in function of circumference radius $R$. 

![Image](image.png)

**Fig. 8:** AUV describing a circumference ($v < 0, r > 0$)
It is empirical that, for surge and yaw speeds \( r, r > 0 \), the propeller \( p1 \) exercise more force than \( p2 \). Assuming that the maximum propeller force is \( f_{p_{\text{max}}} \), as in the previous section, we can compute numerically (36) in order to \( f \), for \( f_{p1} = f_{p_{\text{max}}} \). We show this result in the next figure as the maximum surge speed as function of circumference radius \( R \).

**VI. CONCLUSIONS**

We begin with the AUV MARES presentation, a vehicle designed by the Ocean Systems Group. Investigation in control, oceanographic studies and environmental monitoring areas is the principal aim of this underwater vehicle. We continue with the presentation of hydrodynamic of a body inserted in a fluid, discriminating the different forces that actuate on it. We follow the commonly used hydrodynamic theory found in [1]. In the coefficients determination section, we compute the added mass and drag coefficients. Though we have relative certainty on the first ones, the others can diverge from experimental results knowing that their theory is based in empirical and experimental formulas for some object forms. The analysis of the motion limits, allows us to approach the behavior of Mares in certain maneuvers. These results are important to know the force requirements and for controllers design.

**REFERENCES**